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폐쇄직렬 생산시스템의 점근적 성능해석

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Asymptotic Performance Analysis of Closed Serial Production Systems

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Abstract

This paper formulates a problem of analysis and design of serial production lines, closed with respect to the number of carriers available in the system for parts transportation between operations. For two machines - two buffers systems, the paper gives an asymptotic solution and shows that optimization of the system with respect to the number of carriers available and the capacity of the feedback buffer may lead to substaintial improvements of system's performance.

1. INTRODUCTION

Consider a manufacturing system defined by the following assumptions:

(i) The system consists of M machines m_i, $i = 1, ..., M$, arranged in the consecutive order and $M-1$ buffers, B_i seperating each two machines, m_i and m_{i+1}.

(ii) The machines have identical cycle time T. The time axis is slotted with the slot duration T. Each machine begins its operation at the beginning of the time slot.

(iii) Each buffer is characterized by its capacity, N_i, $i = 1, ..., M-1$, where N_i is a positive integer.

(iv) Machine m_i is starved during a time slot if buffer B_{i-1} is empty at the beginning of this time slot, machine m_i is blocked during a time slot if at the beginning of this time slot buffer B_i is full and machine m_{i+1} is either down or blocked. Machine m_1 is never starved, machine m_M is never blocked.

(v) Machine m_i, being not blocked and not starved, produces a part during any time slot with probability $q_i = 1-\epsilon k_i$ and fails to do so with probability ϵk_1, $i = 1, ..., M$, where $0 < \epsilon \ll 1$ and $k_i > 0$ is independent of ϵ. The k_i's are called the loss parameters.

Manufacturing systems defined by assumptions (i)-(v) are referred to as asymptotically reliable, *open* serial production lines. An asymptotic method for their analysis and design has been developed in [1]. In many practical situations, however, serial production lines are *closed*, *i.e.*, have a feedback loop with respect to carriers on which the parts (jobs) are transported from one machine to another. This is, in particular, the case in assembly and painting operations in the automobile industry where car bodies and engine blocks are transferred between operations on carriers, and the number of carriers in the system is constant. To account for this situation, introduce the following assumptions:

(vi) The jobs are transported within the system on carriers. Each job is placed on a carrier at the input of machine m_1 and is removed from the carrier at the output of machine m_M. Empty carriers are returned to the empty carrier buffer, B_M, and are supplied to the input of m_1 instantaneously, given that B_M is not empty. The total number of carriers in the system is S, where $M \le S \le \sum_{i=1}^{M} N_i$.

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Manufacturing systems defined by (i)-(vi) are referred to as asymptotically reliable closed serial production lines. Performance of closed lines can be characterized by their production rate, PR_c, *i.e.*, the average number of jobs produced in the steady state by the last machine, m_M, and work-in-process, WIP_c, *i.e.*, the average number of jobs in the system at the steady state. The problem of analysis of these lines is formulated as follows: Given, k_1, N_1, ..., k_M, and S, find $PR_c(k_1, N_1, ..., k_M, N_M, S)$ and $WIP_c(k_1, N_1, ..., k_M, N_M, S)$. The probem of design is: Given $k_1, N_1, ..., k_{M-1}, N_{M-1}, k_M$, find the smallest S and N_M so that

$$\begin{aligned} PR_c(k_1, N_1, ..., k_M, N_M, S) = PR_o(k_1, N_1, ..., k_{M-1}, N_{M-1}, k_M) \\ WIP_c(k_1, N_1, ..., k_M, N_M, S) = WIP_o(k_1, N_1, ..., k_{M-1}, N_{M-1}, k_M), \end{aligned} \quad (1)$$

i.e., find the best conditions under which the feedback does not impair the performance of the line (PR_o and WIP_o in (1) denote the production rate and the work-in-process of the open line, respectively).

The purpose of this paper is to give a solution to these problems for $M = 2$. Specifically, we show below that the problem of analysis of closed lines can be reduced to that of open ones (Section 2). Thus the methods of [1] become applicable to closed serial production systems. For the purpose of design, we show how to choose S and N_2 so that (1) is asymptotically satisfied (Section 3).

Closed Markovian queuing systems, which the problem at hand belongs to, have been studied in a number of articles (see [2]-[11]). Most of them, however, address non-blocking systems with infinite buffers [2], [4]-[10]. For finite buffers, the joint steady state distribution of buffer occupancy has been calculated in [3]. An application of mean value analysis has been reported in [11].

In spite of these achievements, analytic methods for performance evaluation of closed Markovian queuing system are still missing. This paper is intended to contribute to this end.

2. ASYMPTOTIC ANALYSIS

Consider an open serial production line defined by (i)-(v) with $M = 2$ and the buffer capacity N. Let $h(n)$, $n = 0, 1, ...$, be the occupancy of the buffer at the beginning of slot [n, n+1). Introduce

$$v_j(n) = Prob\{h(n) \geq j\}, \quad j = 1, ..., N. \quad (2)$$

Let v_j be the steady state value of $v_j(n)$, *i.e.*, the probability that there are at least j parts in the buffer at the steady state. Define the following function

$$Q(\alpha, N) = \frac{1-\alpha}{1-\alpha^N}, \quad \alpha \in R_+. \quad (3)$$

Theorem 1 [1]: The performance of open serial production lines defined by (i)-(v) with $M = 2$ is as follows:

(a) The steady state distribution of buffer occupancy is:

$$\begin{aligned} v_1 &= 1 - \epsilon k_1 Q(\tfrac{k_2}{k_1}, N) + O(\epsilon^2) \\ v_N &= Q(\tfrac{k_1}{k_2}, N) v_1 + O(\epsilon) \\ v_i &= Q^{-1}(\tfrac{k_1}{k_2}, N - i + 1) v_N + O(\epsilon), \; i = 2, ..., N-1. \end{aligned} \quad (4)$$

(b) The production rate is

$$\begin{aligned} PR_o(k_1, N, k_2) &= 1 - [k_2 + k_1 Q(\tfrac{k_2}{k_1}, N)]\epsilon + O(\epsilon^2) \\ &= 1 - [k_1 + k_2 Q(\tfrac{k_1}{k_2}, N)]\epsilon + O(\epsilon^2). \end{aligned} \quad (5)$$

Formula (4) can be used to calculate the system's work-in-process. Indeed, since

$$WIP_o(k_1, N, k_2) = \sum_{i=1}^{N} i(v_i - v_{i+1}) = \sum_{i=1}^{N} v_i,$$

from (4), it follows that

$$WIP_o(k_1, N, k_2) = \frac{NQ\left(\frac{k_1}{k_2}, N\right) k_2 - k_1}{k_2 - k_1} + O(\epsilon). \quad (6)$$

We show below that performance evaluation of closed lines can be reduced to that of open lines, where the effective buffer capacity, N_e, depends on the relationship between N_1, N_2 and S.

Theorem 2: The performance of closed serial production lines defined by (i)-(vi) with $M = 2$ is characterized as follows:

$$PR_c(k_1, N_1, k_2, N_2, S) = PR_o(k_1, N_e, k_2) + O(\epsilon) \quad (7)$$

$$WIP_c(k_1,N_1,k_2,N_2) = \begin{cases} \max(0, S-N_2-1) + WIP_o(k_1, N_e, k_2) + O(\epsilon), & \text{if } N_1 \le N_2 \\ S - [\max(0, S-N_1-1) + WIP_o(k_2, N_e, k_1)] + O(\epsilon), & \text{if } N_1 > N_2 \end{cases} \quad (8)$$

where N_e, the effective buffer size, is given by

$$N_e = \begin{cases} S-1, & \text{for } 2 \le S \le \min(N_1, N_2) \\ \min(N_1, N_2), & \text{for } \min(N_1,N_2) < S \le \max(N_1,N_2) \\ N_1 + N_2 - S + 1, & \text{for } \max(N_1,N_2) < S \le N_1 + N_2. \end{cases} \quad (9)$$

Theorem 2 can be interpreted as follows:

1. Since $N_e \le N_1$ and PR_o and WIP_o are monotonically increasing functions of N_e, performance of a closed line cannot supersede that of the corresponding open line. If S and N_2 are chosen so that $N_e < N_1$, the feedback impairs the performance of the open line.

2. Alike open lines, closed lines under consideration are equivalent to a single, aggregated machine characterized, in isolation, by the parameter

$$\begin{aligned} q_{aggregation} &= 1 - [k_2 + k_1 Q(\tfrac{k_2}{k_1}, N_e)]\epsilon + O(\epsilon^2) \\ &= 1 - [k_1 + k_2 Q(\tfrac{k_1}{k_2}, N_e)]\epsilon + O(\epsilon^2), \end{aligned}$$

3. When $S = 2$ and $S = N_1 + N_2$, the aggregated loss parameter is the sum of the losses of both machines no matter how large N_1 is:

$$k_{aggregation} = k_1 + k_2.$$

For $2 < S < N_1 + N_2$, function $Q(\alpha, N_e)$ describes the attenuation of perturbations (failures) introduced by machines.

4. From (7),

$$PR_c(k_1, N_1, k_2, N_2, S) = PR_c(k_1, N_2, k_2, N_1, S).$$

However, from (8),

$$WIP_c(k_1, N_1, k_2, N_2, S) \le WIP_c(k_1, N_2, k_2, N_1, S), \quad \text{if } N_1 \le N_2.$$

It means that large S and N_1 with small N_2 will result in large WIP whereas large S and N_2 with small N_1 will result in small WIP but the same production rate.

3. DESIGN

Proposition 1: Under assumptions (i)-(v) with $M = 2$, the asymptotic solution of the design problem (1), *i.e.*, the smallest S and N_2 which, for given k_1, N_1 and k_2, guarantee that

$$\begin{aligned} PR_c(k_1, N_1, k_2, N_2, S) &= PR_o(k_1, N_1, k_2) + O(\epsilon^2) \\ WIP_c(k_1, N_1, k_2, N_2, S) &= WIP_o(k_1, N_1, k_2) + O(\epsilon^2) \end{aligned} \quad (10)$$

is given by

$$S = N_2 = N_1 + 1. \quad (11)$$

Proof: Follows directly from Theorem 2.

From (7)-(9), it is clear that equalities (10) also take place for

$$N_2 > N_1 + 1, \quad N_1 + 1 < S \le N_2. \quad (12)$$

This solution, however, is inferior to (11) since (12) results in the same performance but requires a large number of carriers and feedback capacity.

4. CONCLUSIONS

1. Analysis of asymptotically reliable closed serial production lines with $M = 2$ can be reduced to that of the corresponding open lines and the effective buffer capacity is defined by the number of carriers in the system and the capacity of the buffers in both feedforward and feedback paths.

2. The optimal design of the system with respect to the number of carriers and the capacity of the feedback buffer can lead to substaintial improvements in the performance characteristics.

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