Force acting on a high Tc superconductor at 77K

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Abstract
The force acting on high Tc superconductors at 77K is measured and analyzed numerically.Both values are compared, and the difference between them is discussed. The forces, acting on a superconducting disk (thickness:1[mm],diameter:12[mm]) in an axially-symmetric magnetic field produced by a solenoid or a permanent magnet ring, are measured at 77K. The disk is an YBCO high Tc superconductor. The discrete surface current method(DSCM) is formalized for an axially-symmetric magnetic field. The forces of the superconducting disk in the magnetic field are analyzed using the DSCM, assuming that the disk is a perfect diamagnetic body. When the bottom side of the disk is separated 8[mm] from the top side of the solenoid, and the magnetic field applied on the center of the bottom side of the disk is 96[G], the measured value and the calculated value of the force are 96 and 496[mgf], respectively. The difference between them is caused by a non-perfect diamagnetism of the high Tc superconductor at 77K. It is proposed that a real force acting on high Tc superconductors at 77K can be estimated on the basis of a measured magnetic susceptibility of the high Tc superconductor at 77K and a calculated force of a perfect diamagnetic body.

1. Introduction

In recent years, various high Tc superconductors have been discovered[1,2], and their critical temperatures are higher than 77K, which is the temperature of liquid nitrogen. Liquid nitrogen as a coolant has many advantages over liquid helium[3]. The relative heat of vaporization of liquid is about 60 times greater than that of liquid helium. Furthermore, there would be a large saving in cost: a litre of liquid helium costs between $5 and $10, while a litre of liquid nitrogen costs only about $0.25. Hence, the combined greater heat of vaporization and lower price of nitrogen results in a reduction by a factor of 1200 in the daily cost for operating a superconducting device or system. Therefore, it is expected that the high Tc superconductors are applied to various fields of science[4].

In applications of superconductivity phenomena, there are magnetic suspension[5] and magnetic levitation[6] using Meissner effect. The application of Meissner effect have advantages over other magnetic suspension or levitation system[7]: there are no needs of mechanical supports and controls for stable levitation, compared with levitation using permanent magnets and electromagnets. However, Meissner effect has not been used widely because of a cooling by liquid helium and a complex thermal insulation system.

The force produced by Meissner effect of low Tc superconductors (perfect diamagnetic body) has been analyzed and/or experimented[8-10]. In YBCO high Tc superconductors, levitation of a magnet over a type II superconductor has been analyzed analytically in a complete Meissner model and a flux penetration model[11]. In high Tc superconductors, it is known that it is difficult to acquire the perfect diamagnetism at 77K, and they are often used in the range of Hc_1 to Hc_2. In order to apply the force acting on high Tc superconductors at 77K, the force have to be analyzed and measured, and both values should be compared and discussed.

In section 2 of this paper, the forces of the superconducting disk in the magnetic field pro duced by a solenoid or a permanent magnet ring are measured. The disk is an YBCO high Tc superconductor, and it is cooled by liquid nitrogen.

In section 3, the discrete surface current method (DSCM)[10] is formalized for an axially-symmetric magnetic field. The forces of a superconducting disk in the magnetic field are analyzed using the DSCM, assuming that the disk is a perfect diamagnetic body.

In section 4, the measured value and the calculated value of the forces are compared. The difference between them is discussed, and it is proposed that a real force acting on high Tc superconductors at 77K can be estimated on the basis of a measured magnetic susceptibility of the high Tc superconductors at 77K and a calculated force of a perfect diamagnetic body.

2. Experimentals

2.1 Experimental setup

The force on the high Tc superconductor is measured as shown in Fig.1. The superconducting disk is hanging from a digital scale, with which the disk is weighed. The disk is cooled at 77K by liquid nitrogen. An axially-symmetric magnetic field is produced by a solenoid or a permanent magnet ring magnetized in an axially the z direction. The axes of the disk and the magnetic field coincide. Since the disk repulses the magnetic flux by Meissner effect, the repulsion of the magnetic flux causes a repulsive force. The repulsive force F(B) acting on the disk at a magnetic field B, is obtained by subtracting a value W(B) indicated on the scale at B from a value W(0) at zero field.

$$F(B) = W(0) - W(B) \qquad (1)$$

,when a separation,d between a bottom side of a disk and a top side of a solenoid or a magnet is fixed.

The force is measured as a function of a magnetic field B applied on a disk. The strength of the magnetic field is changed either by adjusting a separation distance,d, or by adjusting a current of a solenoid.

In Fig.2, a superconducting disk is an YBCO high Tc superconductor whose Tc is about 90K. The measured critical current density at 77K is 176[A/cm²]. The measured magnetic susceptibilities at 77K and 4.2K are -0.23 and -0.68, respectively, in the magnetic field, 10[G]. The diameter and thickness of the disk are 12 and 1[mm], respectively. The inner and outer diameters and length

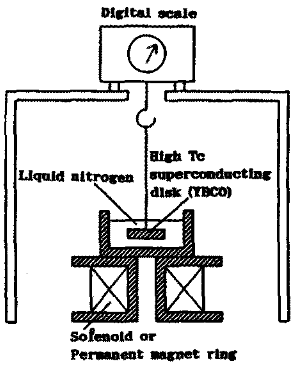

Fig.1 Experimental Setup.

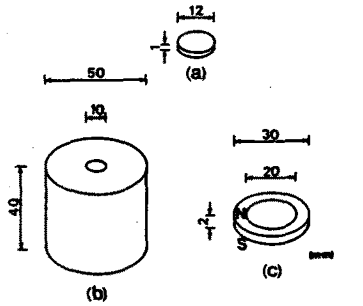

Fig.2 Superconductor, solenoid and permanent magnet. a) YBCO high Tc superconductor b)air-cored solenoid c)permanent magnet

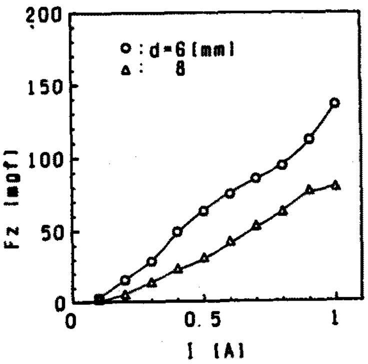

Fig.3 Measured force in a solenoid as a function of a solenoid current. A separation is fixed at 6 and 8[mm].

of the air-cored solenoid are 10, 50 and 40[mm], respectively. The inner and outer diameters and thickness of the permanent magnet are 20, 30 and 2[mm], respectively. The magnet is a rare earth magnet and its coercivity is 640[kA/m]. It is magnetized in an axial direction.

2.2 Experimental results

2.2.1 Magnetic field produced using a solenoid

The forces are measured as a function of a solenoid current when a separation,d is fixed at 6 or 8[mm]. As shown in Fig.3, the forces increase as the current increases from 0.1 to 1[A]. When the magnetic fields are produced using the solenoid, the magnetic fields are measured without the superconducting disk at the center of the bottom side of the disk, and shown in Fig.4.

In Fig.3, while the solenoid current is small, namely the magnetic fields applied on the disk are low, the forces roughly increase in proportion to a square of the current. However, as the current increases more than 0.5[A], the forces increase slowly. The forces are larger when the separation is 6[mm] than when the separation is 8[mm]. The reason is why the magnetic fields applied on the disk are higher when the separation is 6[mm]. When the separations are 6 and 8[mm], the forces at 1[A] are 149 and 96[mgf], respectively. At this time, the magnetic fields without the disk are 117 and 96[G], respectively, at the center of the bottom side of the disk.

2.2.2 Magnetic field produced using a permanent magnet

The force and magnetic field are measured as a function of a separation from 4 to 7[mm]. The magnetic field is produced using a permanent magnet ring. In Fig.5, as the separation becomes large, namely, the magnetic field applied on the disk becomes low, the force decreases. When the separation is 6[mm] and the magnetic field applied on the disk is 130[G], the force is 147[mgf]. The force referred to the magnetic field is less than that of a solenoid, because the configuration of the magnetic flux line produced by a permanent magnet ring is different from that of a solenoid.

3. Numerical analysis using DSCM

3.1 Formulation of DSCM

Assuming that an axis of a superconducting disk is coincident with an axis of an axially-symmetric magnetic field, the magnetic field in a three dimensional cylindrical coordinate can be formalized to the field in a two dimensional R-Z coordinate. The magnetic fields at an arbitrary point, P in Fig.6, produced by a current ring, are expressed as Eq.(2) and (3)[12].

$$B_r(r,z)=\frac{\mu I}{2\pi}\frac{(z-b)}{r\{(a+r)^2+(z-b)^2\}^{1/2}}\left\{-K+\frac{a^2+r^2+(z-b)^2}{(a-r)^2+(z-b)^2}E\right\} \quad \cdots\cdots(2)$$

$$B_z(r,z)=\frac{\mu I}{2\pi}\frac{1}{\{(a+r)^2+(z-b)^2\}^{1/2}}\left\{K+\frac{a^2-r^2-(z-b)^2}{(a-r)^2+(z-b)^2}E\right\} \quad \cdots\cdots(3)$$

$$k^2=\frac{4ar}{(a+r)^2+(z-b)^2} \quad (4)$$

,where $K(k^2)$ and $E(k^2)$ are the first and second elliptical integrals, respectively.

Since the discrete surface currents in the DSCM[10] are placed on the surfaces of a superconductor, the current rings are placed discretely and equidistantly on the surfaces. The contour points are placed between the discrete surface currents as shown in Fig.7. Assuming that the superconductor is a perfect diamagnetic body, the boundary condition is that the normal components of the magnetic field on the superconductor's surfaces are zero. It is naturally satisfied that the amount of the surface currents is always zero because the shape of the discrete surface currents is a ring.

The values of the discrete surface currents are determined to satisfy the boundary condition. The force acting on the superconductor in the mag-

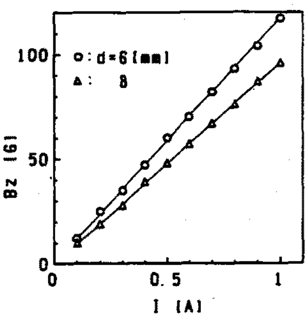

Fig.4 Measured magnetic field as a function of a solenoid current. A separation is fixed at 6 and 8[mm], and a magnetic field is measured at a center of a bottom side of a superconductor without a superconductor.

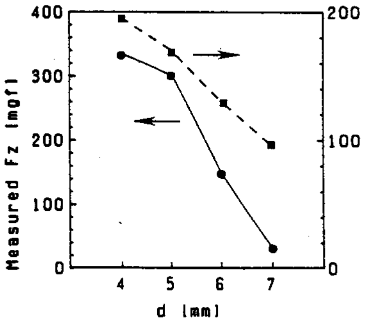

Fig.5 Measured force and magnetic field in a permanent magnet ring as a function of a separation. ●:measured force, ■:measured magnetic field.

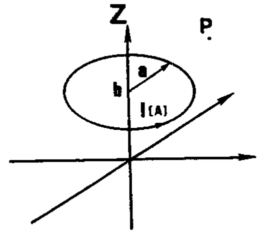

Fig.6 Axially-symmetric magnetic field in a cylindrical coordinate.

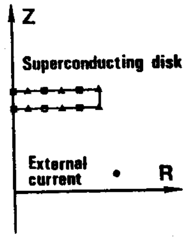

Fig.7 Configuration of external line currents, discrete surface currents and contour points in the DSCM in an axially-symmetric magnetic field. ●:external line current, ▲:discrete surface current, and ◻:contour point.

netic field is obtained by Eq.(5).

$$F_z = -\int_{z_1}^{z_2}\int_0^a I_{sc}(r,z)\cdot B_r(r,z)\cdot 2\pi r\cdot dr\cdot dz \quad (5)$$

,where I_{sc} is a discrete surface current.

3.2 Numerical analysis

The force is analyzed numerically using the DSCM. Forty surface current rings are placed discretely and equidistantly on the top and bottom sides of the superconducting disk, respectively. The contour points are placed between the surface currents on the surfaces of the disk.

Using a solenoid, the forces are analyzed as a function of a solenoid current in a range of 0.1 to 1[A], when a separation is fixed. As a result, it is found that the forces increase exactly in proportion to a square of the current from 0.1 to 1[A]. At 1[A], the calculated forces, when the separations are 6 and 8[mm], are 714 and 496[mgf], respectively. At this time, the magnetic fields without the disk are 111 and 94[G], respectively, at the center of the bottom side of the disk.

Using a permanent magnet ring, the force is analyzed as a function of a separation in a range of 4 to 7[mm]. The magnet ring is replaced by four equivalent magnetizing current rings at the inside and outside surfaces of the magnet, respectively [12]. When the separation is 6[mm], the calculated force is 1351[mgf].

4. Discussion

The measured force is compared with the calculated force. In Fig.8, the ratio of the measured force to the calculated force is shown as a function of a magnetic field. The magnetic field is measured without the disk at the center of the bottom side of the disk. When the magnetic field is 100[G], the measured and calculated forces are 0.88 and 4.96[mgf/mm^2], respectively. The measured force is about a fifth of the calculated force at 100[G]. When the magnetic field is lower than 100[G], the ratio increases to 40[%]. As the magnetic field increases, the ratio decreases. The reason that the measured force is a third or fifth of the calculated force is caused by a non-perfect diamagnetism of the high Tc superconductor at 77K. It is said that levitation height, i.e., the repulsive force, of a magnet over a high Tc superconductor depends on a thickness of a superconductor[11], while a force acting on a perfect diamagnetic body is independent of a thickness of a superconductor: a calculated force is constant regardless of the thickness. Since a thickness of a superconducting disk is fixed at 1[mm] in this paper, it is expected that the ratio would increase as a superconductor becomes thicker than 1[mm].

As shown in Fig.9, a magnetization of a superconductor at 77K is measured as a function of an applied magnetic field. A magnetic susceptibility, χ, is obtained on the basis of a magnetization M[emu] and an applied magnetic field H[G] in Fig.9.

$$\chi = 4\pi M'/H \quad (6)$$

,where M'[emu/cm^3] is a magnetization per volume. A perfect diamagnetism means that χ is -1. However, χ is about -0.25 until 50[G], and χ begins to decrease greater than 60[G]. At 400[G], χ decreases to -0.05. It seems that Hc_1 is about 50[G]. As shown in Fig.9, the high Tc superconductor at 77K is not a perfect diamagnetic body for all of a range of H.

Let's estimate a real force, acting on a high Tc superconductor, using a calculated force and a measured magnetic susceptibility. It is supposed that an infinitely long cylinder, whose magnetic susceptibility is χ, is placed parallel to an external line current I, in a free space. The force interacted between the cylinder and the line current is obtained by the image method[12]. The image current in the cylinder is $(\mu_r-1)/(\mu_r+1)$ times I. The external current is $(3\mu_r+1)/(\mu_r+1)$ times I. Therefore, the force is influenced by the magnetic susceptibility variation as long as the configuration is fixed. The factor,K, is shown in Eq.(7).

$$K = \frac{(3\mu+1)(\mu-1)}{(\mu+1)^2} = \frac{(3\chi+4)\chi}{(\chi+2)^2} \quad (7)$$

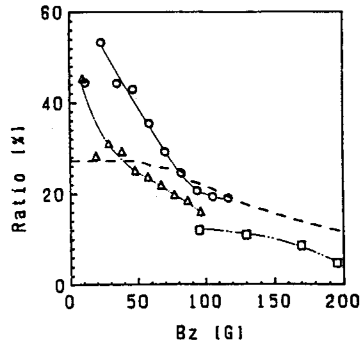

Fig.8 Ratio of measured force to calculated force as a function magnetic field. o:a separation is 6[mm] with a solenoid, ▲:a separation is 8[mm] with a solenoid, ◻:with a permanent ring magnet, and———:estimated ratio using a measured magnetic susceptibility.

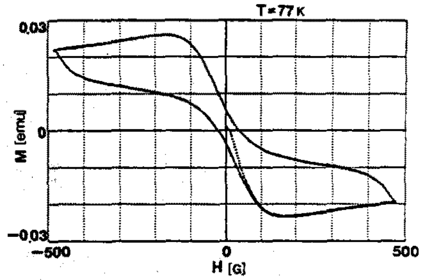

Fig.9 Magnetization of the YBCO superconductor as a function of a magnetic field.

,where $\mu_r=1+\chi$, and the distance between the cylinder and the line current is far greater than the radius of the cylinder.
When a magnetic susceptibility of the cylinder is zero, the factor is zero. When the cylinder is a perfect diamagnetic body, the factor is -1, where the negative sign means repulsive.

The factor,K of Eq.(7) is calculated as a function of an applied magnetic field on the basis of the measured magnetic susceptibility in Fig.9. The factor is plotted in Fig.8(a broken line) as a percentage to a force acting on a perfect diamagnetic body. When the magnetic field is 50[G], the factor is -0.27. The factor begins to decrease greater than 60[G].

Although the magnetic susceptibilities of a diamagnetic body and a superconductor are same χ(χ is not -1), the magnetic property of a superconductor is different from that of a diamagnetic body. The magnetic susceptibility is not uniform in the superconductor. The magnetic susceptibility is zero or -1 in the superconductor, and the magnetic field on the superconductor's surface has only a normal or a tangential component. On the other hand, in the diamagnetic body, the magnetic susceptibility is uniform in all the volume. It is possible that the magnetic field on the boundary has a normal and a tangential component, simultaneously.

However, if it is proposed that a high Tc superconductor at 77K, a non-perfect diamagnetic body, is regarded as a diamagnetic body, a real force acting on a high Tc superconductor at 77K in a magnetic field can be estimated on the basis of a measured magnetic susceptibility and a calculated force.

5. Conclusions

The force acting on a high Tc superconductor is measured, and analyzed numerically using the DSCM. The forces, acting on an YBCO disk in an axially-symmetric magnetic field, are measured at 77K. The forces are analyzed using the DSCM, assuming that the disk is a perfect diamagnetic body. The measured and calculated forces per area, acting on a high Tc superconductor at 77K, are 0.88 and 4.96[mgf/mm^2], respectively, at about 100[G]. As a magnetic field increases, the difference between the values of a measured and a calculated force becomes large. The difference between them is caused by a non-perfect diamagnetism of the high Tc superconductor at 77K. It is proposed that a real force acting on high Tc superconductors at 77K can be estimated on the basis of a measured magnetic susceptibility of the high Tc superconductors at 77K and a calculated force of a perfect diamagnetic body.

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References

[1] J.G.Bednorz and K.A.Muller, Possible high Tc superconductivity in the Ba-La-Cu-O system, Z.Phys.B-Condensed Matter, 64 (186) 189-193.
[2] S.Uchida, et al.,High Tc superconductivity of La-Ba-Cu Oxides, Jpn.J.Appl.Phys., Vol.26, No.1 (1987) L1-L2.
[3] M.Nissenoff,Superconducting electronics: current status and future prospects, Cryogenics, Vol.28, No.1 (1988) 47-56.
[4] D.J.Scalpino, et al.,New research opportunities in superconductivity, Cryogenics, Vol.28, No.11 (1988) 711-723.
[5] G.Binnig, et al.,Tunneling through a controllable vacuum gap, Appl.Phys.Lett., Vol.40(2), No.15 (1982) 178-180.
[6] V.A.Arkadiev,A floating magnet, Nature, Vol.160 (1947) 330.
[7] B.V.Jayawant,Electromagnetic levitation and suspension techniques, Edward Arnold, London, 1981, pp.34-36.
[8] I.Simon,Forces acting on superconductors in magnetic fields, J.Appl.Phys., Vol.24, No.1 (1953) 19-24.
[9] A.H.Boerdijk,Levitation by static magnetic fields, Phillips Technical Review, Vol.18, No.4 (1956) 125-127.
[10] Y.-K.Kim, et al.,Analysis of a magnetically levitated micro linear actuator using Meissner effect, Papers of Technical Meeting of IEE of Japan on Linear Drives, LD-88-2 (1988) 9-18 (in Japanese).
[11] F.Hellman, et al.,Levitation of a magnet over a flat type II superconductor, J.Appl.Phys., Vol.63(2), No.15 (1988) 447-450.
[12] W.R.Smythe,Static and dynamic electricity, Mc Graw-Hill, New York, 3rd ed., 1968.