한국콘크리트학회학술발표논문집
제 2 권 2 호 1990년 11월 10일

반복하중에 의한 콘크리트 손상 모델링

Modeling of Concrete Damage Subjected to Repeated Loadings

鄭 英 秀 * 金 己 奉 **
Chung, Young Soo Kim, Ki Bong

요 약

지진하중에 의한 콘크리트의 손상및 파괴의 적절한 평가를 위한 많은 연구가 수행되고 있지만 현재까지 손상을 당한 부재의 잔여강도를 예측할 수 있는 적절한 손상 모델이 없는 실정이다. 본 논문은 콘크리트의 손상에 관한 기본적인 현상들을 면밀히 조사연구한 후 콘크리트의 low-cycle 피로현상을 해석모델링하여 반복하중 하에서의 손상모델을 제안하였다. 제안된 모델은 콘크리트가 파괴에 도달할 시 싸이클수 대신에 부재의 흡수에너지 능력을 주요변수로 택하였다. 특히 본 손상모델의 정확성은 기 제안한바 있는 반복하중 작용시의 콘크리트 부재의 해석적인 이력모델[3]을 사용하여 예증하였다.

1.0 Introduction

The concrete of damage permeates many branches of concrete engineering. As concrete is subjected to loading of increasing intensity, it undergoes different phases of damage, from microcracking up to ultimate failure. Our capability to simulate this process mathematically is desirable because it allows us to predict the residual strength and serviceability of damaged or aged concrete structures. Of particular concern are those members that have been subjected to several damaging load cycles such as may be inflicted by a major earthquake.

A considerable effort has been expanded by researchers to develop models of concrete damage. In recent study[3] Chung et al have critically evaluated 17 such models, many of which are either of an empirical nature or were derived originally for metal structures. Most of these models are not well suited to predict the residual strength of damaged concrete members.

It is the purpose of this paper to review some basic facts about concrete damage and to use this knowledge to systematically construct a new model that is capable of simulating reasonably well the strength and stiffness degradation that accompanies the damage process. Such a model may then form the basis for rational seismic risk evaluation of concrete structures.

* 정회원 중앙대학 공과대학 토목과 조교수
** 중앙대학 공과대학 토목과 조교수

2.0 Damage and Failure of RC Members

Any attempt of devising mathematical models to quantify damage in a rational way should set out with a clear and precise definition of damage, because "damage" is a widely used word that describes all kinds of different phenomena and is prone to subjective interpretation. In this paper, damage of a RC member shall be defined to signify a specific degree of physical degradation with precisely defined consequences regarding the member's capacity to resist further load. A damage index is usually defined as the damage value normalized with respect to the (arbitrarily defined) failure level so that a damage index value of unity corresponds to failure.

As an illustration, Fig. 1 shows a typical response of a reinforced concrete cantilever beam to progressively increasing load cycles[6]. The stiffness of the member decreases gradually, once the yield capacity of the member has been exceeded. It takes a significant further increase in loading until the strength deteriorates as well, i.e. when the force necessary to cause a given tip deflection decreases in subsequent cycles. Fig. 1 also demonstrates the difficulty of defining failure. Thus, any failure definition such as "a strength reduction of 25% of the first yield load level" is arbitrary. Even then, such a de-

This paper was supported in part by NON DIRECTED RESEARCH FUND, Korea Research Foundation, 1990.

finition is not sufficiently precise, since the apparent residual strength may increase with further displacement increase, Fig. 1.

The response of reinforced concrete to load is complicated by the complex interaction between steel and concrete. This is reflected by the numerous possible failure modes in flexure, shear or bond. Conventional reinforced concrete design philosophy calls for such member detailing that all but a few ductile failure modes are precluded. For dynamically applied cyclic loads it is difficult to predict the failure mode even for "properly" detailed members, because of the effect of the strain rate and the load reversals. Bond deterioration and shear cracking typically progress more rapidly under cyclic loading than flexural strength degradation. As a result, reinforced concrete members subjected to earthquake-type loads are more likely to fail in bond or shear than in flexure, even if properly designed for monotonically applied loads.

The progressive accumulation of damage in a material up to the point of failure under repeated load application is generally known as "fatigue". Each load cycle inflicts a certain amount of irreversible damage and can be campared to the passage of some time unit of the life span of the material. If the material is subjected to a history of varying stress levels, the prediction of the fatigue life is much more difficult. In this case, it is common practice to utilize Miner's hypothesis,

$$\sum_i \frac{n_i}{N_i} = 1 \tag{1}$$

where N_i is the number of cycles with stress level S_i leading to failure, and n_i is the number of cycles with stress level S_i actually applied. This Eq (1) assumes that the accumulation of damage is linear and independent of the loading sequence. In studies of low-cycle fatigue of reinforced concrete the number of load cycles to failure is typically replaced by the cumulative dissipated energy, which is often normalized with regard to the energy stored when the member is stressed to the yield level. In reality, the amount of energy dissipated in each cycle decreases with progressive damage until failure. In analogy to a S-N curve, given the necessary experimental data, it would be possible to present a relationship between constant deformation level D_i and total energy dissipation capacity E_i. In a S-N relationship, N_i is a function of stress level S_i and can be determined experimentally. For metals, such a function, drawn on log scales, is generally approximated by a straight line. Substituting the energy dissipation capacity E_i for N_i, and deforamtion level D_i for stress level S_i, the corresponding D_i-E_i relationship is far from linear, Fig 2. In fact, for very low deformation levels, the load-deformation relationship remains linear, and no energy is dissipated at all.

It can be observed from some laboratory experiments[4] that the failure mode is closely related to the formation of initial cracks that eventually may become critical. Consider the two idealized load histories of Fig 3. It is conceivable that the four low-level load cycles of history "a" introduce a cracking pattern which results in a different kind of damage due to the final strong load cycle, than if this same strong load cycle were to be applied to the undamaged member, as in history "b". Thus, not only the total energy dissipation capacity of the member is dependent on the load history, but its failure mode might be as well.

3.0 Previous Damage Models

Numerous models have been proposed in the past to represent damage of structural members or entire structures. Some of these were derived for metal structures. Because of fundamental differences between reinforced concrete and homogeneous materials such as steel, these models are not directly applicable to reinforced concrete. Other models are based on empirical damage definitions[8]. These all but disregard the mechanics of the materials involved when subjected to cyclic load, and therefore do not lend themselves to rational predictions of the strength reserve and response characteristics of a structure with a specified degree of damage.

Several investigators have introduced energy indices which are functions of a few selected parameters[4]. Other notable examples are the damage ratio introduced by Lybas and Sozen[5], and the flexural damage ratio and normalized dissipated energy used by Banon[2] as basic damage state variables to derive contours of equal proabability of failure. Of the more recent damage models, the widely cited model of Park and Ang[7] should be noted,

$$D_e = \frac{\delta_{max}}{\delta_u} + \frac{\beta}{Q_y \delta_u} \int dE \qquad (2)$$

where δ_{max} = maximum deformation experienced so far, δ_u= ultimate deformation under monotonic loading, Q_y = calculated yield strength, dE = dissipated energy increment. $\beta = (-0.357 + 0.73\ l/d + 0.24\ n_o + 0.314\ P_t)\ 0.7^{\rho_w}$ with l/d = shear span ratio, n_o = normalized axial force, ρ_w = confinement ratio, P_t = longitudinal steel ratio. In Ref 3, Chung et al critically reviewed these and many other damage models.

4.0 A New Damage Model

4.1 Stiffness Degradation

Under load reversals, a RC member experiences a progressive stiffness due to concrete cracking and bond deterioration of the steel-concrete interface primarily in the plastic hinge. The model of Roufaiel and Meyer[9] is used to simulate this behavior. It takes into account the finite size of plastic regions. Fig 4 illustrates the various branches of hysteretic behavior: 1) elastic loading and unloading; 2) inelastic loading; 3) inelastic unloading; 4) inelastic reloading during closing of cracks; and 5) inelastic reloading after closing of cracks. In a reversed load cycle with high shear, previously opened shear cracks tend to close, leading to an increase in stiffness and a characteristic "pinched" shape of the moment-curvature. Roufaiel and Meyer have modeled this effect by introducing the "crack-closing" moment M^+_p, associated with curvature ϕ_p^+. If shear stresses are negligible and the hysteresis loops are stable during cyclic loading, no pinching is likely to occur and branches 4 and 5 will form a single straight line.

4.2 Strength Degradation

In addition to stiffness degradation, RC members experience strength degradation under cyclic loading beyond the yield level. Atalay and Penzien[1] had noticed some correlation between commencement of strength degradation and the spalling of the concrete cover. But Hwang's experiments[4] showed that strength degradation can start at considerably lower load levels. Even for loads slightly above the yield level, damage and strength degradation can be observed, provided a sufficiently large number of load cycles is applied. It is, therefore, suggested that strength degradation is initiated as soon as the yield load level is exceeded, and the strength degradation accelerates as the critical load level is reached. For this purpose, a strength drop index, S_d, is proposed[Fig 5],

$$S_d = \frac{\Delta M}{\Delta M_f} = \left(\frac{\phi - \phi_y}{\phi_f - \phi_y}\right)^w \qquad (3)$$

where ΔM = moment capacity(strength) reduction in a single load cycle up to curvature ϕ_y, ΔM_f = fictitious moment capacity(strength) reduction in a single load cycle up to failure curvature ϕ_f. For analysis purposes, the strength drop is measured from the second branch of the primary moment-curvature curve. The actual strength reductions in a single load cycle are indicated by the shaded area in Fig 5. For the parameter ω, calibration studies [3] suggest a value of about 1.5 to 2.0. Denoting the strength drop, $\Delta M = S_d \Delta M_f$, in a single load cycle to some curvature ϕ, as

$$\Delta M=[(\phi_f-\phi_y)p(EI)_e+M_y-M_f]\left(\frac{\phi-\phi_y}{\phi_f-\phi_y}\right)^w, \qquad (4)$$

the residual strength after this one load cycle, Fig 4, is given by

$$m_1(\phi) = M(\phi) - \Delta M$$
$$= M_y+(\phi-\phi_y)p(EI)_e-\Delta M \qquad (5)$$

In order to incorporate this concept of strength degradation into the hysteresis model, an imaginary point with coordinates $(\overline{\phi_x}, \overline{M_x})$, is introduced, at which the load-deformation curve is aimed during reloading, Fig 5. Details are given in Ref 3.

4.3 Definition of Failure

For RC members undergoing cyclic loading, several investigators[1,4,7] have defined failure as the point where the member strength(moment) has dropped below 75% of the initial yield strength(moment). But if the member is subsequently loaded beyond this maximum displacement(curvature), its moment can be observed to increase well above the 75% level[4], even though it has already been assumed to have "failed"[Fig 1]. For this reason it is necessary to relate the failure definition to the actual strength reserve or residual strength, which is a function of the experienced loading histroy.

First, the failure moment M_f and the corresponding curvature ϕ_f is defined to be the curvature at which the concrete's crushing strain is reached. Given the complete stress-strain curves for steel and concrete and the cross-sectional dimensions, it is relatively straightforward to compute the monotonic moment-curvature curve, by determining the moment M_i associated with any curvature, ϕ_i[3]. The failure moments for other curvature levels are assumed as

$$M_{fi} = M_f \frac{2\Phi_i}{\Phi_i + 1.0} \qquad (6)$$

where M_{fi} = failure moment for given curvature level ϕ_i. M_f = failure moment for monotonic loading, $\Phi_i = \phi_i/\phi_f$ (curvature ratio), and ϕ_f = failure curvature for monotonic loading. According to Fig 6, the failure moment M_{fi} decreases with smaller curvature levels ϕ_i, i.e. larger strength drops from the monotonic loading curve are needed to lead to failure. If the total strength drop down to the failure moment M_{fi} at some curvature ϕ_i is known, the number of cycles for this curvature level needed to cause failure, can be determined.

4.4 New Damage Index

Based on the above definition of failure, a new damage index, D_e, is proposed as a measure of damage sustained by RC members undergoing inelastic cyclic loading. It combines a modified Miner's hypothesis with damage modifiers, which reflect the effect of the loading history, and it considers the fact that RC members typically respond differently to positive and negative moments:

$$D_e = \sum_i \alpha_i^+ \frac{n_i^+}{N_i^+} + \alpha_i^- \frac{n_i^-}{N_i^-} \qquad (7)$$

where i = indicator of displacement (curvature) level, $N_i = (M_i - M_{fi})/\Delta M_i$ = number of cycles to cause failure at curvature level i, n_i = number of cycles actually applied at curvature level i, α_i = damage modifier, and + and - are indicators of loading sense. The loading history effect is captured by including the damage modifier α_i, which, for positive moment loading, is defined as

$$\alpha_i^+ = \frac{\sum k_{ij}^+}{n_i^+ \times \overline{k_i}^+} \frac{\phi_i^+ + \phi_{i-1}^+}{2\phi_i^+} \qquad (8)$$

$$= \frac{\sum M_{ij}^+ / n_i^+}{M_{i1}^+ - 0.5(N_i^+ - 1)^+ \Delta M_i^+} \frac{\phi_i^+ + \phi_{i-1}^+}{2\phi_i^+}$$

where $k_{ij}^+ = M_{ij}^+/\phi_i^+$ is the stiffness during the j-th cycle up to the load level i, $\overline{k_i}^+ = \sum k_{ij}^+ / N_i^+$ is the average stiffness during N_i^+ cycles up to load level i, and $M_{ij}^+ = M_{i1}^+ - (j-1)\Delta M_i^+$ is the moment reached after j cycles up to load level i. The definition of Eq (8) needs some explanation. The energy that is dissipated during a single cycle up to a given curvature level decreases for successive cycles. That means the damage increments also decrease. In a constant amplitude loading sequence, the first load cycle will cause more damage than the last one. Therefore, the α_i-factor decreases as load cycling proceeds, being a function of the stiffness ratio. The factor $(\phi_i^+ + \phi_{i-1}^+)/2\phi_i^+$, is necessary to normalize the damage increments in the case of changing load amplitudes. For negative loading, the damage modifier is defined similarly.

To illustrate the accuracy, with which the proposed mathematical model of Eq (7) can simulate hysteretic response of RC members, many experimental results have been reproduced numerically in Ref 3. Agreement between numerical and experimental results was in general excellent. Fig 7 represents an example that is typical for the kind of agreement achieved. Table 1 contains the cumulative damage indices computed for the same specimens tested by Hwang and Ma et al[4,6]. It is noteworthy that in all but the last case the damage index computed after test termination correlates reasonably well with 1.0, which corresponds to our definition of failure. In some cases, the testing proceeded well beyond this point, e.g. Specimen S22. This means that testing had continued beyond the point of (artificially defined) failure. Other specimens, most notably B35, appear not to have failed at the time the test was terminated.

5.0 Conclusions

A new damage model and associated damage index have been developed which are believed to be more rational than previously proposed models and take into account factors such as loading sequence which are usually ignored in others. An accurate determination of damage is essential for meaningful nonlinear dynamic analysis of concrete structures, because the damage index is closely tied to the strength reserve of a member, after it has undergone large inelastic cycles.

6.0 References

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Table 1. Numerical Cumulative Damage Indices

Nos of	Specimen									
Cycles	S12	S14	S22	S23	S24	S32	S33	S34	R5	B35
1	0.0254	0.0569	0.0707	0.0210	0.0664	0.0722	0.0059	0.0461	0.0041	0.0231
2	0.1250	0.2227	0.2597	0.0685	0.2278	0.2537	0.0225	0.1846	0.0154	0.0697
3	0.2485	0.2644	0.4513	0.1808	0.2863	0.4601	0.0870	0.2234	0.0389	0.1733
4	0.3740	0.2937	0.6422	0.3546	0.3202	0.6545	0.1956	0.2404	0.1043	0.3379
5	0.4871	0.4580	0.8220	0.4080	0.4763	0.8313	0.2156	0.3863	0.1876	0.5816
6	0.5963	0.6439	0.9956	0.4513	0.6453	1.0003	0.2303	0.5445	0.2745	
7	0.7016	0.6704	1.1557	0.5957	0.6950	1.1436	0.3271	0.5740	0.3605	
8	0.8019	0.7040	1.3018	0.7490	0.7296	1.2613	0.4360	0.5993	0.4509	
9	0.9035	0.8326	1.4336	0.7971	0.8552		0.4533	0.6226	0.6354	
10	0.9935	0.9718	1.5552	0.8336	0.9943		0.4061	0.7271	0.8524	
11	1.0721	0.9978		0.9565			0.5491	0.8438	1.0771	
12	1.1395	1.0156		1.0844			0.6392	0.8722	1.3877	
13	1.1900	1.1044					0.6540	0.8910		
14	1.2435	1.1976					0.6647	0.9790		
15	1.2837	1.2142					0.7307			
16	1.3215	1.2252					0.8029			
17	1.3600	1.2837								
18		1.2925								

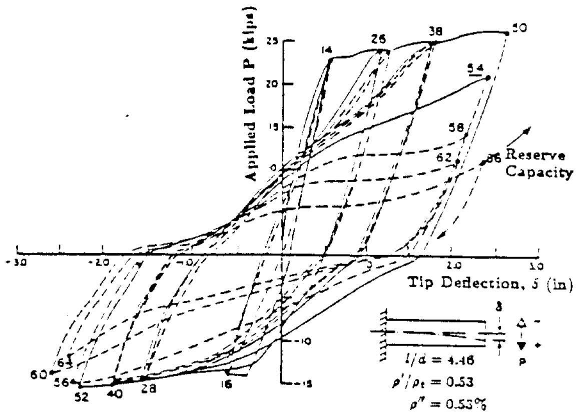

Fig. 1 – Typical Inelastic Response of RC Member (Ref. 6)

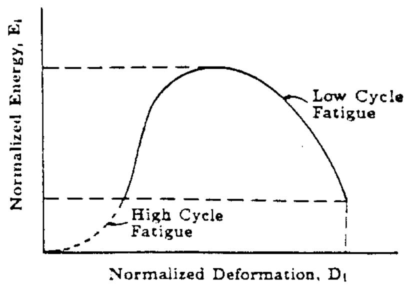

Fig. 2 – E_i–D_i Curve for RC Member

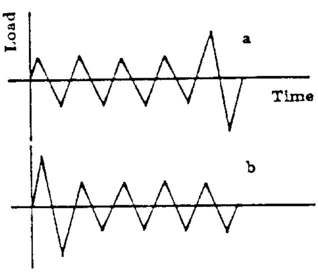

Fig. 3 – Two Different Load Histories

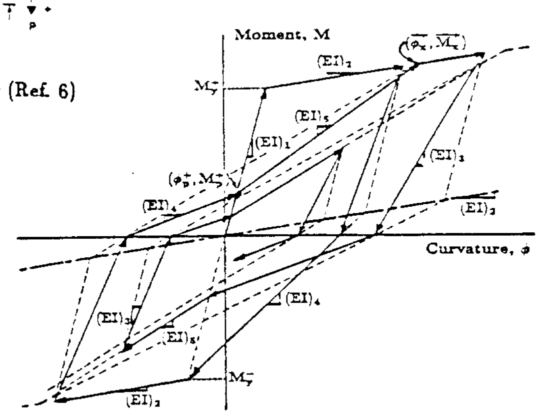

Fig. 4 – Typical Hysteretic Moment-Curvature Relationship

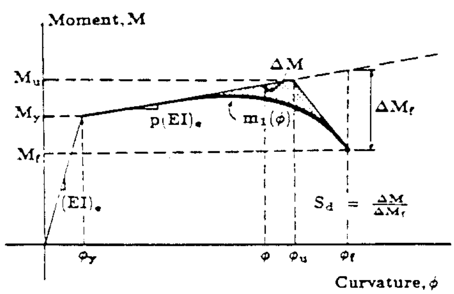

Fig. 5 – Strength Degradation Curve

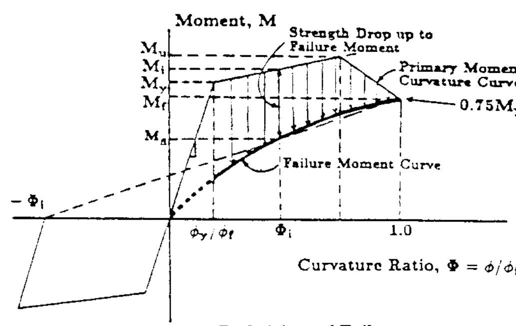

Fig. 6 – Definition of Failure

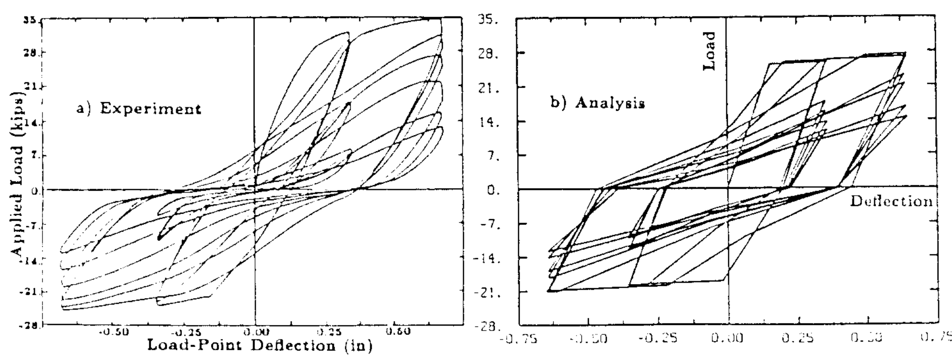

Fig. 7 – Experimental and Analytic Load-Deformation Curves for Beam S2-3 tested by Hwang (Ref. 4)