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A General Dynamic Iterative Learning Control Scheme with High-Gain Feedback

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A general dynamic iterative learning control scheme is proposed for a class of nonlinear systems. Relying on stabilizing high-gain feedback loop, it is possible to show the existence of Cauchy sequence of feedforward control input error with iteration numbers, which results in a uniform convergence of system state trajectory to the desired one.

1 Introduction

Recently, iterative learning control schemes for a class of repeatable dynamic systems have been studied. Besides the simplicity and straightforwardness, iterative learning control does not require exact description of system dynamics so that it can be applied to the various systems which are hard to control under varying operation conditions. For example, iterative learning techniques are applied to robot motion control problems [3,5,6,7,8]. Amongst them Arimoto et.al. used a general learning method to a robot manipulator in which time derivative of system output error is used to modify the control input torque for the next trial[2]. To overcome difficulties and constraints in the choice of control gain matrix in Arimoto's method, Oh et.al. adopted an parameter estimator with iteration number[8].

On the other hand, high gain feedback concept was introduced to establish desired tracking error bound for system state trajectory [3,4,6,7]. As a torque generator Miller et.al.[7] utilized CMAC memory [1] to learn approximate inverse model of robot in appropriate regions of the state space. T. Kuc and K. Nam also utilized a high-gain feedback control and a version of CMAC memory with a mapping rule to reduce the memory size, and demonstrated convergence of the system response to a desired one by computer simulation results. The simulation results shows possibility of incorporating high gain feedback with learning control for a class of repeatable nonlinear dynamic systems.

In this paper we study a possible learning control scheme which is aided by high gain feedback to stabilize the uncertain system state so that a learning(update) rule modifies control input for the next iteration. A brief outline of the paper is as follows. In section II we describe the target system dynamics and problem formulation. Motivations are also explained for a possible approach to the problem solving. Then the existence of a feedback law for local stabilization of the target dynamic system is explained through a conceptual discussion of Liapunov transformation in section III. Moreover, a constant high-gain feedback controller which stabilizes the nonlinear system in the global sense is designed for the closed feedback loop. Relying on the stablizing feedback law, we see that it is possible to construct a dynamic iterative learning control scheme for the target system in section IV. Convergence analysis of the overall learning control scheme is given in section V, which forms a uniform convergence rate with respective to a certain kind of function norm. In Section VI the overall structure of the proposed control scheme and learning procedure is illustrated briefly. Section VII contains concluding remarks.

2 System dynamics and problem formulation

Consider a class of nonlinear dynamical systems described by the following set of equations.

$$\dot{x}(t) = f(x(t)) + g(x(t))u \tag{1}$$

where

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$f = \begin{bmatrix} x_2 \\ f_1(x(t)) \end{bmatrix}$$

$$g = \begin{bmatrix} 0 \\ g_1 \end{bmatrix}$$

$x(t) \in R^{2n}$, $f_1 \in R^n$, $g_1 \in R^{n\times n}$ Lipshitz continuous, and g_1 positive or negative definite for all $t \in [0, t_f]$. That is,

$$|f(x^1(t)) - f(x^2(t))| \leq \kappa|x^1(t) - x^2(t)|$$

$$||g_1(x^1(t)) - g(x^2(t))|| \leq \alpha|x^1(t) - x^2(t)| \quad 0 \leq t \leq t_f$$

where $|\cdot|$ denotes euclidean norm and a matrix norm is defined as

$$||A|| = inf\{M : |Ax| \leq M|x| \text{for all } x \neq 0\}$$

for $v \in R^n, A \in R^{n\times n}$.

Suppose that we are to solve the following.

Problem Statements: Let $x_d \in W$ being a closed and bounded subset of R^{2n} and $dist(\partial W, x_d) \geq s > 0$ for a pre-specified state trajectory $x_d(t)$ of the system (1). For the system dynamics (1), which may contain uncertainty of structural or parametric, we want to find a piece-wise continuous control input $U_d(t)$ for the dynamic system (1) to track the desired state trajectory $x_d(t)$ within a given error bound of $\epsilon(0 \leq \epsilon < s)$ for all $t \in [0, t_f]$.

In general, however, since the existence of an explicit solution is rarely the case in nonlinear control problems of tracking, even if the system dynamics are known exactly, it is not easy to find a control $U_d(t)$ explicitly for the target trajectory $x_d(t)$. Moreover, there exists chances that a precise description of system model is not known due to structural uncertainties in system dynamics or parametric uncertainties. Under such an ambiguous environments if we are to control tightly the system trajectory to maintain a specified error bound of tolerance all the control time, with one of usual approaches commonly utilized as well as adaptive control method, it is not easy to achieve the goal without any difficulty at first hand. To counteract possible difficulties caused by uncertain operating conditions and other constraints in practical situation we are going to construct *a dynamic iterative learning control* method, which is one of simple and straight forward strategies to shoot the problem.

By iterative learning we mean that via a learning scheme and an update rule for a control input to the system (1) the closed loop operation or open loop operation is repeated iteratively with the same initial condition for the time interval of $[0, t_f]$. Therefore, we are aiming at reducing the trajectory errors progressively as iteration number j increases.

3 Existence of stabilizing state feedback

As one might suppose, in order to introduce an iterative learning process to the system (1) a stable closed loop operation should be guarenteed at the initial stage of learning. Hence, our goal in this section is to design a constant linear state feeback to stabilize target system (1).

3.1 A local state feedback

When we consider only a local property of the nonlinear system, it is worth while to discuss a conceptual existence of simple feedback in local sense. That is, we pay a special attention to the local properties of the nonlinear dynamic system model (1). Linearizing system (1) around the desired state trajectory $x_d(t)$ at the $j-th$ trial yields an error equation of linear time-varying system which is an approximation of a perturbed system of (1).

$$\delta\dot{x}^j(t) = A(t)\delta x^j(t) + B(t)\delta U^j(t) \qquad (2)$$

where

$$A(t) = \begin{bmatrix} 0 & I \\ A_1 & A_2 \end{bmatrix}$$

$$B(t) = \begin{bmatrix} 0 \\ g_1 \end{bmatrix}_{x_d}$$

$$\begin{aligned} A_1 &= \frac{\partial f}{\partial x_1}|_{x_d(t)} + \frac{\partial g_1}{\partial x_1}|_{x_d(t)}U_d \\ A_2 &= \frac{\partial f}{\partial x_2}|_{x_d(t)} + \frac{\partial g_1}{\partial x_2}|_{x_d(t)}U_d \\ \delta x^j(t) &= x_d(t) - x^j(t) \\ \delta U^j(t) &= U_d(t) - U^j(t) \end{aligned}$$

for $t \in [0, t_f]$.

Then, most of the local properties of the nonlinear system (1) can be deduced from the linear system (2). In other words, if the linear system (2) is asymtotically stable to the desired trajectory with $\delta U^j(t) = 0$, then the nonlinear system (1) is asymptotically stable to the same trajectory. If this is not so for $\delta U^j(t) = 0$, but a feedback control $F\delta x^j(t)$ can be found to make the linear system (2) asymptotically stable for any $t \in [0, t_f]$, then the same feedback control will make the nonlinear system (1) asymptotically stable. Moreover, since the system matrices $A(t), B(t)$ are periodic with period T of t_f with respect to iteration such that

$$\begin{aligned} A(t+T) &= A(t) \\ B(t+T) &= B(t) \qquad 0 \leq t \leq T \end{aligned}$$

, there exists a linear time invariant system equivalent to the periodic time-varying system (2) via Liapunov transformation which preserves stability characteristics. In order words, considering the periodicity imposed by the iterative learning process to the system (1) , we may find a linear feedback law which stabilizes the system in local sense, if a Liapunov transform is known for the controllable pair $\{A(t), B(t)\}$.

3.2 A global state feedback

Although we have discussed the existence of a local feedback law via Liapunov transformation, it is not easy to construct a Liapunov transform in practice. However, as a precise generalization of Gusev's result [4], it is shown that a stable closed-loop operation can be achieved by introducing a high-gain concept to feedback loop.

Theorem 1: Suppose that the following conditions are satisfied for all $t \in [0, t_f]$

i) $|f_1(x^1(t)) - f_1(x^2(t))| \leq \kappa_1 |x_1^1(t) - x_1^2(t)| + \kappa_2 |x_2^1(t) - x_2^2(t)|$

iii) $0 < \lambda_1 I \leq g_1 \leq \lambda_2 I$

Then there exists a constant high-gain state feedback control such that the initial tracking error of iterative learning scheme is bounded with a given accuracy $e > 0$

$$|x(t) - x_d(t)| \leq e < s \qquad t \in [0, t_f]$$

The state feedback and gain matrix are given by the following equations and inequalities.

$$u = K\tilde{x} \tag{3}$$

where

$$\begin{aligned} \tilde{x} &= x - x_d \\ K &= \left[K_1 \vdots K_2\right]_{n \times 2n} \\ K_1 &= -adb^{-1} I_{n \times n} \\ K_2 &= -db^{-1} I_{n \times n} \\ a &> 0 \\ b &\geq \lambda_2 \\ d &> (\frac{\kappa_1}{a} + 2(a + \kappa_2))\frac{b}{\lambda_1} \end{aligned} \tag{4}$$

Further, tracking error bound is conditioned by

$$\sqrt{1 + 4a^2} \frac{\hat{d}}{a(d - (\frac{\kappa_1}{a} + 2(a + \kappa_2))\frac{b}{\lambda_1})} \leq e < s \tag{5}$$

where $\hat{d} = |g_d U_d|$.
The proof can be given in similar manner as in [4].

Remark: 1) Condition *i)* is always satisfied for the class of Lipshitz continous function stated in section II.
2)By letting $b = \lambda_2$ in (4), we get

$$d > (\frac{\kappa_1}{a} + 2(a + \kappa_2))BR = (\frac{\kappa_1}{a} + 2(a + \kappa_2))(\frac{BW}{\lambda_1} + 1) \tag{6}$$

where $BR = \lambda_2 / \lambda_1$ and $BW = \lambda_2 - \lambda_1$ denote input gain bandratio and bandwidth. Therefore, the inequality (6) implies that d is lower bounded by BR, or equivalently BW.

3) In case that g_1 is negative definite the same argument as in *theorem1*) can be applied by multiplying negative sign to the feedback gain matrices.

4 A general dynamic iterative learning control scheme

Utilizing a high-gain feedback concept seems to be very attractive as one of useful approaches to the given nonlinear tracking problem. However, from (5) we see that it may not be possible for the initial tracking error e to be driven to zero with only high-gain state feedback. That is, in practical application, we can not let the feedback gain d indefinitely large due to the actuator saturation and noise vulnerability etc. , so that initial tracking error bound given by (5) should be traded off with gain limitation. This problem of trading off in between setting feedback gain and specifying initial error bound can be overcome by combining an iterative learning process, which is another reason for why we prefer a simple stragety of iterative learning control in nonlinear tracking problem. At the initial stage of learning an appropriate gain is chosen not to be too large, which may results in an initial error bound of e much greater than specified tolerance ϵ. Then learning process pull the state attractor to the inner region of nominated error bound ϵ. It is shown in the next section that a dynamic iterative learning scheme works to guarentee convergence to the target trajectory, even though initial error exists due to finite feedback gain.

Now, we are in a position to outline a dynamic iterative learning scheme. Recalling that our control objective is to track a given desired state trajectory through an iterative learning, we need to find an adjustable learning method for the control input. As one of simple and feasible candidates for the piece-wise continuous control input and update rule, we set at the $j-th$ trial for on-line learning control

$$U^j(t) = \quad H^j(t) + E^j(t). \tag{7}$$

$$E^j(t) = \quad K[x^j(t) - x_d(t)] \tag{8}$$

$$H^{j+1}(t) = \quad H^j(t) + \beta E^j(t) \tag{9}$$

, where $x_d(t), x^j(t)$ for $j = 1, 2, \ldots$ denote desired and actual state trajectories and $H^j(t)$ is feedforward control input to be adjusted, $K \in R^{n \times 2n}$ is a high gain feedback matrix given in (3) and $U, H, E \in R^n$ with a positive constanst $\beta (< 1)$. Further, as initial conditions we let $x_d(0) = x^j(0)$ for all j and $H^1(t) = 0$ for all $t \in [0, T]$. With respect to this kind of control law for the system (1) we give a uniform convergence analysis in next section.

Remark) To get an insight to the control law given above and delineate an intuitive reasoning for why we choose such an update rule as the one in (9), define an index functional.

$$J_t = \frac{1}{2} \sum_{r=1}^{\infty} |U_d(t) - H^r(t)|^2 \tag{10}$$

where $U_d(t)$ is a desired control input to track $x_d(t)$ for $t \in [0,T]$. Applying gradient descent rule to the index(), we obtain

$$\begin{aligned} H^{j+1}(t) &= H^j(t) - \beta\frac{\partial J_t}{\partial H^j(t)}, \\ &= H^j(t) + \beta(U_d(t) - H^j(t)), \end{aligned} \quad (11)$$

where β is the training factor of constant. Hence if U_d is known, learning can be proceeded with the above learning rule (11). However, since precise system dynamics are not known apriori we can not use (11) directly. Instead, we consider the state error equation to replace (11) with a readily available equation of known terms only. To be more specific, considering linearized system (2) and substituting (7),(8) for $U^j(t)$ and $E^j(t)$ yields the following.

$$\delta\dot{x}^j(t) + (B(t)K - A(t))\delta x^j(t) = B(t)(U_d(t) - H^j(t)) \quad (12)$$

Hence $U_d - H^j$ may be considered as control input error which should be removed for the state error to converge. Recalling that we have shown that the existence of high-gain feedback in *theorem 1)* and $\{A(t), B(t)\}$ is a controllable pair , we can also find a sufficiently large $K_{n\times 2n}$ so that the linearized system dynamics (12) can be also dominated by the dynamics introduced via state feedback. That is, time-varying terms $\{A_1(t), A_2(t)\}$ may be suppressed by feedback dynamics imposed on by $\{B(t)K_1, B(t)K_2\}$ respectively, so that this closed loop operation induces a uniformly asymptotic stability of autonomous part of the system (12) for all $t \in [0,T]$ and equaivalently gurantees a boundedness of tracking error around the desired trajectory at initial stage of training. Therefore, if we combine an update rule for feedforward control input $H^j(t)$ so that the error bias of control input $U_d(t) - H^j(t)$ may be canceled, the overall closed loop system can be made to converge asymtotically with iteration number. Motivated by this observation and observing $B(t)$ nonsinglar, we replace the unknown input error $U_d(t) - H^j(t)$ in (11) by $K\delta x^j(t)$ of the only term known and dominant in the system dynamics (12), which leads to an update rule with known terms only,

$$H^{j+1}(t) = H^j(t) + \beta E^j(t) \qquad j = 1,2,3,\ldots \quad (13)$$

where $0 < \beta < 1$,$H^1(t) = 0$ for $t \in [0,T]$. Roughly speaking, this update rule can be considered as an approximation of unknown control input errors by known high gain feedback error terms. As a training factor of positive constant, generally β is chosen smaller than unity due to sensitivity. However, in case that system operation is smooth so that the difference in derivatives of state trajectories with iteration number may be sufficiently small , it is shown that $0 < \beta < 2$ for the system state error to reduce asymtotically in local sense (see [6]).

5 Convergence of the learning control scheme

In order to show the validity of this control scheme we define a vector valued norm and matrix norm for the control time period.

$$\begin{aligned} |v|_m &= max_{0\le t\le T}|v| \\ ||A||_m &= max_{0\le t\le T}||A|| \end{aligned}$$

where $A = (a_{ij})_{n\times n}, v = (v_1, \ldots, v_n)$ for $m \le n$.

Then, the above learning control scheme is shown to be convergent in the following statements.

Theorem 2: Consider a vector valued function norm

$$|v(t)|_\mu = sup_{t_0\le t\le T}\{e^{-\mu t}|v(t)|\} \quad (14)$$

and assume that $|U_d(t)|_\mu \le U_0$ for all $t \in [0,T]$ and $x_d(0) = x^j(0)$ for all $j = 1,2,\ldots$. Then, with the set of iterative learning control scheme (7),(8),(9) for the system (1) there exists a cauchy sequence of control input error in the sense that for positive constants μ and $\rho(0 \le \rho < 1)$ the following inequality holds

$$|U_d(t) - H^{j+1}(t)|_\mu \le \rho|U_d(t) - H^j(t)|_\mu \qquad 0 \le t \le T \quad (15)$$

Proof) Firstly, note that $U_d(t)$ is bounded and since overall closed loop system is stable along the desired trajectory, δx^j is bounded so that H^j for all $j = 1,2,\ldots$ is bounded. Hence $|U_d - H^j|$ is bounded for all j.

Substituting (7),(8) for U^j and E^j in state error equation (16), we have

$$\begin{aligned} x_d(t) - x^j(t) &= \int_0^t (f_d + g_dU_d)d\tau - \int_0^t (f^j + g^jU^j)d\tau \\ &= \int_0^t ((f_d - f^j) + (g_d - g^j)U_d - g^jK(x_d(\tau) - x^j(\tau)))d\tau \\ &+ \int_0^t g^j(U_d(\tau) - H^j(\tau))d\tau \end{aligned} \quad (16)$$

where

$$f_d = f(x_d(\tau))$$
$$f^j = f(x^j(\tau))$$

Multiplying both sides of (16) by $e^{-\mu t}$

$$\begin{aligned} e^{-\mu t}(x_d(t) - x^j(t)) &= \int_0^t e^{-\mu(t-\tau)}e^{-\mu\tau}\{(f_d - f^j) + (g_d - g^j) \\ U_d &- g^jK(x_d(\tau) - x^j(\tau))\}d\tau \\ &+ \int_0^t e^{-\mu(t-\tau)}g^je^{-\mu\tau}(U_d(\tau) - H^j(\tau))d\tau \end{aligned} \quad (17)$$

This yields the following inequality

$$|x_d(t) - x^j(t)|_\mu \le ||g||_m\frac{1 - e^{-\mu t}}{\mu}|U_d(t) - H^j(t)|_\mu$$

$$+\int_0^t e^{-\mu(t-\tau)}(\kappa_m + \alpha_m|U_d|_m + ||gK||_m)|x_d(\tau) - x^j(\tau)|_\mu d\tau \tag{18}$$

Applying Belmann-Gronwall's lemma it becomes

$$|x_d(t) - x^j(t)|_\mu \le \rho_0|U_d(t) - H^j(t)|_\mu \tag{19}$$

, where

$$\rho_0 = \frac{1-e^{-\mu t}}{\mu}||g||_m exp(\frac{1-e^{-\mu t}}{\mu}(\kappa_m + \alpha_m|U_d|_m + ||gK||_m)) \tag{20}$$

Note that boundedness of right hand side of the inequalities (18) is guarrented by the stability along the desired trajectory.

On the other hand, from (1),(7),(8),(9)

$$\begin{aligned}\dot{x}_d(t) - \dot{x}^j(t) &= f_d - f^j + g_d U_d(t) - g^j U^j(t)\\ &= (f_d - f^j) + g_d U_d(t) - g^j(H^j(t) + E^j(t))\\ &= (f_d - f^j) + g_d U_d(t) - g^j(H^{(j+1)}(t) + (1-\beta)E^j(t))\\ &= (f_d - f^j) - (1-\beta)g^j K(x_d(t) - x^j(t))\\ &\quad + g^j(U_d(t) - H^{(j+1)}(t)) + (g_d - g^j)U_d \end{aligned} \tag{21}$$

It becomes

$$\begin{aligned} U_d(t) - H^{j+1}(t) &= g^{\#}(\dot{x}_d(t) - \dot{x}^j(t)) - g^{\#}((f_d - f^j)\\ &\quad + (g_d - g^j)U_d) + (1-\beta)K(x_d(t) - x^j(t)) \end{aligned} \tag{22}$$

where $g^{\#}$ denotes a generalized inverse which always exists, for g is full rank for all $t \in [0, T]$.

Since f is bounded and control input U is a piecewise continuous function of x, from the closed loop system for (1) we may possibly assume the following:

$$|\dot{x}_d(t) - \dot{x}^j(t)| \le \zeta(t)|x_d(t) - x^j(t)| \tag{23}$$

With this inequality multiplying both sides of (22) by $e^{-\mu t}$ and taking function norm leads to the inequality

$$|U_d(t) - H^{j+1}(t)|_\mu \le \omega_0|x_d(t) - x^j(t)|_\mu \tag{24}$$

where

$$\begin{aligned}\omega_0 &= ||g^{\#}||_m(\kappa_m + \zeta_m + \alpha_m|U_d|_m) + (|1-\beta|)||K||_m\\ \zeta_m &= max_{t_0 \le t \le t_f}\zeta(t)\end{aligned}$$

Combining this with (19) yields

$$|U_d(t) - H^{j+1}(t)|_\mu \le \rho|U_d(t) - H^j(t)|_\mu \tag{25}$$

where $\rho = \rho_0\omega_0$. Note that since ω_0 is finite , we can choose a sufficiently large μ in ρ_0 to yield $0 \le \rho < 1$. Hence, applying inductive relation to (25) yields the desired results.

$$\begin{aligned}|U_d(t) - H^{j+1}(t)|_\mu &\le \rho^j|U_d(t) - H^1(t)|_\mu\\ &= \rho^j|U_d(t)|_\mu\\ &\le \rho^j U_0\end{aligned} \tag{26}$$

Therefore, we conclude that $|U_d - H^j|_\mu \to 0$ uniformly as $j \to \infty$ for all $t \in [0, T]$. This completes the proof.

In addition to *theorem* 2) we have the following corollarys as a natural reasoning of the theorem.

Corollary 1) With assumptions in *theorem*1) the system state converges uniformly, that is, $|x_d(t) - x^j(t)|_\mu \to 0$ uniformly as $j \to \infty$ for all $t \in [0, T]$.

Corollary 2) For the system (1) with the learning control scheme (7),(8),(9) there exists a cauchy sequence of feedforward control input $\{H^j(t)\}$ with respect to iteration axis.

Proof of Cor.1),2) *Cor*.1) is obvious from the inequality (19) and the result of *theorem*2). That is, since the error bias input of closed loop system converges uniformly asymtotically as $j \to \infty$, the overall closed loop operation is also uniformly asymtotically stable. Alternatively, *Cor*.1) can be proved in a similar maner as in *theorem*2). *Cor*.2) results from the update rule (9) and *Cor*.1). That is, $|H^{j+1}(t) - H^j(t)|_\mu \to 0$ uniformly as $j \to \infty$ for all $t \in [0, T]$.

Remark) The most essential feature of the control scheme is that a precise description of system dynamics is not necessary ,which results in flexibility and adaptability of controller under varying operation conditions. What we should concern in application is only to bound the system states within a positively invariant stable regions by taking full advantage of high gain feedback from which feedforward controller pulls the attractor to the equilibrium state via control input error bias reduction. This feature can be compared with other types of general learning schemes : most of general learning rules do not involve any system dynamics at all, or even if they do, not exlpicitly. On the contrary, the above update rule (9) contains dominant system dynamics via high gain feedback. This is why we call the control method *a dynamic iterative learning control.*

6 Synthesis of dynamic iterative learning control scheme

The overall control scheme can be organized as in Fig.1. It consists of four main blocks. That is, a system state trajectory planner, high gain feedback controller, an update rule, feedforward controller. This is a kind of hybrid type controller in which high gain feedback controller stabilizes the uncertain dynamics and feedforward control input sequence generator reduces the system state error progressively with iteration number. After perfect training, the feedforward controller would learn an inverse dynamics of the plant and then play a major role in tracking control, while the feedback controller takes the main role at the initial stage of learning where a large tracking error may exist. Note that the overall learning control scheme is self-organized via stable closed loop operation to complete the control objectives. That is, it needs no external input, as it does in other learning methods[2,3,8], to perturb the states of plant for learning adaptation except a command state trajectory. This feature adds robustness to the controller by avoiding an initial perturbation input which may collapse the stable operation of the learning controller. In addition, we mention that the feedforward controller can be equiped with one of neural network architectures as long as its convergence property is suitable for real time control. In this application, we utilized a version of CMAC in[6]. Also note that to prevent memory size from increasing indefinitely with control time duration or sampling frequency , we may adopt the same kind of mapping rule as in[6] defined for the memory cells to store and update feedforward control input sequence as distributed data. We summerize the iterative learning procedure briefly in the following.

Step 1. Determine $x_d(t)$ a desired state trajectory and $e, \epsilon (< s)$ initial and final tracking error bound.

Step 2. For a given initial error tolerance e, design high-gain feedback and feedforward learning block structure.

Step 3. Set $j = 0$.

Step 4. Set $j = j + 1$.

Step 5. Start the learning process.

Step 6. If

$$|x^j - x_d|_m \leq \epsilon$$

,then stop.

Else go to *Step4*.

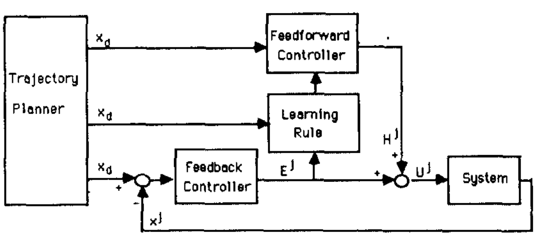

Fig.1 Learning Controller with High-Gain Feedback

7 conclusion

We proposed a general dynamic iterative learning control method with high-gain feedback loop for a class of repeatable nonlinear systems. In the control scheme the most dominant closed loop system dynamics is fully taken into account via feedforward input learning procedure. Moreover, since any external input is not necessary at the initial stage of learning, robustness characteristic might be enhanced in comparison to the other type of general learning methods. This self-organizing ptoperty and role transition property mark two main features of the hybrid type learning controller. Although we focused on the given nonlinear system class (1) ,this learning control scheme can also be adapted effectively for a class of repeatable or periodic uncertain linear systems as long as they are controllable. More emphases for further study should be laid on the effect of disturbances to the performance of the learning scheme and extension of the target class of nonlinear systems for learning control to a more general class.

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