3-D Vibration Analysis of Floating Structures Like Ships Using FEM-BEM

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1 Introduction

In the vibration analysis of structures in fluid such as ships and offshore structures, the hydrodynamic added mass considerably affects the result of analysis. Therefore correct evaluation of the hydrodynamic added mass effect is reguired for an accurate analysis. But the correct evaluation of the effect is not simple because the added mass varies with the mode shape of vibration as well as the configuration of the structure.

The universal method employed to evaluate added mass in ship hull vibration is Lewis' method via the introduction of 3 dimensional correction factor. But this conventional method is valid only for beam-like vibration.

For the calculation of 3 dimensional complex vibration behavior, 3 dimensional finite element model of structure is necessary. Therefore, for the analysis of structure-fluid coupled system the method which is directly appicable to finite element structural analysis is required for the evaluation of added mass.

The main objective of this work is to develop the computer program for the structure-fluid interactive vibration of 3 dimensional structure with complicated configuration such as ships, offshore structures and so on. For this objective following study was performed in this work ;

1. A matrix formulation for hydrodynamic added mass is derived by using boundary element method. This matrix is directly applicable to the equation of motion of the finite element model of structure.

2. The equation of motion of structure-fluid coupled vibration is derived by the combination of structure finite element method and fluid boundary element method.

3. A computer program for analysis of the structure-fluid coupled vibration is developed. And the utility of the program is verified through numerical calculations.

2 Hydrodynamic Added Mass Matrix

2.1 Boundary integral equation

It is assumed that the surrounding fluid is ideal, the floating structure performs simple harmonic vibration of small amplitude with frequency ω, and the water depth is deep enough. With these assumptions the governing equations of this linear boundary value problem are formulated in terms of velocity potential $\Phi = Re[\phi(x, yz)e^{-i\omega t}]$ as follows. Where, cartesian coordinate system as shown in Fig.1 is employed.

Laplace equation :

$$\nabla^2\phi(x,y,z) = 0 \quad in\ fluid\ domain \tag{1}$$

Homogeneous boundary conditions :

$$\frac{\partial\phi}{\partial z} - K\phi = 0 \quad ;\ free\ surface\ condition \tag{2}$$

$$\lim_{z\to\infty}\frac{\partial\phi}{\partial z} = 0 \quad ;\ bottom\ condition \tag{3}$$

$$\lim_{r\to\infty} r^{\frac{1}{2}}(\frac{\partial\phi}{\partial r} + iK\phi) = 0 \quad ;\ radiation\ condition \tag{4}$$

where K is the wave number and $r = (x^2 + y^2)^{\frac{1}{2}}$.

Inhomogeneous boundary condition :

$$\frac{\partial\phi}{\partial n} = -i\omega U\cdot n \quad on\ wetted\ surface \tag{5}$$

where U is the displacement vector on the wetted surface and n is the normal vector on wetted surface into the fluid domain.

We employ the Green function G(P,M;K) where M denotes the source point and P the field point. It satisfies Laplace equation and homogeneous boundary conditions. Applying Green's theorem to the potential and the Green function in the fluid region we find the following boundary integral equation :

$$\frac{\phi(P)}{2} + \int_S \phi(M)\frac{\partial G(P,M)}{\partial n_M}dS_M = \int_S \frac{\partial\phi(M)}{\partial n}G(P,M)dS_M, \quad P\in S_w \tag{6}$$

The Green function G(P,M;K) can be expressed as follow:

$$G(P,M;K) = -\frac{1}{4\pi}[\frac{1}{r} + \frac{1}{r'} + H(P,M;K)] \tag{7}$$

$$r = \sqrt{(x_P - x_M)^2 + (y_P - y_M)^2 + (z_P - z_M)^2}$$

$$r' = \sqrt{(x_P - x_M)^2 + (y_P - y_M)^2 + (z_P + z_M)^2}$$

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where H(P,M;K) is a complex valued harmonic function regular in the lower half space. It is well known that the solution of (6) with Green function (7) becomes undetermined at irregular frequencies. Hong [9,10] proposed the improved integral equation which possesses nontrivial solutions at irreqular frequencies

$$\frac{\phi(P)}{2} + \int_{S_w \cup S_P} \phi(M) \frac{\partial G'}{\partial n_M} dS_M = \int_{S_w} \frac{\partial \phi(M)}{\partial n} G'(P,M) dS_M, \quad P \in S_w \cup S_P \tag{8}$$

The associated Green function $G'(P,M;K)$ is

$$G'(P,M;K) = -\frac{1}{4\pi}\{\frac{1}{r} + \frac{1}{r'} + H(P,M;K)[1 - \delta(z_P - 0)\delta(z_M - 0)]\} \tag{9}$$

This improved equation is obtained by imposing the following condition at the waterplane inside the body.

$$\int_S \phi(M) \frac{\partial G(P,M)}{\partial n} - \int_S \frac{\partial \phi(M)}{\partial n} G(P,M) = 0 \quad , P \in S_p$$

If we have an another assumption that the harmonic frequency ω is high compared with gravity acceleration g (that is, high frequency approximation), the homogeneous boundary conditions (2),(3) and (4) are replaced with followings.

$$\nabla \phi(\infty) \rightarrow 0 \tag{10}$$

$$\phi(x,y,0) = 0 \tag{11}$$

Eq.(10) represents that the vibrating body does not affect on the fluid at infinity far from the body. And Eq.(11) is the free surface boundary condition in case of $\omega \gg 1$.

For high frequency approximation integral equation is the same as Eq.(6) and the Green function G(P,M) is as follow:

$$G(P,M) = -\frac{1}{4\pi}[\frac{1}{r} - \frac{1}{r'}] \tag{12}$$

It is noted that the Green function (12) does not depend on ω (or K) in contrast with (7) and (9) and also that (7) and (9) becomes (12) when ω approaches infinity.

In this work the Green function (7) and (9) are taken in Ref.[9].

2.2 Hydrodynamic force matrices

Using the discretized model of structure for F.E. analysis, we can convert the intergral equation into algebraic equation. If it is assumed that ϕ and $\frac{\partial \phi}{\partial n}$ are constant in an element (that is, constant element), the integral equation (6) is converted as follow:

$$\frac{\phi_i}{2} + \sum_{j=1}^{N} \phi_j \int_{S_j} \frac{\partial G_{ij}}{\partial n_j} dS_j = -i\omega \sum_{j=1}^{N} U_j \cdot n_j \int_{S_j} G_{ij} dS_j \quad for\ i = 1, \cdots, N \tag{13}$$

where,

N : total number of elements on wetted surface of structure

U_j : vibratory displacement of j-th element

n_j : outer normal vector of j-th element

In Eq.(13) the body boundary condition (5) was substituted.

Matrix form of Eq.(13) is

$$A\phi = -i\omega B U_n \tag{14}$$

where,

$$A = [a_{ij}], \quad a_{ij} = \frac{1}{2}\delta_{ij} + \int_{S_j} \frac{\partial G_{ij}}{\partial n_j} dS_j$$

$$B = [b_{ij}], \quad B_{ij} = \int_{S_j} G_{ij} dS_j$$

$$\phi = \{\phi_j\}$$

$U_n = U_{nj}$, normal component of vibratory displacement $i,j = 1,2,\cdots,N$

For the case of improved integral equation (8), same equation that Eq.(14) is formulated where,

$$A = [a_{ij}], \quad a_{ij} = \frac{1}{2}\delta_{ijkl} + \int_{S_j} \frac{\partial G'_{ij}}{\partial n_j} dS_j$$

$$B = [b_{ik}], \quad B_{ik} = \int_{S_j} G'_{ij} dS_j$$

$$\phi = \{\phi_j\}$$

$$U_n = \{U_{nk}\}$$

$$i,j = 1,2,\cdots,N+NF$$

$$k,l = 1,2,\cdots,N$$

NF is the total number of elements on waterplane of structure.

From the linearized Bernouille equation and velocity potential Φ, the hydrodynamic pressure on the wetted surface of structure is

$$P(x,y,z,t) = -\rho \frac{\partial \Phi}{\partial t} - \rho g U_z = \rho Re[i\omega\phi e^{-i\omega t}] - \rho g Re[U_z e^{-i\omega t}] \tag{15}$$

Then, the virtual work due to hydrodynamic pressure is

$$\delta W = -\int_{S_w} P \delta(U_n) dS_w = -\rho Re[\int_{S_w} (i\omega\phi - gU_z) e^{-i\omega t} \delta(U_n) dS_w] \tag{16}$$

For the discrete model Eq.(16) is expressed as

$$\delta W = \sum_{j=1}^{N} Re[\{(-i\omega\rho S_j \phi_j)\delta U_{nj} + (\rho g S_j U_{zj})\delta U_{nj}\} e^{-i\omega t}] \tag{17}$$

where S_j is the area of j-th element.

Matrix form of the above equation :

$$\delta W = Re[-i\omega\rho\{\delta U_{nj}\}^T S\{\phi_j\}e^{-i\omega t} + \rho g\{\delta U_{nj}\}^T S\{U_{zj}\}e^{-i\omega t}] \quad (18)$$

where $S = [\ diag\ S_j\ ]$

From the Eq.(14)

$$\{\phi_j\} = -i\omega A^{-1}B\{U_{nj}\} \quad (19)$$

From the Eq.(18) and Eq.(19), the virtual work is

$$\delta W = -\omega^2\rho Re[\{\delta U_{nj}\}^T SA^{-1}B\{U_{nj}\}e^{-i\omega t}] + \rho g Re[\{\delta U_{nj}\}^T S\{U_{zj}\}e^{-i\omega t}]$$

Therefore hydrodynamic force can be expressed as follow

$$\{F_f\} = Re[-\omega^2\rho SA^{-1}B\{U_{nj}\}e^{-i\omega t} + \rho g S\{U_{nj}\}e^{-i\omega t}] \quad (20)$$

The first term of Eq.(20) becomes complex when the Green function in the boundary integral equation is complex. This term can be rewritten in terms of real part and imaginary part as follow:

$$\omega^2\overline{M}\{U_{nj}\}e^{-i\omega t} + i\omega\overline{C}\{U_{nj}\}e^{-i\omega t} \quad (21)$$

where,

$$\overline{M} = [m_{ij}] = -\rho S Re[A^{-1}B] \quad (22)$$

$$\overline{C} = [c_{ij}] = -\omega\rho S Im[A^{-1}B] \quad (23)$$

The second term of Eq.(20) is always real and is expressed as follow:

$$-\overline{K}\{U_{nj}\}e^{-i\omega t} \quad (24)$$

where,

$$\overline{K} = [k_{ij}] = [\rho g \cdot n_{zj}] \quad (25)$$

The matrices $\overline{M}$,$\overline{C}$ and $\overline{K}$ can be defined as hydrodynamic added mass, damping and added stiffness matrices respectively.

It is noted that the hydrodynamic damping matrix $\overline{C}$ becomes null for the case of high frequency approximation. It is the reason that the first term of Eq.(20) is real in that case.

3 Natural Vibration Analysis of Coupled System

The equation of natural vibration for the F.E. model of structures without fluid effect is written as follow:

$$M_s\ddot{U} + K_sU = 0 \quad (26)$$

Where the subscript s denotes for structural system. Imposing the hydrodynamic force (20) on the above equation, we can obtain the equation of coupled system.

As an usual formulation of natural vibration the hydrodynamic damping can be neglected. And in general the hydrodynamic stiffness which means boyancy spring is negligible compared with strucutral stiffness.

In order to impose the hydrodynamic addad mass on Eq.(26), coordinate transformation into F.E. coordinates is necessary.

$$U_n = TU \quad (27)$$

The transformed hydrodynamic added mass matrix becomes as follow:

$$\overline{M}_f = T^t\overline{M}T \quad (28)$$

Therefore the equation of coupled system can be written as follow:

$$(M_s + \overline{M}_f)\ddot{U} + K_sU = 0 \quad (29)$$

The bandwidth of the matrix in the above equation becomes large because of $\overline{M}_f$. Therefore, to solve the above eigenvalue problem for large systems, mode superposition technique is much more efficient than direct method although some accuracy loses due to modal truncation.

Let the natural frequency without fluid effect be ω_i and the corresponding mode shape ψ_i. Then following orthogonality conditions exist.

$$\psi^t M_s\psi = [\ulcorner \tilde{M}_i \lrcorner] = \tilde{M}$$
$$\psi^t K_s\psi = [\ulcorner \omega^2\tilde{M}_i \lrcorner] = \tilde{K} \quad (30)$$

In mode superposition U is expressed as a linear summation of mode shapes ψ, i.e.,

$$U = \psi Q \quad , \quad where,\ Q\ ;\ modal\ coordinates$$

From the above relations the equation transformed into modal coordinates can be obtained

$$(\tilde{M} + \tilde{M}_f)\ddot{Q} + \tilde{K}Q = 0 \quad (31)$$

Where,

$$\tilde{M}_f = \psi^T\overline{M}_f\psi$$

This equation is expressed as following eigenvalue equation from the fact that $Q = qe^{-i\omega t}$

$$\{K - \Omega^2(\tilde{M} + \tilde{M}_f)\}q = 0 \quad (32)$$

It is noted that the hydrodynamic added mass depends on the frequency except for the high frequency approximation. In this case iteration method can be used to solve the nonlinear eigenvalue problem.

4 Numerical Investigations and Remarks

In order to check the utility of the methods described in the previous chapters when applied to ship vibration analysis, some numerical investigations were performed.

4.1 Hydrodynamic added mass for the rigid motion

Numerical calculations were carried out for the two models (hemisphere and half circle cylinder) to check the accuracy of calculated added mass and to investigate the frequenay dependency of added mass.

Hemisphere model (Fig.2)

1. In the case of high frequency approximation
2. In the case of using improved integral equation
3. In the case of using integral equation (6)

For the above three cases, Fig.3 shows the added mass coefficient (C_m = added mass/$\frac{1}{2} \cdot \frac{4}{3}\pi\rho r^3$) with respect to nondimensionalized frequency when the hemisphere oscillates rigidly with unit amplitude.

In Table 1 calculated results for high frequency approximation are compared with the results in Ref.[6] and also with exact solution.

Half circle cylinder (Fig.4)

1. In the case of high frequency approximation
2. In the case of using integral equation (6)

For the above two cases, Fig.5 shows the added mass coefficient (C_m = added mass/$\frac{1}{2} \cdot \pi\rho a^2 L$) with respect to nondimensionalized frequency.

From these results it can be said that

- In the low frequency ranges several irregular frequencies exist. They can be eliminted by using the improved integral equation. But waterplane of the structure must be modelled into many elements in order to insure the accuracy in some extended frequency ranges.

- High frequency approximation is more simple than the other cases and gives the good results except for the very low frequency ranges. Therefore high frequency approximation can be adopted in almost ship vibration analysis except for special cases.

- From present method hydrodynamic added mass can be calculated with a good accuracy.

4.2 Natural vibration of half circle cylinder

Under the assumption of high frequency approximation, natural frequencies of the model in Fig.6 were calculated and compared with those by conventional Lewis' method in Table 2. Fig.7 shows the corresponding mode shapes.

All these calculations were carried out with the program VIBDET. This is the finite element vibration analysis program developed by KRISO. The main function of this program is to analyse the structural vibration by substructure modal synthesis method.

Through this work the function to analyse the structure - fluid coupled system was implemented in the program VIBDET.

5 Conclusion

The major results in this work can be stated as follows;

- Finite element method combined with boundary element method is useful in the vibration analysis of complex structures in fluid such as ships, offshore structures and so on.

- High frequency approximation can be adopted in the almost coupled vibration analysis except for special cases.

- For the very low frequency system improved integral equation can be used. But large number of elements must be added on the waterplane of structure to insure the accuracy in some extended frequency ranges.

- Program VIBDET was developed, which can be used efficiently to analyse the 3-D complex vibration behavior of general structures in fluid like ships.

References

1. Brebbia, C.A., The Boundary Element Method for Engineers, Pentech Press, London, 1979.
2. Banerjee, P.K., Shaw, R.P., Developments in Boundary Element Methods-1 and 2, Applied Science Publishers, 1982.
3. Y.Matsuura, H.Kawakami, Calculation of Added Virtual Mass and Added Moment of Inertia of Ship Hull Vibration by Finite Elament Method, JSNA of Japan Vol. 124.
4. K. Ohta, K. Kagawa, I.Honda, Vibration Analysis of Fluid Structure Interaction Problem (1st report, 2nd report) Trans. of JSME (Series C), Vol. 51, No. 470. 1985.
5. K.T. Chung, On the Vibration Analysis of the Floating Elastic Body Using the Boundary Integral Method, Proceedings of the 19th Workshop on Ocean Engineering in Korea, 1987.
6. S.Hylarides, W.S. Vorus, The Added Mass Matrix in Ship Vibration, Using Distribution Related to the Finite Element Grid of the Ship Structure, International Ship building Progress, Vol. 29, No.330, 1982.
7. Y.Masuura, K.Matsumoto, K.Arima, A.Kinoshita, A Calculation Method for Estimating the Natural Frequency of Fluid Structure Vibration, Hitachi Zosen U.D.C 534.1 : 69.034
8. R.E. Taylor, J.B.Waite, The Dynamics of Offshore Structures Evaluated by Boundary Integral Technigues, International Journal for Numerical Method in Engineering Vol. 13, 1973.

9. D.C. Hong, On the Improved Green Integral Equation Applied to the Water Wave Radiation Diffraction Problem, Journal of The Society of Naval Architects of Korea, 1987.

10. Development of Design Technology for a Semi-Submersible Drilling Rig (I), Report of MOST UCN076-929.D, 1987.

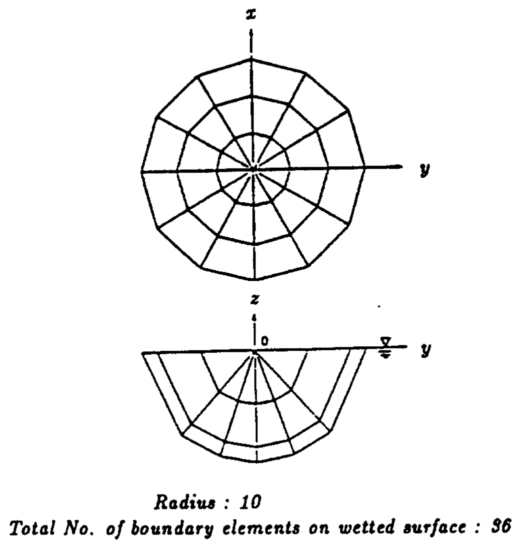


Fig.2 Model of hemisphere

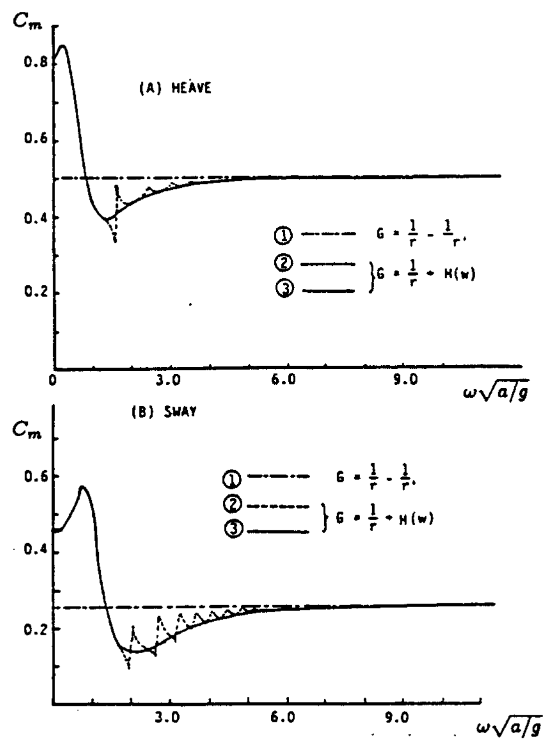
Table 1 Added mass coefficient of hemisphere

Method \ Added mass	Mxx	Myy	Mzz
Present Method for high frequency approximation	0.260	0.260	0.501 (+0.2%)
Hylarides at al[6] (source distribution method)	0.250	0.250	0.491 (-1.8%)

* Mzz, exact : 0.5

Table 2 Comparison of the calculated natural frequencies(Half circle cylinder, L/B=5.0)

Mode Shape	In Air (Hz)	Using Lewis'method		Present Method (Hz)
		3D. corr. factor	freq. (Hz)	
2-node	5.450	0.585	4.328	4.350
3-node	8.153	0.520	6.612	6.277
4-node	10.83	0.460	8.963	8.862
5-node	13.47	0.409	11.35	10.88
6-node	16.08	0.367	13.75	13.62
7-node	18.62	0.332	16.13	15.62



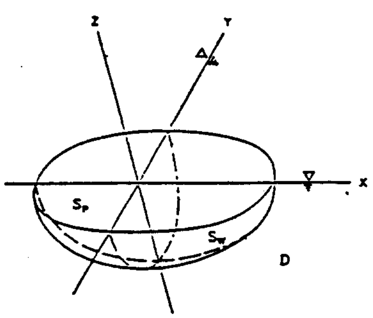


Fig.1 Coordinate system - *The free surface coincides with x-y plane*

Fig.3 Added mass coefficient of hemisphere

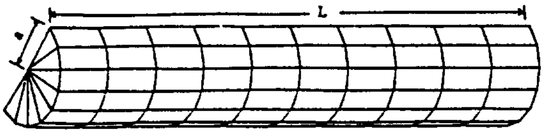


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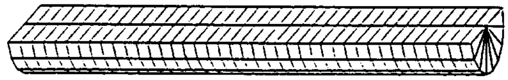
Fig.4 Model of half circle cylinder



Total No. of nodal points : 410

Total No. of boundary elements : 338

Fig.6 Vibration analysis model of half circle cylinder *(vertical vibration)*

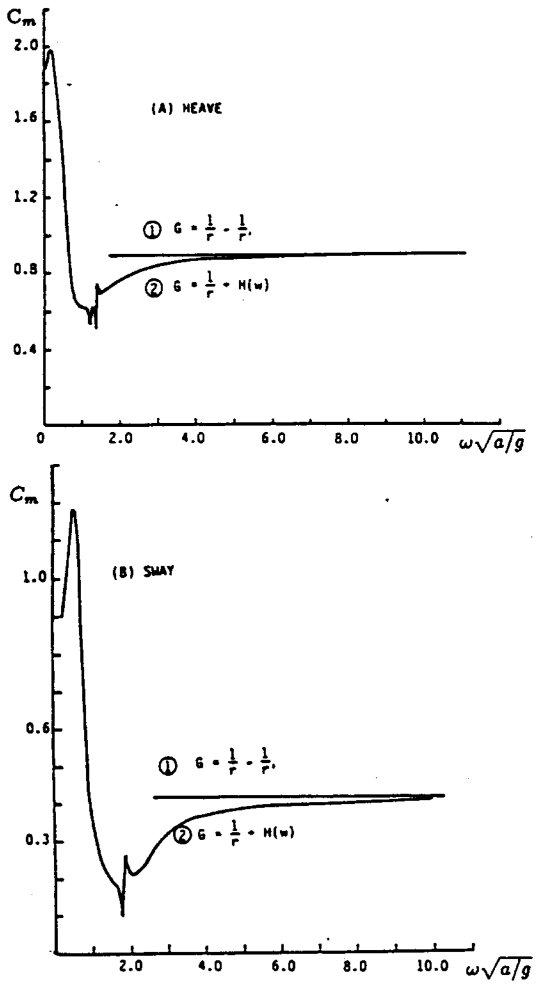


Fig.5 Added mass coefficient of half circle cylinder

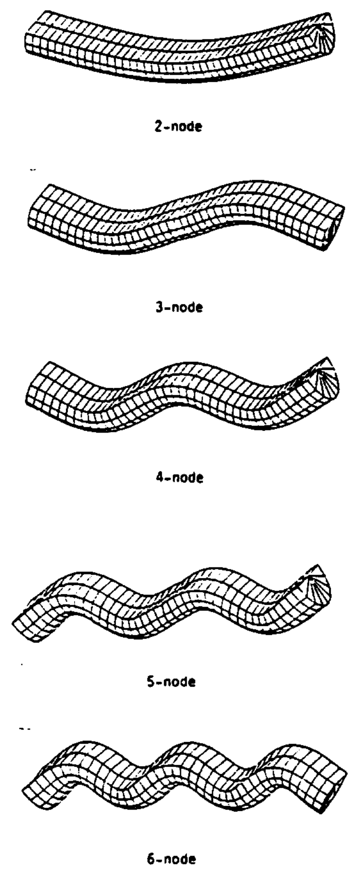


Fig.7 Mode shapes of the half circle cylinder