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ROBUST CONTROL OF A ROBOT MANIPULATOR BY MEANS OF SLIDING OBSERVERS

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Abstract : In this paper a robust control design is developed for the control of a multi-joint manipulators using sliding observer. The sliding observer is introduced to estimate the angular velocity of the links under the disturbance input. The feedback control is designed by the use of the estimated value of the angular velocity $\dot{\theta}$. The VSS control laws is introduced to ensure the robustness concerning the disturbance inputs. To illustrate the effectiveness of the proposed method, a computer simulation is performed for a two-joint manipulator.

1. Introduction

The sliding mode control theory has been investigated mostly Soviet literature [1] where it has been used to stabilize a class of nonlinear systems. It theoretically features excellent robustness properties in the face of parameter uncertainty and disturbances. Moreover the controller design became very simple so that its application to the control of robot arm was examined by several authors [2)~5)].

The above stated method is considered as a kind of state feedback control. More exactly speaking, it requires at least to detect the velocity of the output. In solving this problem, the use of the velocity sensor or numerical differentiation is first considered. However there are several flaws such that sensor is expensive and the numerical differentiation is not effective under the existence of the noise. On the contrary, the use of the nonlinear observer is said to give good results [6)~10)].

In this paper, we deals with a design method for robust multi-joint manipulator control system having unknown disturbance. The controller is designed based on the variable structure control theory(VSS) and the estimation of the velocity, which is necessary to constructed the VSS controller, is performed by using the sliding mode observer which was first proposed by Slotine [8]. The stability problem of the closed-loop system is also considered and sufficient condition for the system to be stable are given.

2. Statement of the Problem

We consider an n-degree of freedom manipulator. Let us denote the relative angular position between the link at the j-th joint as θ_i ($i=1,\cdots,n$). Then its dynamics [12] are represented by

$$A(\Theta)\ddot{\Theta} + B(\Theta,\dot{\Theta})\Theta + G(\Theta) + D(t) = Q(t) \qquad (2.1)$$

where

$$B(\Theta,\dot{\Theta}) = F(\Theta)\begin{bmatrix}\dot{\theta}_1\dot{\theta}_1\\ \vdots \\ \dot{\theta}_n\dot{\theta}_n\end{bmatrix} = F(\Theta)E(\dot{\Theta}). \qquad (2.2)$$

Here $A(\Theta)$ is an $n \times n$ non-singular matrix of inertia, $B(\Theta,\dot{\Theta})$ is an $n \times n$ centrifugal and Coriolis matrix ,$G(\Theta)$ is an n-th order gravity force and $D(t)$ is an n-th order unknown disturbance input vector.

Let

$$x_{i1} = \theta_i, \quad x_{i2} = \dot{\theta}_i, \quad y_i = x_{i1}. \qquad (2.3)$$

Then, from (2.1), we have

$$\dot{x}_i(t) = A_i x_i(t) + b_i f_i^T(y)E(x) + b_i g_i(y) + b_i d_i(t) + b_i u_i(t) \qquad (2.4a)$$

$$y_i(t) = c_i^T x_i(t) \qquad (2.4b)$$

where

$$x_i = \begin{bmatrix}x_{i1}\\x_{i2}\end{bmatrix},\ A_i = \begin{bmatrix}0 & 1\\0 & 0\end{bmatrix},\ b_i = \begin{bmatrix}0\\1\end{bmatrix},\ c_i = \begin{bmatrix}1\\0\end{bmatrix}$$

$$E(x) = \begin{bmatrix}x_{12}x_{12}\\x_{12}x_{22}\\ \vdots \\ x_{n2}x_{n2}\end{bmatrix},\quad x = \begin{bmatrix}x_1\\x_2\\ \vdots \\ x_n\end{bmatrix},\quad y = \begin{bmatrix}y_1\\y_2\\ \vdots \\ y_n\end{bmatrix}.$$

Note that Θ is available for measurement and $Q(t)$ is constructable . Hence $y_i(t)$ and $u_i(t)$ are detectable ,and $f_i(y)$ and $g_i(y)$ can be regarded as known functions. The disturbance $d_i(t)$ is unknown but here we assume that the maximum value;

$$M_{d_i} = \max_t |\ d_i(t)\ | \qquad (2.5)$$

is known.

The control objective is to determine the control input $u_i(t)$ such that the output $y_i(t)$ follows the reference input $r_i(t)$ under the assumption that the output $y_i(t)$ and the input $u_i(t)$ are available for measurement.

3. Sliding Observer

Corresponding to (2.4), sliding observer is given by

$$\dot{z}_i(t) = D_i z_i(t) + h_i y_i(t) + b_i f_i^T(y) E(z) + b_i g_i(y) + b_i u_i(t) - k_i \mathrm{sgn}(e_{i1}) \tag{3.1}$$

where

$$z_i = \begin{bmatrix} z_{i1} \\ z_{i2} \end{bmatrix}, \quad A_i = \begin{bmatrix} -\alpha_{i1} & 1 \\ -\alpha_{i2} & 0 \end{bmatrix}, \quad k_i = \begin{bmatrix} k_{i1} \\ k_{i2} \end{bmatrix}$$

and

$$e_{i1} = z_{i1} - y_i, \quad e_{i2} = z_{i2} - x_{i2}. \tag{3.2}$$

Let

$$e_i(t) = z_i(t) - x_i(t) \tag{3.3}$$

be a state estimation error vector. Then, from (2.4a) and (3.1), the following error equation is obtained.

$$\dot{e}_i(t) = D_i e_i(t) + b_i f_i^T(y)(E(z) - E(x)) + b_i d_i(t) - k_i \mathrm{sgn}(e_{i1}). \tag{3.4}$$

Here D_i is determined from the relation,

$$D_i = A_i - h_i c_i^T \tag{3.5}$$

where h_i is design parameter vector. From (2.4), it is apparent that (A_i, c_i) is an observable pair. Hence we can select an adequate vector h_i which stabilizes D_i[6]. Then, the existence of the sliding mode is given by the following theorem.

[Theorem 1]

Let us choose the sliding curve such that

$$S_i = e_i = 0. \tag{3.6}$$

Then if there exist a time $T \geq 0$ such that

$$|e_{i2}(t)| \leq k_{i1}, \quad \text{for } t \geq T \tag{3.7}$$

where $e_{i2}(t)$ is a solution of (3.4), the sliding mode condition

$$S_i \dot{S}_i < 0 \tag{3.8}$$

is satisfied for all $t \geq T$.

(As to the proof, see *Appendix 1.*)

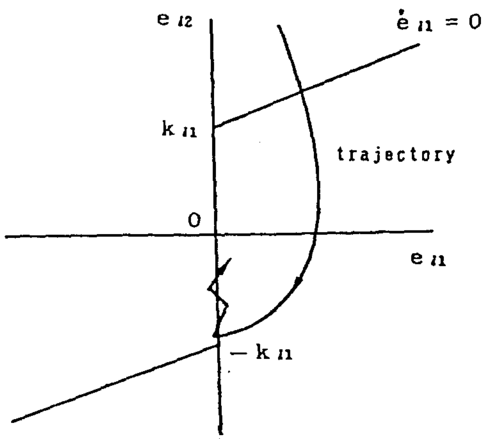

Fig.1 Phase plane trajectory

Theorem 1 has the following meanings. Suppose that the trajectory of the error system reaches the region $|e_{i2}| \leq k_{i1}$ at $t = T$ and stays subsequently in this region. Then, the sliding mode occurs so that the motion of the trajectory of the error system is fixed on the sliding surface: $e_{i1} = 0$. (See Fig.1)

From the above stated facts, the problem results in finding the arrival condition to the region (3.7) and the staying condition in the region after arriving. These condition are shown in theorem 2.

[Theorem 2]

Let Ω be a closed domain of R^{2n} containing the origin and suppose that the nonlinear terms $E(x)$ in (2.4) is Lipschitzan. That is, there exists a constant μ such that for any pair of points $x_1, x_2 \in \Omega$ can gets

$$\| E(x_1) - E(x_2) \| \leq \mu \| x_1 - x_2 \|. \tag{3.9}$$

Then, if (i) there exist positive symmetric matrix P_i and $Q_i \in R^{2\times2}$, stable matrix $D_i \in R^{2\times2}$, vector l_i and $k_i \in R^2$, non-negative scalars α_i and β_i such that

$$P_i D_i + D_i^T P_i = -l_i l_i^T - Q_i \tag{3.10}$$

$$P_i k_i = \beta_i c_i + \alpha_i D_i^T c_i + \sqrt{2\alpha_i c_i^T k_i}\, l_i \tag{3.11}$$

$$c_0 - 2\mu\lambda_2 M_f > 0 \tag{3.12}$$

$$\frac{\lambda_3}{\lambda_1} + \frac{2\lambda_2^2 M_d}{\lambda_1(c_0 - 2\mu\lambda_2 M_f)} < \min_i(k_{i1}) \tag{3.13}$$

where

$$\begin{aligned} &c_0 = \min_i[\lambda(Q_i)], \ \lambda_1 = \min_i[\lambda(P_i)], \ \lambda_2 = \max_i[\lambda(P_i)] \\ &\lambda_3 = \| a \|, \quad a = [2\alpha_1, \cdots, 2\alpha_n] \\ &M_f = \max \| BA^{-1}(\Theta)F(\Theta) \| \\ &M_d = \max \| BA^{-1}(\Theta)D(t) \| \\ &B = \mathrm{diag}[b_1, \cdots, b_n] \end{aligned} \tag{3.14}$$

and (ii) initial conditions x_0 and z_0 in (2.4) and (3.1) satisfy (3.9), the trajectory of the solution $e_i(t)$ approaches the region : $|e_{i2}(t)| \leq k_{i1}$ within the finite time and stays there subsequently.

(As to the proof, see *Appendix 2.*)

4. Sliding Mode Control System

Let

$$\varepsilon_i(t) = r_i(t) - y_i(t) \tag{4.1}$$

be the tracking error for i-th link. Further, we introduce a stable 2nd-order polynomial:

$$\Phi(p) = p^2 + \alpha_1 p + \alpha_2 \tag{4.2}$$

where $p = d/dt$ denotes the differential operator. Then from (4.1), we have

$$\Phi(p)\varepsilon_i(t) = \Phi(p) r_i(t) - c_i^T \Phi(A_i) x_i - g_i(y) - f_i^T(y) E(x) - d_i(t) - u_i(t) \tag{4.3}$$

$$\Phi(A_i) = A_i^2 + \alpha_1 A_i + \alpha_2 I \tag{4.4}$$

Now, we construct the control input u_i such as

$$u_i(t) = \Phi(p)r_i(t) - c_i^T\Phi(A_i)z_i - g_i(y) - f_i^T(y)E(z) - u_{i1}(t) \tag{4.5}$$

Then,(4.3) can be rewritten as

$$\ddot{\varepsilon}_i + \alpha_1\dot{\varepsilon}_i + \alpha_2\varepsilon_i = \tilde{w}_i(t) + u_{i1}(t) \tag{4.6}$$

where

$$\tilde{w}_i(t) = c_i^T\Phi(A_i)e_i + f_i^T(y)(E(z) - E(x)) - d_i(t) \tag{4.7}$$

Note that ,from (2.5),(3.9) and theorems 1 and 2, it can easily be shown that there exists a constant $M_{w_i} > 0$ such that

$$|\ \tilde{w}_i(t)\ | \le M_{w_i} \tag{4.8}$$

The remaining problem is its construct a control input $u_{i1}(t)$ which guarantees the stability of $\varepsilon_i(t)$ under the existence of the unknown disturbance $\tilde{w}_i(t)$.

Now we set the switching line such that

$$S_{ui} = \dot{\varepsilon}_i + c_{si}\varepsilon_i = 0 \tag{4.9}$$

Then sliding mode occurs provided that the sliding mode condition

$$S_{ui}\dot{S}_{ui} < 0 \tag{4.10}$$

is satisfied. That is, along the sliding line $S_{ui} = 0$, tracking error $\varepsilon_i(t)$ follows the equation

$$\dot{\varepsilon}_i = -c_{si}\varepsilon_i \tag{4.11}$$

Hence it converges to zero as time increases. The attainability condition to the switching line becomes

$$S_{ui}\ddot{\varepsilon}_i < 0 \tag{4.12}$$

The control $u_{i1}(t)$ which satisfies the condition (4.10) and (4.12) is given by theorem 3.

[**Theorem 3**]

Let us construct the control input $u_{i1}(t)$ such that

$$u_{i1}(t) = \begin{cases} 0, & 0 \le t < T \\ \hat{\psi}_{i1}\hat{\dot{\varepsilon}}_i + \hat{\psi}_{i2}\varepsilon_i + \hat{\psi}_{i3}, & t \ge T \end{cases} \tag{4.13}$$

where

$$\hat{\psi}_{i1} = -M_{i1}\,\mathrm{sgn}(\hat{\dot{\varepsilon}}_i\hat{S}_{ui}) \tag{4.14}$$
$$\hat{\psi}_{i2} = -M_{i2}\,\mathrm{sgn}(\varepsilon_i\hat{S}_{ui}) \tag{4.15}$$
$$\hat{\psi}_{i3} = -M_{w_i}\,\mathrm{sgn}(\varepsilon_i\hat{S}_{ui}) \tag{4.16}$$
$$\hat{\dot{\varepsilon}}_i = \dot{r}_i(t) - \hat{z}_{i1}(t) \tag{4.17}$$
$$\hat{S}_{ui} = \hat{\dot{\varepsilon}}_i + c_{si}\varepsilon_i \tag{4.18}$$

$$M_{i1} \ge \max_i(\alpha_1\ ,\ c_{si} - \alpha_1) \qquad M_{i2} \ge \alpha_2 \tag{4.19}$$

Then

$$(i)\ \ |\ \varepsilon_i(t)\ | < \infty \qquad 0 \le t < T \tag{4.20}$$
$$(ii)\ \ S_{ui}\dot{S}_{ui} < 0 \qquad t \ge T \tag{4.21}$$
$$(iii)\ \ S_{ui}\ddot{\varepsilon}_i < 0 \qquad t \ge T \tag{4.22}$$

(As to the proof, see *Appendix 3.*)

5. Numerical Simulation

We shall implement a simulation to illustrate the effectiveness of the proposed method. Consider a two-links manipulator as shown in Fig.2. Equation of motion is represented by

$$\begin{aligned} &\frac{4}{3}ml^2\ddot{\theta} - \frac{4}{3}ml^2\dot{\phi}^2\cos\theta\sin\theta - mgl\sin\theta + d_1w_1 = q_1 \\ &\frac{4}{3}ml^2\ddot{\phi}\sin^2\theta + \frac{8}{3}ml^2\dot{\theta}\dot{\phi}\cos\theta\sin\theta + d_2w_2 = q_2 \end{aligned} \tag{5.1}$$

By choosing $y_1 = x_{11} = \theta$ and $y_2 = x_{21} = \phi$, we can drive the state equation (2.4)

Simulation was executed under the assumption that the parameters have some mismaches (40% for mass, 10% for length). Tracking error ε_1 and ε_2 for step response input are shown in Figs 3 and 4. It is noted that an approximated control law

$$\mathrm{sat}(\psi) = \begin{cases} \psi/\rho, & |\ \psi\ | \le \rho \\ \mathrm{sgn}(\psi), & |\ \psi\ | > \rho \end{cases} \tag{5.2}$$

is used instead of pure relay control law : $\mathrm{sgn}(\psi)$, to avoid the generation of chattering [2].

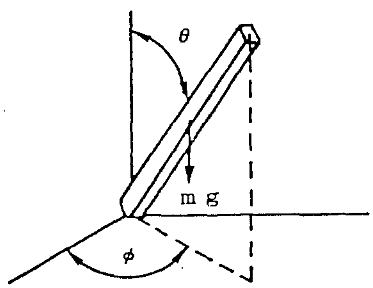

Fig.2 Two-links manipulator

Reference

1) V. L. Utkin : Variable Structure Systens with Sliding Modes, *IEEE Trans. Autom. Control*, AC-22-2, 212/222, (1977)

2) J. J. Slotine and S. S. Sastry : Tracking Control of Non-linear Systems Using Sliding Surface, with Application to Robot Manipulators, *Int. J. Control*, 38-2, 465/492 (1983)

3) K. D. Young : Controller Design for a Manipulator Using Theory of Variable Structure Systems, *IEEE Trans. SMC* 8-2, 101/109 (1978)

4) H. Tokumaru , N. Takagi, K. Mano and Z. Iwai : Design of Robust and Finite-Time-Settling Control for Multi-joint Manipulator Using Variable Structure Control, *Japan-USA Sympo.on FA, JACC and ASME*, 69/76 (1986)

5) K. D. Young : Application of Sliding Mode Constrained Robot Motion Control, *ACC*, 912/917 (1988)

6) D. G. Luenberger : Observing the State of a Linear system, *IEEE Trans. Mili. Erectronics*, MIL-8-2, 74/80 (1964)

7) F. E. Tau : Observing the State of Non-linear Dynamic Systems, *Int. J. Control*, 17-3, 471/479 (1975)

8) J. J. E. Slotine, J. K. Hedrick and E. A. Misawa : On Sliding Observer for Non-linear Systems, *Trans.ASME, J. Dyn. Syst. Meas. Control*, **109**, 245/252 (1987)
9) R. Swiniarski : Variable Structure Observers for Linear and Nonlinear Uncertain Systems, *ACC*, 2149/2159, (1988)
10) J. Xu, H. Hashimoto and F. Harashima : An VSS Observer for Nonlinear Systems, *Proc. of the 27th SICE Annual Conf.*, **II**, 777/780 (1988)
11) J. Xu, H. Hashimoto and F. Harashima : Trajectory Control of Servo System with VSS Observer, *Proc. of the 27th SICE Annual Conf.*, **I**, 435/436 (1988)
12) A. Balestrino, G. De Maria and L. Sciavicco : An Adaptive Model Following Control for Robotic Manipulators, *Trans. ASME, J. Dyn. Syst. Meas. Control*, **105**, 143/151 (1983)

Appendix 1

From (3.4) and (3.6), we have

$$S_i\dot{S}_i = -\alpha_{i1}e_{i1}^2 - (k_{i1}|\, e_{i1}\, | - e_{i2}e_{i1}) \qquad (A1.1)$$

Hence from the assumption (3.7), the relation (3.8) is obtained. Q.E.D

Appendix 2

Let

$$V = e^T Pe + \sum_{i=1}^{n} 2\alpha_i c_i^T e_i \mathrm{sgn}(c_i^T e_i) \qquad (A2.1)$$

be a candidate of the Lyapunov function, where $P = \mathrm{diag}[P_i]$. Then from (3.4) and (A2.1).

$$\begin{aligned}\dot{V} = \sum_{i=1}^{n} [&e_i^T(P_iD_i + D_i^T P_i)e_i + 2e_i^T P_i b_i d_i(t)\\ &- 2e_i^T P_i k_i \mathrm{sgn}(e_{i1}) + 2\alpha_i c_i^T D_i e_i \mathrm{sgn}(e_{i1})\\ &- 2\alpha_i c_i^T k_i \mathrm{sgn}(e_{i1})^2] + 2e_i^T P_i b_i f_i^T(y)\{E(z) - E(x)\}.\end{aligned} \qquad (A2.2)$$

Taking into consideration (3.10),(3.11) and (3.14) and using the Lipschtz condition lead to

$$\dot{V} \le -\gamma_1 \| e \|^2 + \gamma_2 \| e \| \qquad (A2.3)$$

where

$$\begin{aligned}\gamma_1 &= c_0 - 2\mu\lambda_2 M_f\\ \gamma_2 &= 2\lambda_2 M_f.\end{aligned} \qquad (A2.4)$$

From (A2.1),

$$\lambda_1 \| e \|^2 \le V \le \lambda_2 \| e \|^2 + \lambda_3 \| e \| \qquad (A2.5)$$

holds. Hence from (A2.3) and (A2.5)

$$\dot{V} \le -\frac{\gamma_1}{\lambda_2}V + \frac{\gamma_1\lambda_3 + \gamma_2\lambda_2}{\lambda_2}\sqrt{\frac{V}{\lambda_1}} \qquad (A2.6)$$

is obtained. Thus

$$\begin{aligned}V \le (\frac{\beta}{\alpha})^2 + \frac{2\beta}{\alpha}(\sqrt{V(0)} - \frac{\beta}{\alpha})\exp(-\frac{\alpha}{2}t)\\ + (\sqrt{V(0)} - \frac{\beta}{\alpha})^2 \exp(-\alpha t).\end{aligned} \qquad (A2.7)$$

From this

$$V \le (\frac{\gamma_1\lambda_3 + \gamma_2\lambda_2}{\gamma_1\sqrt{\lambda_1}})^2 \qquad (A2.8)$$

holds as t tends to infinity, where

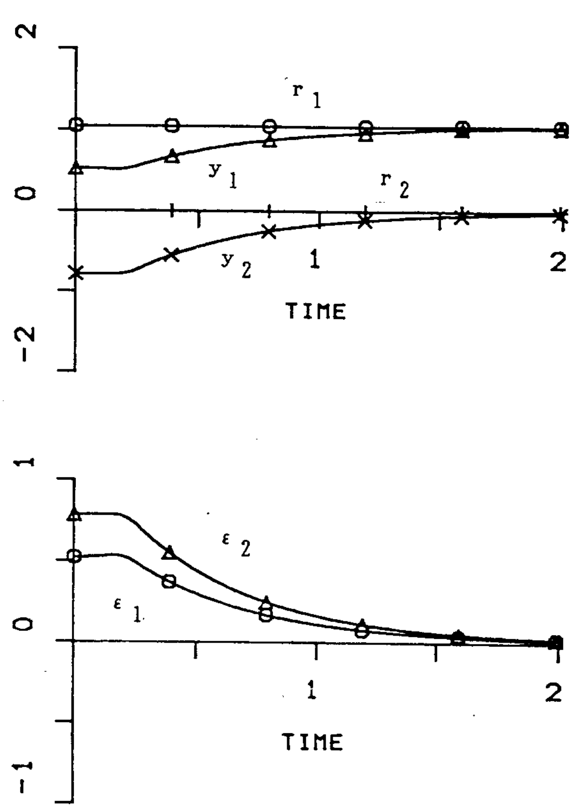

Fig.3 Control results

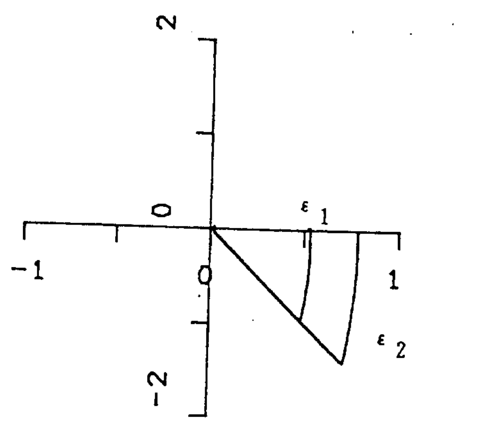

Fig.4 Phase plane trajectory ε_1 and ε_2

$$\alpha = \frac{\gamma_1}{\lambda_2}, \quad \beta = \frac{\gamma_1\lambda_3 + \gamma_2\lambda_2}{\lambda_2\sqrt{\lambda_1}} \qquad (A2.9)$$

From (A2.4),(A2.5) and (A2.8), we can conclude that the trajectory $e(t)$ goes toward the region :

$$\left\| e \right\| \leq \frac{\lambda_3}{\lambda_1} + \frac{2\lambda_2^2 M_d}{\lambda_1(c_0 - 2\mu\lambda_2 M_f)} \qquad (A2.10)$$

Therefor, if the condition (A2.7) is satisfied, $e(t)$ reaches the region

$$\left\| e \right\| \leq \min_i(k_{i1}) \qquad (A2.11)$$

within the finite time. Hence (3.7) holds. Moreover, It is shown from (A2.8) that trajectory stays in the region (A2.10) after arriving at this region. Finally, we can shown from (A2.1) and (A2.7) that the trajectory satisfies the relation :

$$\left\| e \right\| \leq \sqrt{\frac{1}{\lambda_1}\Big\{(\frac{\beta}{\alpha})^2 + \frac{2\beta}{\alpha}(\sqrt{V(0)} - \frac{\beta}{\alpha})\exp(-\frac{\alpha}{2}t) + (\sqrt{V(0)} - \frac{\beta}{\alpha})^2\exp(-\alpha t)\Big\}}. \qquad (A2.12)$$

Thus, trajectories starting from the initial region (3.9) stays in Ω for $t \in [0, \infty)$. Q.E.D

Appendix 3

(i) From (4.6) and (4.13),

$$\ddot{\varepsilon}_i + \alpha_1\dot{\varepsilon}_i + \alpha_2\varepsilon_i = \tilde{w}_i(t) \qquad (A3.1)$$

holds. Taking into consideration (4.8) and stable polynomial $\Phi(p)$, (4.20) is obtained.

(ii) On the interval $t \geq T$, sliding observer has the sliding mode, i.e,

$$e_i = 0, \qquad \dot{e}_i = 0 \qquad (A3.2)$$

or

$$\dot{z}_{i1} = \dot{x}_{i1}, \quad z_{i1} = x_{i1}. \qquad (A3.3)$$

Hence, from (4.17) and (4.18),

$$\hat{\dot{\varepsilon}}_i = \dot{\varepsilon}_i. \qquad \hat{S}_{ui} = S_{ui} \qquad (A3.4)$$

holds. Then, (4.14)~(4.16) can be represented such as

$$\hat{\psi}_{i1} = -M_{i1}\mathrm{sgn}(\dot{\varepsilon}_i S_{ui}) \qquad (A3.5)$$

$$\hat{\psi}_{i2} = -M_{i2}\mathrm{sgn}(\varepsilon_i S_{ui}) \qquad (A3.6)$$

$$\hat{\psi}_{i3} = -M_{wi}\mathrm{sgn}(\varepsilon_i S_{ui}). \qquad (A3.7)$$

Further, from (4.6), (4.9), (4.13) and (A3.4), we have

$$S_{ui}\dot{S}_{ui} = S_{ui}[\{\hat{\psi}_{i1} - (\alpha_1 - c_{si})\}\dot{\varepsilon}_i + (\hat{\psi}_{i2} - \alpha_2)\varepsilon_i + \hat{\psi}_{i3} + \tilde{w}_i]. \quad (A3.8)$$

Hence we can obtain (4.21) from (4.8) , (4.19) and (A3.5) ~ (A3.7).

(iii) From (4.6)~(4.13) and (A3.4),

$$S_{ui}\ddot{\varepsilon}_i = S_{ui}[(\hat{\psi}_{i1} - \alpha_1)\dot{\varepsilon}_i + (\hat{\psi}_{i2} - \alpha_2)\varepsilon_i + \hat{\psi}_{i3} + \tilde{w}_i]. \quad (A3.9)$$

is obtained. Then the relation (4.22) is proved by using (4.8), (4.19) and (A3.5)~(A3.7).