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ANALYSIS OF SYMMETRICAL THREE-PHASE INDUCTION MOTOR FED BY PHASE ANGLE CONTROLLED SOURCES

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ABSTRACT

Amethod of analysis of the steady-state performance of induction motor with supply voltage controlled by cyclically-triggered inline thyristors is presented.Phase-variable model and asymmetrical components are not used in this analysis.Instead,Fast Fourier Transform technique and the method of multiple reference frames are employed to obtain the constant-speed performance of I.M. easily.

1.INTRODUCTION

Large scale use of thyristor converters for speed control of induction motors has necessitated introduction of new consideration in the analysis and design of induction motors.Design of the control systems associated with these converters as well as machine and system stability make up only a part of the spectrum of new and interesting areas for investigation.

So far , various methods have been used to analyse the operating characteristics of thyristor-controlled induction motor.For example , differential equation methods have been used most widely wereby both transient and steady-state performance are determined by solving differential equations for each circuit mode taking into account the initial and terminal conditions.

Digital simulation methods can also be regarded as a version of the differential equation method.Although differential equation methods are suited for analysing transient phenomena , they are slightly tedious when studying only steady state phenomena.State variable technique also have been used for analysing induction motors.However , this technique becomes ineffective in some cases due to the difficulty in driving the transition matrix for each mode of operation.

In this paper , it is shown that by a combination of the method of Multiple Reference Frames[1](MRF) and the Fast Fourier Transform (FFT) technique , the steady-state performance of a three-phase induction motor with speed control by means of line-inserted series thyristors can be readily determined. Moreover , since the method of multiple reference frames does not employ phasor or complex impedances , the stator current is calculated algebraically. Therefore , this simplified method offers convenience of calculation which cannot be duplicated by any other method of analysis.

2.REPRESENTATION OF THE INDUCTION MACHINE

When a nonsinusoidal voltage is used to supply an induction machine , the stator phase voltages V_{as} , V_{bs} and V_{cs} may be related to ds-qs voltages in a reference frame fixed in the stator as in Fig.(1) [2] as:

$$V_{qs}^{s} = V_{as} \quad (1)$$

$$V_{ds}^{s} = (1/\sqrt{3})(-V_{bs}+V_{cs}) \quad (2)$$

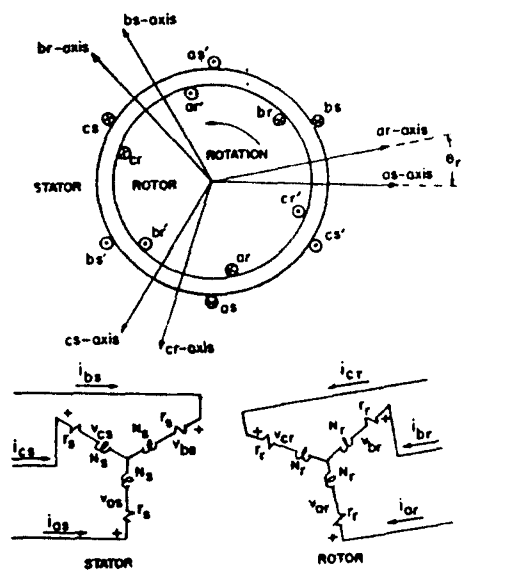

Fig. 1. A 2-pole 3-phase symmetrical machine.

If the phase voltages are periodic in nature , each may be expressed in a Fourier series expansion.

Hence

$$V_{qs}^{s} = \sum_{k=1}^{\infty} (V_{kq\alpha} \cos kWet + V_{kq\gamma} \sin kWet) \quad (3)$$

$$V_{ds}^{s} = \sum_{k=1}^{\infty} (V_{kd\alpha} \cos kWet + V_{kd\gamma} \sin kWet) \quad (4)$$

In these equations We is the electrical angular velocity generally selected to correspond to the fundamental frequency component. The α and γ subscripts denote , respectively , the coefficients of the cosine and sine terms.

The applied voltage in the synchronously rotating reference frame may be expressed as

$$V_{qs}^{e} = V_{qs}^{s} \cos Wet - V_{ds}^{s} \sin Wet \quad (5)$$

$$V_{ds}^{e} = V_{qs}^{s} \sin Wet + V_{ds}^{s} \cos Wet \quad (6)$$

The superscript e denotes variable in synchronously rotating reference frame. By substituting (3) and (4) into (5) and (6) , lengthy expressions for V_{qs}^{e} and V_{ds}^{e} are obtained which are not given here , these expressions contain a constant term and sinusoidally varying terms which form a series of balanced ests of 2-phase voltages. That is to say , balanced sets will appear in the synchronously rotating reference frame regardless of the form of phase voltages.

Now , each of the balanced sets appearing in the synchronously rotating reference frame can be seperately transformed to a reference wherein the applied voltages are constant and hence d.c. circuit theory could be used in the analysis. The calculated quantities can then be transformed from each of these reference frames back to the synchronously rotating reference frame and combined to give the constant-speed performance of the machine . This application of multiple reference frames permits a rigorous steady-state performance analysis with nonsinusoidal stator voltages of any periodic waveform without restoring to the concept of phasors and complex impedances or the method of symmetrical components.

If each of the balanced sets are transformed to the appropriate reference frame , the following expressions are obtained for the reference frame applied voltages:

$$V_{qs}^{+ke} = (1/2)(V_{kq\alpha} - V_{kd\gamma}) \quad (7)$$

$$V_{ds}^{+ke} = (1/2)(V_{kq\gamma} + V_{kd\alpha}) \quad (8)$$

$$V_{qs}^{-ke} = (1/2)(V_{kq\alpha} + V_{kd\gamma}) \quad (9)$$

$$V_{ds}^{-ke} = (1/2)(V_{kq\gamma} - V_{kd\alpha}) \quad (10)$$

It is important to note that once V_{qs}^{s} and V_{ds}^{s} have been established from the phase voltages , the voltages which are to be applied in each of the reference frames may be determined directly from (3) - (10). The superscripts -ke and +ke used in (7) - (10) identify the speed and direction of rotation of the reference frames with respect to the stator.

The equations which describe the symmetrical induction machine in any reference frame may be readily obtained from the equations which describe the machine in an arbitrary reference frame from references [1] and [2]. Since the multiple reference frames are selected so that the applied voltages are constant , the steady - state ds-qs currents in any of the multiple reference frames may be expressed as:

$$i_{qs}^{ne} = (A^{ne}/E^{ne}) V_{qs}^{ne} + (B^{ne}/E^{ne}) V_{ds}^{ne} \quad (11)$$

$$i_{ds}^{ne} = -(B^{ne}/E^{ne}) V_{qs}^{ne} + (A^{ne}/E^{ne}) V_{ds}^{ne} \quad (12)$$

The quantities A^{ne} ,B^{ne} ,and E^{ne} are developed from the machine equation in the arbitrary reference frame in reference [1]. In the above equations the index n is used to denote +k and -K. However , only the reference frames with nonzero applied voltages are to be employed.

Expression for the rotor current may also obtained as

$$i'^{ne}_{qr} = (C^{ne}/E^{ne}) V_{qs}^{ne} + (D^{ne}/E^{ne}) V_{ds}^{ne} \quad (13)$$

$$i'^{ne}_{dr} = -(D^{ne}/E^{ne}) V_{qs}^{ne} + (C^{ne}/E^{ne}) V_{ds}^{ne} \quad (14)$$

If (11-14) are transformed back to the stationary reference frame , thus

$$i_{qs}^{s} = \sum_{k=1}^{\infty} [(i_{qs}^{+ke} + i_{qs}^{-ke}) \cos kwet + (i_{ds}^{+ke} - i_{ds}^{-ke}) \sin kwet] \quad (15)$$

$$i_{ds}{}^{s} = \sum_{k=1}^{\infty} [(i_{ds}{}^{+ke} + i_{ds}{}^{-ke}) \cos kwet -$$

$$(i_{qs}{}^{+ke} - i_{qs}{}^{-ke}) \sin kwet] \quad (16)$$

$$i'_{qr}{}^{s} = \sum_{k=1}^{\infty} [(i'_{qr}{}^{+ke} + i'_{qr}{}^{-ke}) \cos kwet +$$

$$(i'_{dr}{}^{+ke} - i'_{dr}{}^{-ke}) \sin kwet] \quad (17)$$

$$i'_{dr}{}^{s} = \sum_{k=1}^{\infty} [(i'_{dr}{}^{+ke} + i'_{dr}{}^{-ke}) \cos kwet -$$

$$(i'_{qr}{}^{+ke} - i'_{qr}{}^{-ke}) \sin kwet] \quad (18)$$

The stator and rotor currents may be obtained from (15-16) and (17-18) respectively, that is,

$$i_{as} = i_{qs}{}^{s} \quad (19)$$

$$i_{bs} = (-1/2)\, i_{qs}{}^{s} - (\sqrt{3}/2)\, i_{ds}{}^{s} \quad (20)$$

$$i_{cs} = (-1/2)\, i_{qs}{}^{s} + (\sqrt{3}/2)\, i_{ds}{}^{s} \quad (21)$$

$$i'_{ar}{}^{s} = i'_{qr}{}^{s} \quad (22)$$

$$i'_{br}{}^{s} = (-1/2)\, i'_{qr}{}^{s} - (\sqrt{3}/2)\, i'_{dr}{}^{s} \quad (23)$$

$$i'_{cr}{}^{s} = (-1/2)\, i'_{qr}{}^{s} + (\sqrt{3}/2)\, i'_{dr}{}^{s} \quad (24)$$

The instantaneous electromagnetic torque may be given as

$$T = M(m/2)(p/2)(i_{qs}{}^{s} i'_{dr}{}^{s} - i_{ds}{}^{s} i'_{qr}{}^{s}) \quad (25)$$

It is clear that although the expression in the synchronously rotating reference frame from the basis in the drivation, these equations need not be used in the actual application of the method of multiple reference frames.

3.REPRESENTATION OF THE STATOR VOLTAGE WAVEFORMS

With thyristor switches connected in series with the stator winding ,the phase voltages would have different nonsinusoidal waveforms that depends on the firing angles of the thyristors in the three-phases and on the phase angle of the load .In this case Fourier series method would not conveniently be applied to resolve the stator voltages into its harmonic components.

However, it is shown elsewhere [3] that a periodic function f(t) can be expressed by a Discrete Fourier Transform (DFT) rather than by Fourier series. The DFT could resolve any nonsinusoidal waveform into its harmonic components using computation method.

This method requires N^2 multiplications (where N is the number of samples).However ,recent advances in this area have resulted in a class of efficient algorithms known as the Fast Fourier Transform (FFT) , which offer significant reduction in computation time. A side from the algorithm itself , the interpretation of the FFT is the same as that for the DFT.

The three-phase voltages of any controller may be given by Fourier series representation as

$$V_a(wt) = a_o/2 + \sum_{k=1}^{\infty} C_{ka} \cos(kwt + {}_{ka}) \quad (26)$$

$$V_b(wt) = a_o/2 + \sum_{k=1}^{\infty} C_{kb} \cos(kwt + {}_{kb}) \quad (27)$$

$$V_c(wt) = a_o/2 + \sum_{k=1}^{\infty} C_{kc} \cos(kwt + {}_{kc}) \quad (28)$$

where
$a_o/2$ is the d-c component.

C_{ka} , C_{kb} and C_{kc} are the amplituds of the kth harmonic component.
${}_{ka}$, ${}_{kb}$ and ${}_{kc}$ are the phase angles of the kth harmonic component.

Now , instead of using Fourier series the valuse of C_{ka} , C_{kb} , C_{kc}, ${}_{ka}$, ${}_{kb}$, and ${}_{kc}$ could be found easly by using FFT technique, and in order to use the results obtained from FFT, the transformation of (26-28) to qs and ds reference frame fixed in the stator is as follows [1].

$$V_{qs}{}^{s} = (2/3)(V_a(wt) - (1/2)V_b(wt) -$$
$$(1/2)\, V_c(wt)) \quad (29)$$

$$V_{ds}{}^{s} = (1/\sqrt{3})(-V_b(wt) + V_c(wt)) \quad (30)$$

Substituting (26)-(28) into (29) and (30) to get

$$V_{kq\alpha} = (1/3)(\; 2C_{ka} \cos \psi_{ka} - C_{kb} \cos \psi_{kb} -$$
$$C_{kc} \cos \psi_{kc}) \quad (31)$$

$$V_{kq\gamma} = (1/3)(-2C_{ka} \sin \psi_{ka} + C_{kb} \sin \psi_{kb} +$$
$$C_{kc} \sin \psi_{kc}) \quad (32)$$

$$V_{kd\alpha} = (1/\sqrt{3})(-C_{kb}\cos\psi_{kb} + C_{kc}\cos\psi_{kc}) \quad (33)$$

$$V_{kd\gamma} = (1/\sqrt{3})(C_{kb}\sin\psi_{kb} - C_{kc}\sin\psi_{kc}) \quad (34)$$

The d-c components are of neglegable values in 3-phase circuits

4.CONTROLLER PHASE VOLTAGE WAVEFORM

A most widely used controller for 3-phase induction motor composed of inverse parallel thyristor pair for each phase Fig.(2),it is used for different purposes such as starting , and speed control.

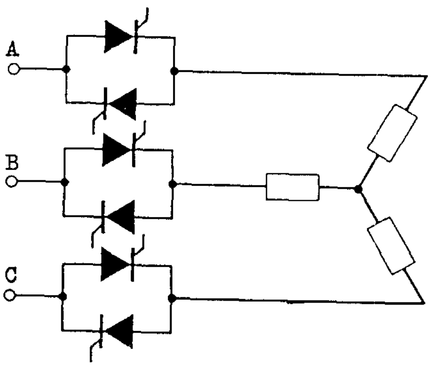

Fig. 2. Six thyristors, three-wire circuit.

The phase voltages waveform of a pair of thyristors feeding single phase (R-L) load is a function of the triggering angles and the phase angle of the load [4] because the extinction angle is a function of both of them as shown

$$\sin(x_\alpha - \phi_1) - \sin(\alpha - \phi_1)e^{-\cot\phi_1(x_\alpha - \alpha)} = 0 \quad (35)$$

where x_α :the extinction angle.
α :the triggering angle.
ϕ_1 :the load phase angle.

The characteristcs of cut-off angle versus triggering angle for different phase angles shown in Fig.(3).

To determine the voltage waveform for such a controller supplying three-phase induction motor , the variation of the phase angle of such a load should be considerd . The phase angle is a function of the slip due to the analysis of the equivalent circuit shown in Fig.(4).

For this circuit the phase angle determined as

$$\phi_1 = \tan^{-1}(r_1^2 + x_1^2 + x_2^2 + x_m(x_1 + x_2) + 2x_1x_2 + (r_2/S)(2r_1 + r_2/S))/(x_m(r_1 + r_2/S)) \quad (36)$$

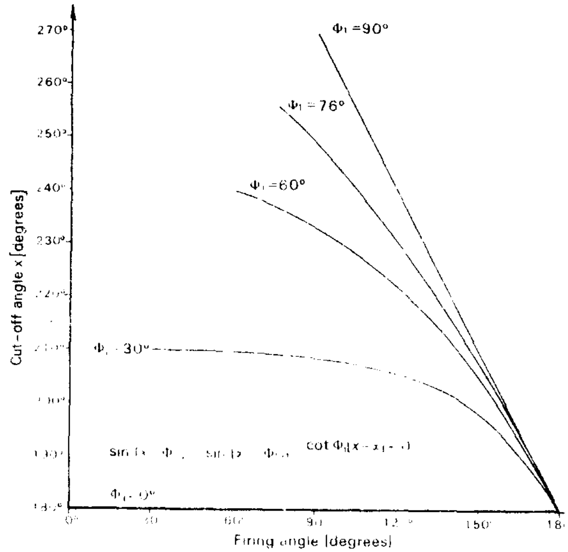

Fig. 3. Extinction angles versus siring angles for various load phase angles.

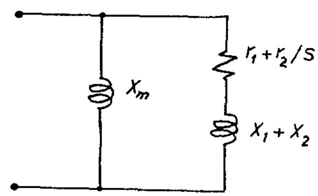

Fig. 4. The equivalent circuit.

Then the relation of the power factor ($\cos\phi_1$) as a function of S is drawn in Fig.(5)which is a nonlinear relation. The cutoff angle then is a function of the triggering angle and the slip.

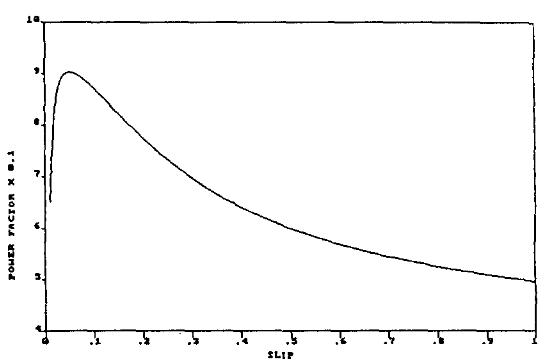

Fig. 5. Power factor versus slip of the induction motor.

This approach leads to the fact that the voltage waveform is a function of the parameters (α & S)due to equations (35,36) cosequently.

5.CALCULATED PERFORMANCE OF THREE-PHASE INDUCTION MOTOR WITH VOLTAGE CONTROLLER:

With the above derivation it would become easy to predict the steady-state operation of an induction machine controlled by semiconductor switching devices. A computer program contains five main steps has been established for this purpose, these are:

1- Simulation of the applied phase voltage waveform.

2- Sampling of these voltages to ($N=2^m$)samples per cycle.

3- Using the FFT subroutine to determine the harmonic components of the applied stator phase voltage waveform.

4- Application of MRF method to determine the performance of the motor.

5- The above four steps are repeated for each slip in certain step to determine the torque slip characteristics.

The present technique is firstly checked by calculating the performance of induction machine when supplied by a 3-phase set of square wave voltages displaced 120 electrical degrees (Example -A- in reference [1]). Exact results have been achived as shown in Fig.(6).

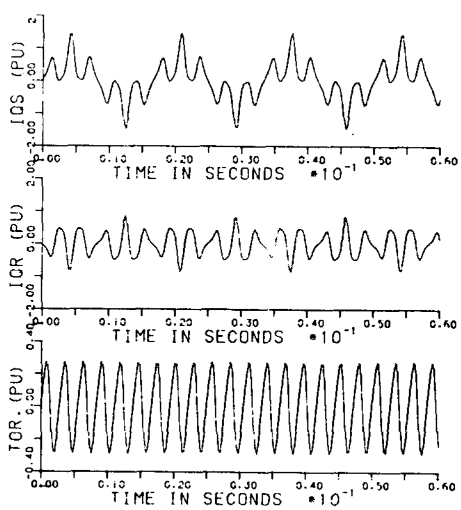

Fig. 6. Results of EX-A ref. [1] using (FFT+MRF) method.

The stator phase voltages , stator phase currents and the instantaneous torque can be calculated for balanced and unbalanced supply voltages in addition to the torque slip characteristics.

In this study the following per unit machine parameters, with 60 HZ as base frequency, are used with a base impedance of 9.45 ohms:

$r_s=0.0453$ $\qquad$ $x_{ls}=0.0775$

$r_r=0.0222$ $\qquad$ $x_{lr}=0.0322$

$x_m=2.024$

This machine is fed through a phase angle controller used as a starter or a speed controller in limitted ranges .So the symmetrical and asymmetrical triggering are studied. An example will be analysed for each case and the results will be discussed.

EXAMPLE-A : SPEED CONTROLLER (Symmetrical Trigg. Mode)

The speed control is achived by using symmetrical triggering mode for each phase .The analysis shows that the effective range of the triggering angle (α) from $(50-90)^0$ in order to achive a constant (rated torque) variable speed operation for a slip range from (0.0265 to 0.1444),as shown in Fig.(7).

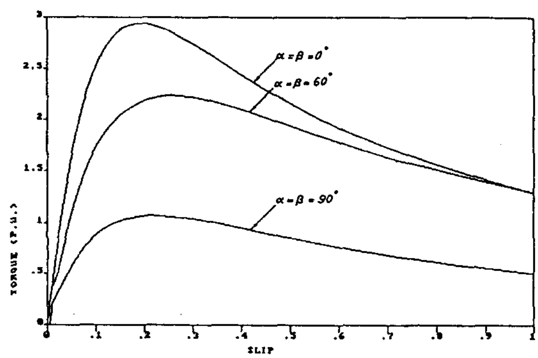

Fig. 7. The torque slip c/c's of symmetrical triggering mode for different trig. angles.

The relation between S and   is drawn in Fig.(8) for rated load .The steady state stator phase voltage, stator phase current, and the instantaneous torque in the case of α-β= 90^0 and the slip=0.1 is shown in Fig.(9).

The most important result is the pulsating torques repeated six times in each cycle. The principal pulsating torques arise from the interaction between the fundamental rotating flux and the harmonic rotor current [5].The spectrum of the stator current shown in

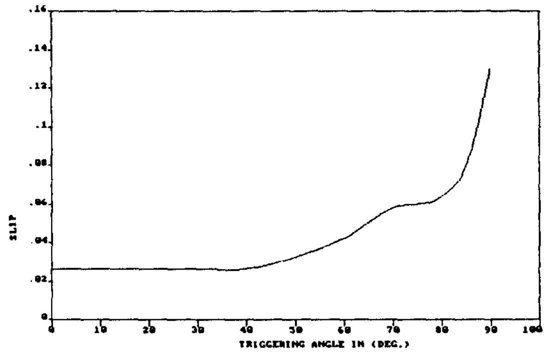

Fig. 8. Slip versus trigg. angle for constant torque operation.

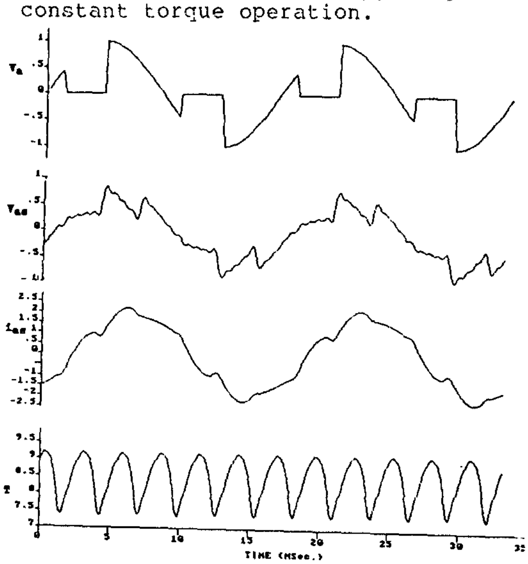

Fig. 9. V_a, V_{as}, i_{as}, and T versus time when $\alpha = \beta = 90^o$ and S=0.1 .

Fig.(10) represents that the 5th harmonic is dominant and rotating in a -ve sequence ,so it will produce a space fundamental m.m.f. wave rotates in five times the fundamental synchronous speed in the opposit direction to the fundamental field. The rotor currents induced by this time harmonic field react with the fundamental rotating field to produce a pulsating torque at six times fundamental frequency.

EXAMPLE-B : THYRODE CONTROLLER (Asymmetrical Trigg. Mode)

This controller is shown in Fig.(11), the control is achived by using phase angle triggering for the first half cycle , so only α is changed while β is kept equal to zero degree (where β is the trigg. angle of the -ve half cycle of the

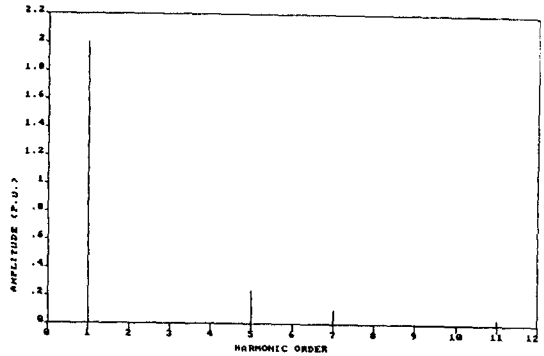

Fig. 10. Spectrum of stator current when α=β=90^o and S=0.1 .

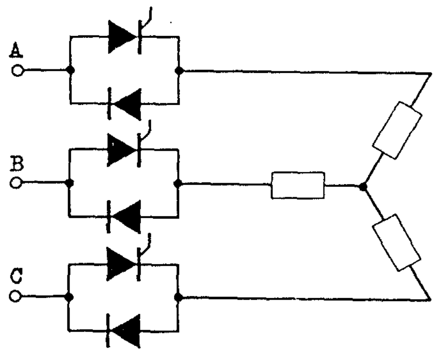

Fig. 11. Thyrode controller.

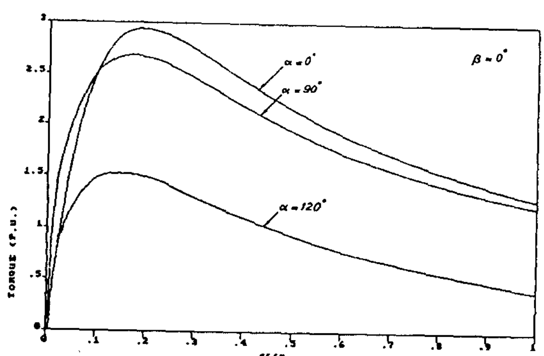

Fig. 12. Torque slip C/C´s of thyrode controller when α=0,90, and 120 deg. .

input voltage) so it represents a special case of asymmetrical triggering mode . The analysis shows that the effective range of the triggering angle is from 50^o to 120^o as shown in Fig.(12).

The steady state performance when α = 90^o and the slip =0.1 shown in Fig.(13). The torque pulsations repeated three times in each cycle.The spectrum of the stator current is shown in Fig.(14) shows that the dominant harmonic is the 2nd which form a -ve sequence system , so it will produce a space fundamental m.m.f. wave rotates in 2 times the fund. sync. speed in the opposit direction to the fundamental field . The rotor current induced by this time harmonic field react with the fundamental rotating field to produce a pulsating torque at three times fund. frequency.

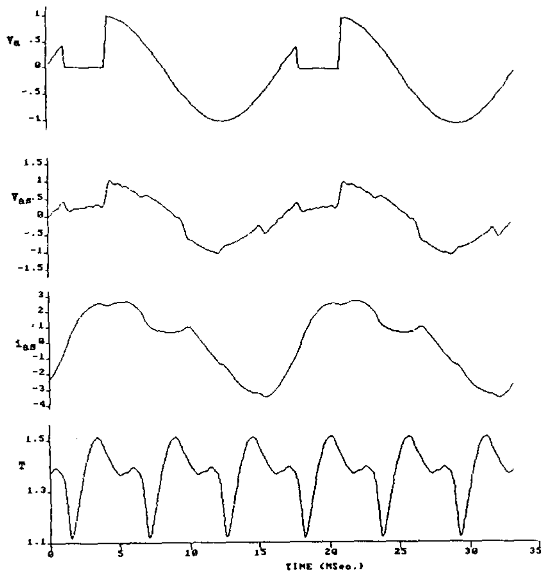

Fig. 13. V_a, V_{as}, i_{as}, and T versus time when $\alpha=90^o$, $\beta=0^o$ and S=0.1 .

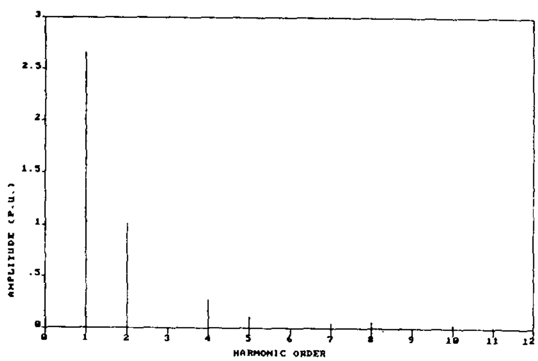

Fig. 14. Spectrum of stator current when $\alpha=90^o$, $\beta=0^o$ and S=0.1 .

6.CONCLUSION

Acomputer aided method of analyzing the steady state modes of unbalanced or nonsinusoidal operation of symmetrical induction machine has been developed. The cooperation of the MRF method and the FFT technique performs an efficient algorithm for such analysis, when the Fourier series method would not conveniently be applied to resolve the input voltages, as in phase angle controllers. In these sources the phase voltage waveform is a function of both the triggering angles and the machine power factor.

The analysis of these controllers shows that the symmetrical triggering mode is the most suitable in speed control, because it performs aregular change in speed with acceptable torque pulsations.While the asymmetrical mode (thyrode) changes the speed in a complicated manner, with torque pulsations of high level comparable with that of the symmetrical mode and of nonuniform shape which will cause a sever effects on the machine structur.

This method is suitable to analyse variabl voltage variable frequency sources (PWM techniques) easly and directly.

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