

# Pattern Recognition as a Consistent Labeling Problem

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**Abstract** *This paper discusses a new method of recognizing patterns employing consistent labeling. A consistent labeling problem is a generalized expression of constraint satisfaction problems. When a pattern is recognized by pattern matching, the matching between a reference pattern and an acquired pattern resolves itself into finding correspondence between the pixels on the former and those on the latter. This can be expressed as a consistent labeling problem. Pattern association, a variation of pattern recognition, is also described employing consistent labeling. The proposed technique is supported by experimental results, yet further studies need to be done before its practical use.*

## 1. INTRODUCTION

Pattern recognition has long been the subject of research in various areas such as factory automation, weather forecasting, medical diagnosis, *etc.*, as well as theoretical researches[1], yet not many techniques have been put to practical use. The main reason why this happens exists in that there are often many variations with each single pattern, visual or auditory. One may easily find out this, if only he calls in mind the way people write characters or pronounce words. To solve this problem, flexible pattern matching techniques have been investigated. Widrow[2] proposes a rubber mask technique to classify human chromosomes in an automatic manner, while Sakoe and Chiba[3] employ dynamic programming for speech recognition. A subset method is studied by Ullmann[4] which examines correspondence between an acquired character pattern and stored reference patterns by the use of local constraint of the patterns concerned.

An early research which employs local constraint and its propagation in constraint satisfaction problems is done by Cherry and Vaswani[5], who show a quick method of obtaining the values of logical variables in a given set of logical constraints. Waltz[6] uses this concept in scene labeling and realizes fast correspondence between geometrical labels and feature points on an image. His technique has

been extended to a relaxation operation by Rosenfeld *et al.*[7], which is an often employed well-established technique in scene analysis. Many types of constraint satisfaction problems have been given formalization as a consistent labeling problem by Haralick and Shapiro[8], and effective ways of solving the consistent labeling problem have been investigated.

Recently, three-dimensional object recognition has drawn much attention particularly in the field of robot vision. The difficulty in three-dimensional object recognition is that it has many – almost infinite appearances according to an observer's position. Here one should note that difference in the appearances includes topological difference in the three-dimensional case. Ishikawa *et al.*[9] restrict a pattern concerned to a dot pattern in the three-dimensional space, and describe the problem of corresponding a dot pattern which receives perspective projection to one of the reference patterns in terms of consistent labeling.

In the present paper, a pattern recognition problem is shown to be described as a consistent labeling problem, and it is then applied to handwritten character recognition. A pattern association problem, which is understood to be a variation of pattern recognition, is also described in terms of consistent labeling, and an algorithm for the association is given with some experimental results.

## 2. A CONSISTENT LABELING PROBLEM

A consistent labeling problem[8] is abstraction of many types of constraint satisfaction problems where the variables which describe the problem are assigned consistent values under several constraints the problem provides. In consistent labeling, those variables are referred to as units and their values as labels.

Let us denote a set of units  $u_i$  ( $i=1,2,\dots$ ) by  $U$  and a set of labels  $l_j$  ( $j=1,2,\dots$ ) by  $L$ . Then the unit constraint relation is defined by  $T \subseteq U^n$  whose element is an  $n$ -tuple of units ( $u_{i1}, u_{i2}, \dots, u_{in}$ ). (Actually, this size doesn't have to be all the same among those elements[10], *i.e.*, the set  $T$  can contain  $k$ -tuples where  $k=1, 2, \dots, n$ . In this particular research, however, the value  $n$  is given a certain fixed value.) The unit-label constraint relation is defined by  $R \subseteq (U \times L)^n$  which contains  $n$ -tuples of a unit-label pair ( $u_{i1}, l_{j1}, u_{i2}, l_{j2}, \dots, u_{in}, l_{jn}$ ). A quadruplet  $(U, L, T, R)$  is called a compatibility model and solved employing depth first search[11]. The solution, called consistent labeling, is a set of  $(u, l)$  pairs consistent to those unit-label constraints given by  $R$ .

The depth first search employed in solving a consistent labeling problem is often time consuming. To avoid this, Haralick *et al.*[12] and Haralick and Shapiro[8] use look ahead operators which eliminate those branches indifferent to the solution on a searched tree. Ullmann *et al.*[10] give an idea of hardware architecture for solving a consistent labeling problem in a parallel way. Since the main purpose of the present paper is to show that a pattern recognition problem can be described as a consistent labeling problem, we leave this point of speeding up untouched and employ a sequential algorithm to find solutions to the problem in the performed experiments.

## 3. A PATTERN RECOGNITION PROBLEM

### 3.1 Interpretation by Consistent Labeling

Pattern recognition is one of the most fascinating fields of study for many scientists as well as engineers, since it provides us with the idea

for realizing an important aspect of the human information processing activities in the brain in an artificial manner. Pattern recognition researches include visual pattern recognition, speech pattern recognition, and syntactic pattern recognition. The present study is devoted to visual pattern recognition. Among various techniques of recognizing patterns, this particular study involves itself in so-called pattern matching.

One of the fundamental techniques in recognizing patterns is template matching in which a standard template of a pattern concerned is superposed on an acquired unknown pattern and their agreement is checked. Naturally, this easily fails in conquering transformation, rotation, distortion, noise, *etc.* of the acquired pattern. To overcome this, we separate an image plane into individual pixels and concentrate on matching pixels between the standard pattern template and the acquired pattern.

Let us take two digital image planes: one contains a reference (or a standard) pattern and referred to as the plane S, while the other provides an acquired (or an input) unknown pattern and referred to as the plane I. Let us denote those pixels on S representing a specific pattern by  $u_i$  ( $i=1,2,\dots$ ) and their set by  $U$ , and the pixels on I by  $l_j$  ( $j=1,2,\dots$ ) and their set by  $L$ . This notation implies that the former represent units and the latter labels. If a pattern on S and a pattern on I are to agree, there must be correspondence between those  $u_i$ 's and  $l_j$ 's, which implies that, for every unit  $u_i$  on S, a certain label  $l_j$  on I is assigned.

Now, given a specific pattern P on S, units  $u_{i1}, u_{i2}, \dots, u_{in}$  on pattern P constrain each other with their positions on S; indeed, in the case of  $n=2$ , if pixel  $u_{i1}$  on plane S corresponds to pixel  $l_{j1}$  on plane I, then  $u_{i2}$  should correspond to  $l_{j2}$  with respect to pattern P. Thus we have the set  $T$  representing the unit constraint relation:

$$T = \{(u_{i1}, u_{i2}, \dots, u_{in})\} \subseteq U^n.$$

The elements in  $T$  and their total number, denoted by  $n(T)$ , are given *a priori*. Obviously, the performance of the pattern matcher proposed in the present paper depends largely upon these factors. For example, if  $n(T)$  is

large, an unknown pattern is specified precisely, but the search process consumes too much time. On the contrary, the small number of  $n(T)$  speeds up the tree search often at the cost of misspecification of an unknown pattern.

Since there are usually lots of variations with a single pattern  $P$ , an  $n$ -tuple of unit constraint relation  $(u_{i1}, u_{i2}, \dots, u_{in})$  with pattern  $P$  may have various combinations of correspondence with the pixels on  $I$ . This in turn determines the set  $R$  giving the unit-label constraint relation with respect to pattern  $P$  and its variations:

$$R = \{ (u_{i1}, l_{i1p}, u_{i2}, l_{i2p}, \dots, u_{in}, l_{inp}), (p=1, 2, \dots), \\ (u_{j1}, l_{j1q}, u_{j2}, l_{j2q}, \dots, u_{jn}, l_{jnq}), (q=1, 2, \dots), \\ \dots, \\ (u_{k1}, l_{k1r}, u_{k2}, l_{k2r}, \dots, u_{kn}, l_{knr}), (r=1, 2, \dots) \} \\ \subseteq (U \times L)^n .$$

Practically, this set is produced employing a number of training patterns of pattern  $P$ .

The judgment if an unknown pattern on plane  $I$  agrees with pattern  $P$  on plane  $S$  can be done by the examination employing  $R$  and those pixels on the unknown pattern if there exists compatible labeling between the units on plane  $S$  and the pixels on the unknown pattern. Thus the pattern matching problem between a specific pattern  $P$  on plane  $S$  and an acquired unknown pattern on plane  $I$  dissolves itself into the consistent labeling problem.

### 3.2 Experimental Results

The performance of the proposed matching technique was examined by handwritten characters recognition. Since the main purpose of the present examination is to show how the proposed technique proceeds in an explicit form, a single *katakana* character is considered for recognition in this particular experiment. A sampled *katakana* on a  $32 \times 32$  image plane is given the following processing; binarization, thinning, extraction of feature points, normalization, and description. A feature point on those strokes of a thinned character includes an edge point, an intersection, and the point having a large curvature. Figure 1 shows the feature points of the *katakana* chosen here for recognition. They

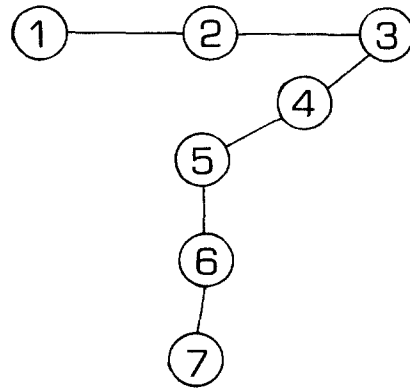


Fig. 1. Feature points on a *katakana* character employed for the experiment.

are given *a priori* and used to define the unit constraint set. In this particular experiment, an  $n$ -tuple is fixed to  $n=2$ , and the set  $T$  is given as follows:

$$T = \{(1,2), (2,3), (3,4), (4,5), (5,6), (6,7)\}.$$

To determine the unit-label constraint set  $R$ , forty handwritten samples of the character were collected. Let us denote the set of these samples by  $S_0$ . Now all the patterns in  $S_0$  are rotated by  $\theta$  around their centroids and the set of these new patterns are denoted by  $S_\theta$ . This we consider in order to take rotational distortion into account. The whole set  $S$  employed in determining the set  $R$  in this particular experiment is then described as follows:

$$S = \bigcup_{i=-3}^3 S_{\theta_i}.$$

Here  $-\pi/10 < \theta_{-3} < \theta_{-2} < \theta_{-1} < \theta_0 \equiv 0 < \theta_1 < \theta_2 < \theta_3 < \pi/10$  holds. Since  $n(S_{\theta_i})=40$ , it follows that  $n(S)=280$ . All of these patterns produce the set  $R$  where  $n(R)=938$ . Employing this unit-label constraint set, nineteen out of twenty-two new samples of the character were recognized successfully. An example of the results is shown in Fig. 2, where all the feature points of the character on the standard image plane have corresponding pixels on the unknown sampled pattern on the input plane.

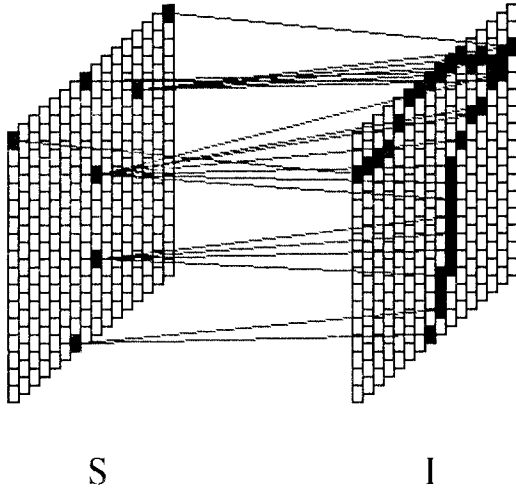


Fig. 2. A sampled character successfully recognized by the proposed matching technique.

## 4. A PATTERN ASSOCIATION PROBLEM

### 4.1 Interpretation by Consistent Labeling

Pattern association is another aspect of human visual information processing activities and is of great importance in recognizing patterns. In the real world, we often encounter a situation where associating patterns is needed. Actually, there are lots of objects in confronting scenes which are occluded by other objects, and, in the evening, the shape of every object we perceive is obscured by darkness.

Pattern association is understood to be a variation of pattern recognition and is also able to be described in terms of consistent labeling. Let us consider the problem of associating a single pattern among patterns  $f_k(i, j)$  ( $k=1, 2, \dots, K$ ) that are stored in memory. An image plane I is assumed to have  $M \times N$  pixels. The pixel at position  $(i, j)$  ( $i=1, 2, \dots, M; j=1, 2, \dots, N$ ) on plane I is denoted by  $p_{ij}$ , and, given a specific pattern on plane I, the gray level at  $(i, j)$  is denoted by  $f_{ij}$ . By the definition,

$$l = (i-1)N + j,$$

and by substituting a new variable  $i$  for  $l$ , we rewrite  $p_{ij}$  and  $f_{ij}$  as  $p_i$  and  $f_i$ , respectively ( $i=1, 2, \dots, MN$ ). These  $p_i$ 's are regarded as units

and  $f_i$ 's as labels. Their sets are denoted by  $U$  and  $L$ , respectively. The unit constraint set  $T$  is defined by choosing  $n$  pixels on plane I, independent of memorized patterns:

$$T = \{p_{i1}, p_{i2}, \dots, p_{in}\}.$$

An alternative choice is to select  $n$  pixels (an  $n$ -tuple) from each memorized pattern so that the chosen  $n$ -tuples are mutually disjoint, but we leave this untouched in this particular paper. A pattern  $f_k(i, j)$  determines the unit-label constraint set  $R_k$ :

$$R_k = \{(p_{i1}, f_{i1}, p_{i2}, f_{i2}, \dots, p_{in}, f_{in}) \mid (p_{i1}, p_{i2}, \dots, p_{in}) \in T\}.$$

Then the state that  $K$  patterns are memorized for association is given by the set  $R$ :

$$R = \bigcup_{k=1}^K R_k.$$

Now we have a compatibility model  $(U, L, T, R)$  with respect to the memorized  $K$  patterns. An acquired unknown pattern  $X$  part of which is missing and therefore from which association occurs is supposed to be given in the same form as unit-label constraint set, *i.e.*,

$$X = \{(p_{j1}, f_{j1}, p_{j2}, f_{j2}, \dots, p_{jn}, f_{jn}) \mid (p_{j1}, p_{j2}, \dots, p_{jn}) \in T, \text{not}((f_{j1} = \text{nil}) \cap (f_{j2} = \text{nil}) \cap \dots \cap (f_{jn} = \text{nil}))\}.$$

The association process begins by producing a new set  $R^*$  from  $R$  and  $X$ :

$$R^* = \{e \mid e = (p_{i1}, f_{i1}, p_{i2}, f_{i2}, \dots, p_{in}, f_{in}) \in R, e \sim^{\forall} e' \in X\}.$$

Here ' $\sim$ ' means *not contradictory*, and *nil* is understood as *don't care* when checking the consistency,  $e \sim^{\forall} e' \in X$ . If unknown pattern  $X$  is to agree with one of those memorized patterns  $f_k(i, j)$  ( $k=1, 2, \dots, K$ ), the very pattern is recalled by solving the consistent labeling problem given by the compatibility model  $(U, L, T, R^*)$ .

### 4.2 Experimental Results

According to the idea stated in the former section, pattern association was performed employing synthetic numeral data. The unit constraint set  $T$  is defined in this experiment as the set of all the pairs of the pixels on a  $5 \times 5$  digital image plane on which those numeral patterns

are presented. The employed numerals are 2, 3 and 8 as shown in Fig. 3, and their respective unit-label constraint sets are denoted by  $R_2$ ,  $R_3$  and  $R_8$ . Then the over-all unit-label constraint set  $R$  for association is given by

$$R = R_2 \cup R_3 \cup R_8.$$

Part of each numeral pattern is presented on the image plane as an unknown pattern  $X$  which reduces the set  $R$  to  $R^*$ . The consistent labeling is then searched on the set  $R^*$ . Figure 4 shows some results of the association.

## 5. DISCUSSION

A consistent labeling problem is an abstracted concept of constraint satisfaction problems[8] which cover a Latin square puzzle, a school time-tabling problem, a scene labeling problem, *etc.* We showed in the present paper that a pattern recognition problem and a pattern association problem are also described as consistent labeling problems. This idea was examined by the experiments employing a handwritten katakana and synthetic numerals, and satisfactory results were obtained. Some problems still remain to be solved, however.

The performance of the proposed pattern matching technique employing consistent labeling largely depends on some factors intrinsic to the technique. The followings are questioned:

- 1) How should we decide the value  $n$  with an  $n$ -tuple of units in the unit constraint set  $T$ ?
- 2) Which pixels on the base plane should we choose to make the  $n$ -tuple?
- 3) How many  $n$ -tuples of units are necessary for realizing reasonable performance with the proposed pattern matching technique?

All of these questions should be solved taking economy into account. In practice, the increase in the number of the  $n$ -tuples in the set  $T$  which will surely assure high performance of the proposed matching technique inevitably results in high cost, because of a large number of the memory chips that must keep those  $n$ -tuples

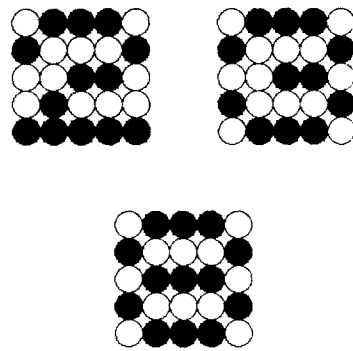


Fig. 3. Three synthetic numerals employed for the experiment.

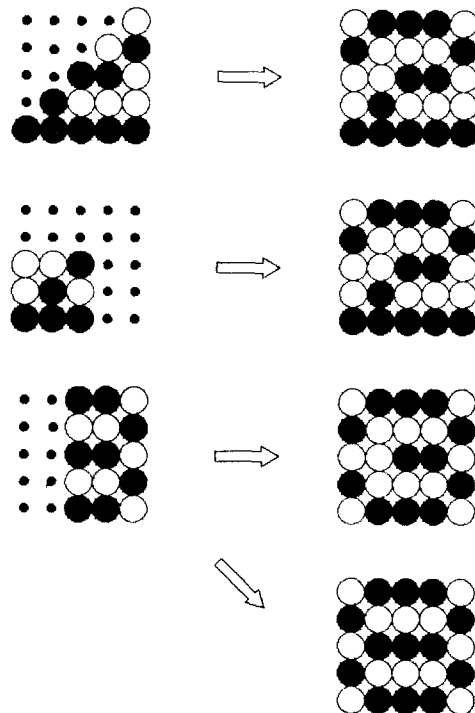


Fig. 4. Examples of the association of the numerals from their incomplete data.

in the sets  $T$  and  $R$  and enormous time for searching consistent labeling on those  $n$ -tuples in  $R$ . Although, at the moment, the matching procedure is realized by a sequential program on the computer, special hardware architecture (e.g., [10]) should be taken into consideration in order to put the proposed matching technique to practical use.

The point of solving a pattern recognition problem in a consistent labeling manner is that the method may overcome various kinds of distortion much better than those techniques already proposed. It has flexibility in the way that it regards a distorted pattern as the collection of a large number of its distorted portions. The human pattern recognition process might employ a similar mechanism with respect to familiar patterns.

## 6. CONCLUSION

Pattern recognition and association problems were described in terms of consistent labeling. The experiments for examining this idea were performed employing handwritten characters and synthetic figures and satisfactory results were obtained. Some problems intrinsic to the performance of the proposed pattern matching technique were discussed. Further studies still need to be done before the proposed technique is put to practical use.

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## REFERENCES

- [1] Tou, J.T., Gonzalez, R.C. : *Pattern Recognition Principles*, Addison-Wesley, Mass.(1974).
- [2] Widrow, B. : "The rubber-mask technique — I. Pattern measurement and analysis", *Pattern Recogn.*, **5**, 3, 175-197(1973).
- [3] Sakoe, H., Chiba, S. : "Dynamic programming algorithm optimization for spoken word recognition", *IEEE Trans. Acoust., Speech & Signal Process.*, **ASSP-26**, 1, 43-49(1978).
- [4] Ullmann, J.R. : "Subset methods for recognizing distorted patterns", *IEEE Trans. Syst. Man, Cybern.*, **SMC-7**, 3, 180-191(1977).
- [5] Cherry, C., Vaswani, P.K.T. : "A new type of computer for problems in propositional logic, with greatly reduced scanning procedures", *Inform. Control*, **4**, 155-168(1961).
- [6] Waltz, D.L. : "Understanding line drawings of scenes with shadows", *The Psychology of Computer Vision* (P. H. Winston, Ed.), McGraw-Hill, New York, 19-91(1975).
- [7] Rosenfeld, A., Hummel, R. A., Zucker, S. W. : "Scene labeling by relaxation operations", *IEEE Trans. Syst. Man, Cybern.*, **SMC-6**, 6, 420-433 (1976)
- [8] Haralick, R. M., Shapiro, L. G. : "The consistent labeling problem: Part 1", *IEEE Trans. Pattern Anal. Machine Intell.*, **PAMI-1**, 2, 173-184(1979).
- [9] Ishikawa, S., Tamura, H., Kato, K. : "On matching distorted patterns by consistent labelling", *Inst. Electron. Inform. Commun. Eng. Tech. Report*, **PRU88-32**, 15-22(1988).
- [10] Ullmann, J. R., Haralick, R. M., Shapiro, L. G. : "Computer architecture for solving consistent labelling problems", *Comput. J.*, **28**, 2, 105-111 (1985).
- [11] Nilsson, N. J. : *Principles of Artificial Intelligence*, Springer-Verlag, Berlin(1980).
- [12] Haralick, R. M., Davis, L., Rosenfeld, A., Milgram, D. : "Reduction operations for constraint satisfaction", *Inform. Sci.*, **14**, 199-219 (1978).