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Crosscorrelation Function of M-arrays of Different Sizes

Masato Sakata and Hiroshi Kashiwagi

Faculty of Engineering, Kumamoto University, Kumamoto, Japan

Abstract: In a two-dimensional positioning system using M-array, the crosscorrelation between the observed M-array and the reference M-array is calculated. In this case, when the distance between the object and the camera is varied, the crosscorrelation value fluctuates. This paper investigates the effect of variation of size of M-array on the crosscorrelation values.

1. Introduction

In two-dimensional (2-D) positioning systems using M-arrays, a reference M-array is placed inside the system, and an identical M-array is pasted on the object to be aligned (referred as 'object M-array'). A camera is fixed at a specifed distance and focused so that the observed M-array is the same size as that of the standard M-array. The principle of the system is to compute the crosscorrelation between the reference M-array and the observed M-array, and the maximum of the two-dimensional crosscorrelation function is sought with X-Y positioning servo system.

In such systems, the size of the observed M-array varies as the distance between the camera and the object M-array varies. Consequently, it is necessary to investigate the effect of such variations on the computed crosscorrelation functions.

To this end, the properties of the crosscorrelation functions of M-arrays of different sizes are studied and they are compared with the crosscorrelation functions of the corresponding M-sequences. The results of simulation and experiment are presented.

2. 2-D Position Alignment System using M-arrays

A 2-D position alignment system using an M-array pattern is shown in Fig. 1.

In this system a pattern of black and white squares to represent M-array is pasted on the object to be aligned, which in turn is mounted on the X-Y table. The object M-array pattern is observed with a camera. The crosscorrelation function between the observed M-array and the reference M-array is computed using the personal computer with a hardware FFT board. By observing the point of maximum crosscorrelation, control signals are transmitted to X-Y table unit. The alignment is done by moving the X-Y table according to the control signals.

Generally, in such systems, if the distance between the camera (in z-direction) and the object M-array varies due to some reason, then the size of the observed pattern fluctuates accordingly.

Therefore, it is necessary to investigate how the variation of size of M-arrays effects the crosscorrelation function of M-arrays.

3. Crosscorrelation Function of M-arrays

Let the reference M-array, that is placed inside the computer be M_0, and the observed M-array pattern be $M_1 = M_0 + Noise$.

If the camera is at a specified distance from the object M-array, the size of the observed pattern is the same as the reference M-array.

In this case, the observed M-array can be considered as free of noise.

Consequently, the computed crosscorrelation function is the crosscorrelation function between two identical M-arrays. On the other hand, if this distance varies due to some unknown reason, the size of the observed M-array proportionally varies.

There are some portions which differ from reference M-array, if the observed M-array is overlaid on the reference M-array for comparison. The portions undergone variation correspond to the noise present in the observed M-array.

Therefore, the crosscorrelation function $\Phi_{M_0M_1}(k,l)$ between the two M-arrays $M_0(i,j)$, and $M_1(i,j)$ is as given in Eq. (1).

$$\begin{aligned}\Phi_{M_0M_1}(k,l) &= \overline{M_0(i,j)\cdot M_1(i,j) + M_0(i,j)\cdot \text{Noise}} \\ &= \Phi_{M_0M_0}(k,l) \qquad (1)\end{aligned}$$

4. Crosscorrelation Function of M-arrays of Different Sizes

In such systems, if the distance between the object M-array and the camera positioned at a speceifed height varies, then the size of the observed M-array differs from that of the reference M-array.

An M-array of 15 rows and 17 columns is obtained from an M-sequence of order $n = 8$ and period $N = 255$. In Fig. 2 is shown this M-array pattern that is used as reference M-array M_0, along with M-array patterns M_1 obtained by distorting this M-array 15%, 0%, and −15% in lateral dimensions. In these patterns, white squares represent 0's and black squares represent 1's. The M-array patterns of different sizes represent observed M-array patterns.

The crosscorrelation function between the standard M-array M_0, and the M-array of different sizes M_1 (−15% ∼ +15% variation) are obtained by simulation. Fig. 3(a),(b),(c) show respectively, the crosscorrelation functions between the standard M-array and M-arrays with 0%, +10%, and +15% variation in lateral dimensions.

It is assumed that each M-array data is made up of four pixels when the distance between the object M-array and the camera takes the specified standard value.

The crosscorrelation function between M-arrays when there is no variation in lateral dimensions is shown in Fig. 3(a) and has peak of $15 \times 17 \times 4 \times 4 = 4080$ at the center of the region. The correlation values at other places in the region is $-4 \times 4 = -16$. These correlation values are normalized by the correlation value 4080 at $(0,0)$.

When the lateral dimensions vary by +10%, there is a variation of +21% in area, implying there by the presence of a proportional amount of noise in the standard M-array.

As a result, the crosscorrelation function shown in Fig. 3(b), gets deformed from that of the usual crosscorrelation function shown in Fig. 3(a). Nevertheless, it can be seen that the position of the maximum peak does not change.

Further, when the lateral dimensions are varied by +15%, the area of the M-array patterns differ by +32%, meaning an increase in the amount of noise included, and the crosscorrelation function is shown in Fig. 3(c).

From the results of simulations, the peaks of the crosscorrelation functions for M-arrays of different sizes with the standard M-array are plotted in Fig. 4. (shown with ○ marks). It is found out that if the variation in lateral dimensions is in the range −15% ∼ +15%, the position of the maximum peak of the crosscorrelation function remains unchanged.

Next, experiments are carried out with an actual system as described previously, and crosscorrelation function between the standard M-array and observed M-arrays are computed. For each observation, the distance between the object M-array and the camera is changed. The results are given in Fig. 4 with □ marks.

The experimental results indicate that if the variation in lateral dimensions is in the range −10% ∼ +10%, then the position of maximum peak can be accurately determined.

It can be seen from Fig. 4. that if size of the M-array varies, then the peak value of the crosscorrelation function decreases.

This is because there exists a distorted portion when the observed M-array of a different size is overlaid on the standard M-arry, which indicates that the observed M-array is contaminated with noise.

5. The relation between size variation and noise in M-array

As explained previously, M-array of 15 rows and 17 columns are constructed using an M-sequence of order $n = 8$ and period $N = 255$ and each data is represented by 4×4 pixels. Therefore, M-array is made up of 60×68 pixels, meaning 4080 pixels in total.

The crosscorrelation functions between this standard M-array and M-arrays obtained by varying the lateral dimensions of the M-array are computed by simulation. The results are shown in Table 1, where the values are normalized by 4080 (total number of pixels).

The ratio of the included noise is obtained by the decrease in the correlation values due to variation in the size. The standard M-array pattern and the M-array pattern obtained by varying the lateral dimensions are overlaid one over the other and adjusted to get maximum crosscorrelation function between them. In that case, it can be observed that both the patterns agree at the center, but not at the peripheral regions. The extent or the ratio by which the two patterns do not agree at these peripheral regions is computed.

If the black and white patterns are arranged as in a checker board, then the data changes alternately. However when M-array patterns are used, then the probability that the two consecutive data are different is 1/2.

The theoretical values represented in Table 1 are obtained in this fashion. If there are no variations in lateral dimensions, then the standard M-array and the observed M-array are identical and the correlation value is 1.0, indicating the absence of noise.

If the lateral dimensions are increased by 5%, the correlation value is 0.67, and the noise ratio is 16.7% with respect to the total area of the M-array, and the corresponding theoretical value is 18.6%.

If the lateral dimensions are increased by 10%, the noise ratio is 30.9%, and the corresponding theoretical value is 32.6%.

Thus, the variation in lateral dimensions correspond to the inclusion of noise at the peripheral region of the M-array. When these dimensions are varied by +5%, the overlay of the two patterns to obtain maximum correlation is shown in Fig. 5. In this figure, the identical data elements are shown by 1's and the data elements that do not match (that represent noise), are shown by 0's. In M-arrays, since black and white patterns appear randomly, only half the noise plays a role of actual noise.

In order to compare the effect of size of M-array with that of M-sequence, let's us consider the relation between the length of the M-sequence and noise. The M-sequence of order $n = 8$ and period $N = 255$, that is used to construct the standard M-array is assumed to have elements of 4 pixels each.

The crosscorrelation between this standard M-sequence and the sequences with different lengths are obtained by simulation and results are shown in Table 2.

When the two sequences are of same length, the value of the crosscorrelation is 1.0.

If the length of the M-sequence is increased by 0.5%, the correlation value is 0.71 and ratio of the noise present is 14.7% with respect to the total length of the sequence. The theoretical value of the noise ratio is 17.6%.

If the length of the sequence is increased by 1.0%, the noise ratio is 29.3%.

Therefore, it can be seen that M-arrays are more robust to noise than M-sequences.

6. Conclusion

If in 2-D position alignment systems, the distance between the camera and the object M-array varies due to some reason, the size of the observed M-array also varies. If the observed M-array is overlaid on the standard M-array, the portions that do not match with the standard M-array can be noticed.

These portions are considered to be the noise present in the observed M-array. The following points were observed by computing the crosscorrelation function between the standard M-array and observed M-arrays of different sizes.

1. If the variation in the size is 10% for M-arrays and 1.0% for M-sequences, the amount of noise present is about 30%. This is because, M-arrays are two dimensional arrangement of M-sequences and hence the noise generated is smaller. Therefore, M-array patterns are advantageous for 2-D position alignment systems.

2. In the case of M-array of 15 rows and 17 columns obtained by an M-sequence of order $n = 8$, it is found that if lateral dimensions are varied by $-10\%, +10\%$, the amount of noise present is 30.9% and 37.9% respectively.

3. By experimental observations it is clear that if the lateral size variations are in the range $-10\% \sim +10\%$, the position of the maximum peak of the crosscorrelation remains unchanged.

References

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2. H. Kashiwagi, M. Sakata, "A Two-dimensional Positioning System by use of M-array", proc *'88 KACC*, pp.782–785, (1988).

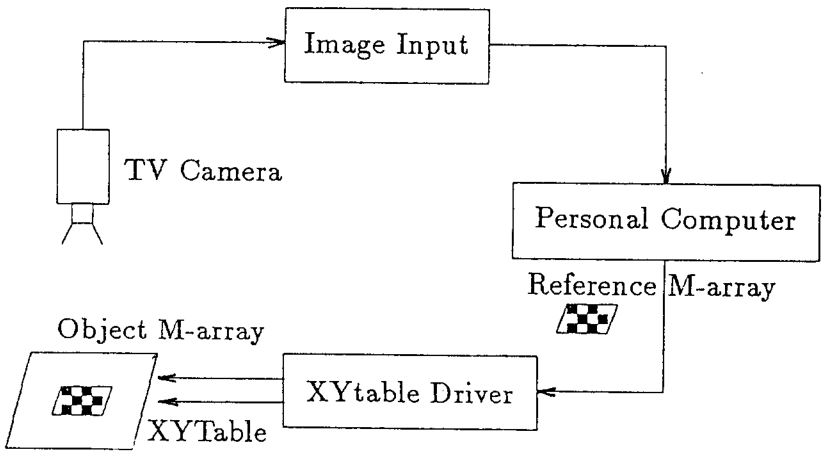

Fig. 1 Block diagram of the 2-D Positioning System

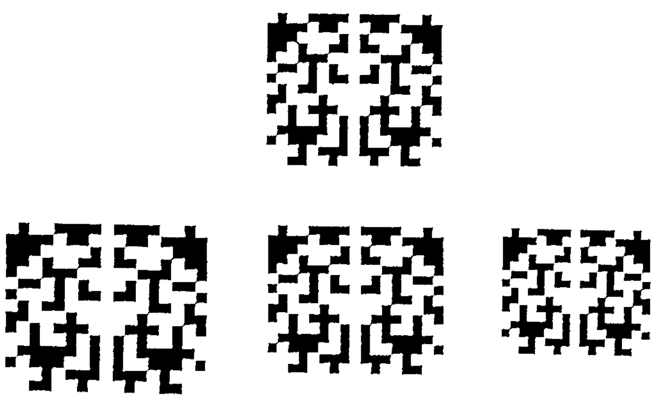

Fig. 2 Reference M-array and Object M-array with 15%, 0% and -15% reduction in lateral dimensions

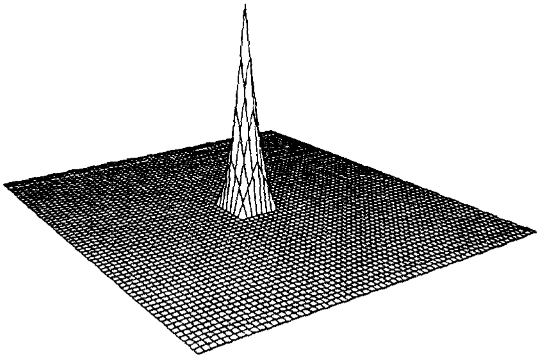

Fig. 3(a) Three dimensional representation of Cross-corralation between reference M-array and object M-array

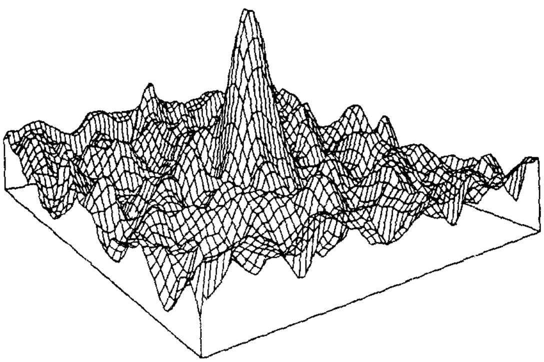

Fig. 3(b) Cross-correlation when lateral dimensions of Object M-array is increased by 10%

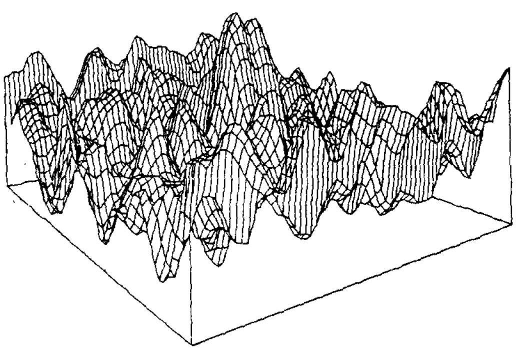

Fig. 3(c) Cross-correlation when lateral dimensions of Object M-array is increased by 15%

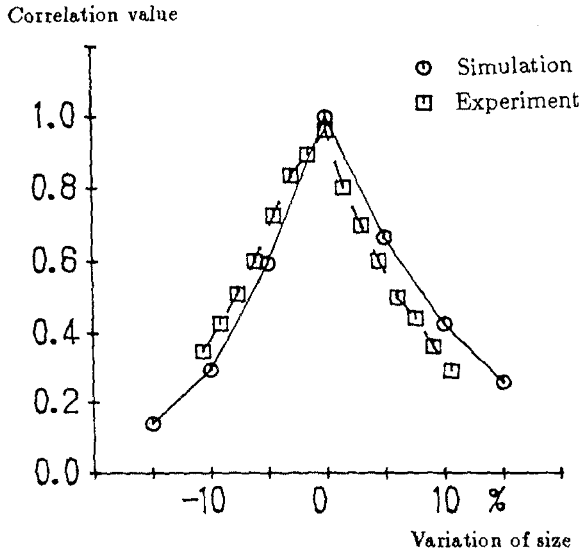

Fig. 4 Variation in correlation function due to variation of the size of M-array

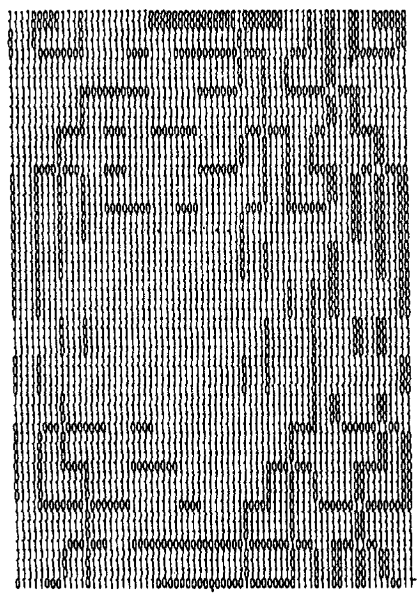

Fig. 5 The noise included in the M-array when the lateral dimensions of the Object M-array is reduced by 5%

Table 1 The noise ratio due to the change in the size of the M-array

Change in size	Correlation value	Noise ratio	Theoretical value of noise ratio
0%	1.0	0%	0%
5	0.67	16.7	18.6
10	0.38	30.9	32.6
15	0.22	39.1	
20	0.15	42.5	

Table 2 The noise ratio due to the change in the size of the M-sequence

Change in size	Correlation value	Noise ratio	Theoretical value of noise ratio
0%	1.0	0%	0%
0.5	0.71	14.7	17.6
1.0	0.41	29.3	28.8
1.5	0.20	36.3	
2.0	0.23	38.7	
3.0	0.20	40.2	