# Power system stabilizer using VSS-MFAC

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#### Abstract

In this paper we present a variable structure systeme-model following adaptive control (VSS-MFAC) method for an uncertain turbo-generator system which is apt to suffer from the unmodeled parameter uncertainties and the external disturbances. The simulation results for the power system stabilizer(PSS) exhibit robust adaptive model-following properties well in the PSS designed by the proposed VSS-MFAC methodology when a step change in the mechanical torque and apparameter variation is applied.

### 1. Introduction

In order to improve the power system stability, additional stabilizing signals are introduced into the excitation control system. The controller that generates the stabilizing signal is called the power system stabilizer(PSS). The turbo-generator systems including exiter and governor in the large-scale power systems are apt to suffer from the uncertain parameter variations and the external desturbances. If the sufficiently severe disturbances occur, the turbo-generator system may be led to rapid acceleration of one or more generating units and will lost their synchronism. Moreover, because of the inherent characteristics of changing loads, the operating condition of the these systems may change very much during daily cycle. As a result, a fixed controller which is optimal under operating condition may no longer be suitable in another status. Recently, the sliding mode control theories have been developed over the last twenty years which offers a effective way of the design of the transient stability controllers of a large scale power system[1]. On the other hand, various adaptive control techniques have been proposed for dealing with large parameter variations[2-4]. In this paper, we introduce the robust VSS-MFAC PSS for the uncertain turbo-generator system which subject to the unmodeled plant uncertainties and the external disturbances.

### 2. Model following adaptive control

The state equation of the parameter uncertain system with external disturbances following the reference model can be defined as

$$\begin{split} \dot{x}_{p}(t) &= \widetilde{A}_{p} x_{p}(t) + \widetilde{B}_{p} u_{p}(t) + H_{pd}(t) \\ &= (A_{p} + \Delta A_{p}(r(t)) x_{p}(t) \\ &+ (B_{p} + \Delta B_{p}(s(t)) u_{p}(t) + H_{pd}(t) \end{split} \tag{1}$$

where  $x_p(t) \in R^n$  is the plant state,  $u_p(t) \in R^m$  is the plant control input,  $r(t) \in R \subset R^p$  represents the plant parameter uncertainty,  $s(t) \in \Phi \subset R^l$  represents the input connection parameter uncertainty, and  $\Delta A(\cdot)$  and  $\Delta B(\cdot)$  are assumed to be continuous matrix functions of appropriate dimensions. And, where  $H_{pd}(t)$  is the external disturbance. It is assumed that

- a) ploynominal  $B_p$  is Hurwitz;
- b) degrees n and m (m < n) of  $A_p$  and  $B_p$ , respectively, are known:
- upper and lower bounds of the unknown plant parameters are available,

that is 
$$A_p - \Delta A_p < \widetilde{A}_p < A_p + \Delta A_p$$
,  $p = 0, \dots, n-1$   
 $B_p - \Delta B_p < \widetilde{B}_p < B_p + \Delta B_p$ ,  $p = 0, \dots, m$ .

And, the reference model state equation for the controlled plant is given in general terms of the system

$$\dot{x}_m(t) = A_m x_m(t) + B_m u_m(t) \tag{2}$$

where  $x_m \in \mathbb{R}^n$  is the reference model state and  $u_m \in \mathbb{R}^r$  is the

reference model input,  $A_m$  is a stable matrix and the pair  $(A_m, B_m)$  is assumed completely controllable.

The plant control input can be represented by

$$u_p(t) = -K_p^T x_p(t) + K_m^T x_m(t) + k_u u_m(t) - \delta_l$$
 (3)

where  $K_p^T = [k_{p_1}, \dots, k_{pn}]$  $K_m^T = [k_m, \dots, k_{mn}]$ 

The error between the model and the plant is  $e(t) = x_m(t) - x_p(t) = 0$  and  $\dot{e}(t) = \dot{x}_m(t) - \dot{x}_p(t) = 0$ . It can easily be shown that

$$\dot{e}(t) = \dot{x}_m(t) - \dot{x}_p(t) = 0$$

$$= [A_m - \widetilde{A}_p + \widetilde{B}_p (K_p^T - K_m^T)] x_p(t)$$

$$+ [B_m - \widetilde{B}_p k_u] u_m(t) + [\widetilde{B}_p \delta_t - H_{pd}(t)] \tag{4}$$

# 3. Variable structure-model following adaptive control

In general, the discontinuous control has the form

$$u_{pl}(t) = \begin{cases} u_{pl}^{+} & \text{if } s_{l}(e) > 0 \\ u_{pl}^{-} & \text{if } s_{l}(e) < 0 \end{cases}$$
 (5)

Suppose the sliding mode exists on all hyperplanes. Then, during sliding

$$s(e) = C^{T}e = 0 \text{ and } \dot{s}(e) = C^{T}\dot{e} = 0$$
 (6)

From  $\dot{s} = 0$ , the equivalent control can be represented by

$$u_{eq}(t) = (C^T B_p)^{-1} C^T [A_m e(t) + (A_m - A_p) x_p(t) + B_m u_m(t) - H_{ed}(t)]$$
(7)

assuming det  $[C^T B_n] \neq 0$ .

By comparing (3) with (7), the condition can be represented by

$$K_{m}^{T} = (C^{T}B_{p})^{-1}C^{T}A_{m}$$

$$K_{p}^{T} = (C^{T}B_{p})^{-1}C^{T}A_{p}$$

$$k_{u} = (C^{T}B_{p})^{-1}C^{T}B_{m}$$

$$\delta_{i} = (C^{T}B_{p})^{-1}C^{T}H_{m}$$
(8)

The equivalent control input  $u_{eq}(t)$  of (7) is expressed as

$$u_{eq}(t) = -K_{p}^{T}x_{p}(t) + K_{m}^{T}x_{m}(t) + k_{\mu}u_{m}(t) - \delta_{t}$$
 (9)

By substituting (8) into (4), the error differential state equation of sliding mode is given by

$$\dot{e}(t) = [A_m - B_p(C^T B_p)^{-1} C^T A_m] e(t) + [(A_m - A_p) + B_p(K_p^T - K_m^T)] x_p(t) (B_m - B_p k_u) u_m(t) + B_p \delta_t - H_{pd}$$
(10)

If the perfact model matching conditions from (10) are satisfied,

$$\dot{e}(t) = [I - B_{p}(C^{T}B_{p})^{-1}C^{T}]A_{m}e(t)$$
 (11)

The control input of the plant is expressed by

$$u_{p}(t) = -[K_{e}^{T}(t) e(t) + K_{x}^{T}(t) x_{p}(t) - k_{u}(t)u_{m}(t) + \delta_{l}(t)] + k \phi \Phi[|s(e)|] sgn[s(e)]$$
(12)

where k is a class K function.

From the above equation (6), the condition can be represented by

$$\dot{ss} = [C^{T}A_{m} + C^{T}B_{p}K_{x}^{T}(t)] e(t) s(e) 
+ C^{T}(A_{m} - A_{p}) + C^{T}B_{p}K_{x}^{T}(t)] x_{p}(t) s(e) 
+ [C^{T}B_{m} - C^{T}B_{p}k_{u}(t)] u_{m}(t)s(e) 
+ [C^{T}B_{p}\delta_{l}(t) - C^{T}H_{pd}(t)] s(e) 
+ C^{T}B_{p}k\phi |s(e)| \Phi[|s(e)|] 
\leq -\eta|s(e)| \Phi[|s(e)|]$$
(13)

where  $\eta$  is a positive constant.

And, the necessary and sufficient condition for the existence of a sliding mode on s = 0 can be represented by

$$[C^{T}A_{m} + C^{T}B_{p}K_{\epsilon}^{T}(t)] e(t)s(\epsilon)$$

$$+ [C^{T}(A_{m} - A_{p}) + C^{T}B_{p}K_{x}^{T}(t)] x_{p}(t)s(\epsilon)$$

$$+ [C^{T}B_{m} - C^{T}B_{p}k_{u}(t)] u_{m}(t)s(\epsilon)$$

$$+ [C^{T}B_{p}\delta_{i}(t) - C^{T}H_{pd}(t)] s(\epsilon)$$

$$+ (C^{T}B_{p}k\delta_{i} + \eta)|s(\epsilon)| \Phi[|s(\epsilon)|]$$

$$\leq 0$$
(14)

The switched gains from (14) can be written as

$$\begin{cases} k_{el}^{+} = \min \left[ -b^{-1} \left( c_{l-1} - a_{ml} \right) \right] & \text{if } e_{l}(t) \, s(e) > 0 \\ k_{el}^{-} = \max \left[ -b^{-1} \left( c_{l-1} - a_{ml} \right) \right] & \text{if } e_{l}(t) \, s(e) < 0 \end{cases}$$
 (15)

$$\begin{bmatrix} k_{nl}^{+} = \min \left[ -b^{-1} (a_{nl} - a_{l}) \right] & \text{if } x_{pl}(t) \, s(e) > 0 \\ k_{nl}^{-} = \max \left[ -b^{-1} (a_{nl} - a_{l}) \right] & \text{if } x_{pl}(t) \, s(e) < 0 \end{bmatrix}$$
(16)

$$\begin{bmatrix} k_u^+ = \max [b^{-1}b_m] & \text{if } u_m(t) s(e) > 0 \\ k_u^- = \min [b^{-1}b_m] & \text{if } u_m(t) s(e) < 0 \end{bmatrix}$$
(17)

$$\begin{cases} \delta_{l}^{+} = \min \left[ -b^{-1} d(t) \right] & \text{if} \quad s(e) > 0 \\ \delta_{l}^{-} = \max \left[ -b^{-1} d(t) \right] & \text{if} \quad s(e) < 0 \end{cases}$$
 (18)

Then the nonlinear term is expressed as an exponentially decaying term

$$\Phi[|s(e)|] = \exp[-1/|s(e)|]$$
 (19)

By substituting(19) into (12), the new control input can be represented by

$$u_{p_{new}}(t) = -[K_e^T(t) e(t) + K_x^T(t) x_p(t) - K_u(t) u_m(t) + \delta_i(t)] + k \phi \exp[-1/|s(e)|] \operatorname{sgn}[s(e)]$$
(20)

Then, the structure of the boundary layer controller is

$$u_p(t) = u_{eq}(t)$$
 if  $|s(e)| < \Delta$  (21)

as the so-called boundary layer of thickness  $\Delta$ . The offset resulting from this approximation is rejected by a proportional-integral (P-I) controller included in the control input.

$$u_{PI}(t) = -K_P s(e) - K_I \int_{t_0}^{t} s(e) d\tau.$$
 (22)

where  $K_P$  and  $K_I$  are the gains of the P-I controller.

From (20-22), consider the new control input  $u_{p_{max}}(t)$ 

$$u_{p_{new}}(t) = -\left( \left[ k_e^T(t) \ e(t) + k_x^T(t) \ x_p(t) - k_u(t) u_m(t) + \delta_l(t) \right] + k \phi \exp\left[ -1 / |s(e)| \right] \operatorname{sgn}[s(e)] \right) \text{ for } |s(e)| \ge \Delta$$
or.

$$u_{p_{max}}(t) = u_{eq}(t) + u_{PI}(t) \text{ for } |s(e)| < \Delta$$
 (24)

# 4. example

The corresponding block diagram for the system of a single synchronous machine infinite bus system connected to a large power system through an external impedance is shown in [1]. Then the corresponding plant matrices for a PSS following the reference model from (1) are given by

$$A_p = \begin{bmatrix} -0.1580 & -5.6930 & -0.0560 & 0.1026 & 0 \\ 0 & 0 & -0.1070 & 0 & 0 \\ -0.1023 & 156.58 & -0.0376 & 0.0725 & 0 \\ -1000 & 0 & 0 & -19 & -1000 \\ -29 & 0 & 0 & -0.600 & -30 \end{bmatrix}$$

$$B_p = \begin{bmatrix} 0 & 0 & 0 & 999 & 29 \end{bmatrix}^T$$

and the plant parameter uncertainties from (1) are generated by the random number generator

$$\Delta A_{p} = \begin{bmatrix} \Delta A_{11} & \Delta A_{12} & \Delta A_{13} & \Delta A_{14} & 0 \\ 0 & 0 & \Delta A_{23} & 0 & 0 \\ \Delta A_{31} & \Delta A_{32} & \Delta A_{33} & \Delta A_{34} & 0 \\ \Delta A_{41} & 0 & 0 & \Delta A_{44} & \Delta A_{45} \\ \Delta A_{51} & 0 & 0 & \Delta A_{54} & \Delta A_{55} \end{bmatrix}$$

$$\Delta B_{p} = \begin{bmatrix} 0 & 0 & 0 & \Delta B_{4} & \Delta B_{5} \end{bmatrix}^{T}$$

where the bounds for the uncertain parameters  $\Delta A_{p_{ij}}(t)$  and the uncertain gains  $\Delta B_{p_i}(t)$  of the system are assumed such that

$$-0.0001 < \Delta A_{p_{ii}} < 0.0001$$
,  $-0.0001 < \Delta B_{p_i} < 0.0001$ 

Then, a step change of a terminal voltage and a mechanical torque is given by

$$H_{pd} = \begin{bmatrix} 0.003 & 0 & 0.003 & 0 & 0 \end{bmatrix}^T$$

And the reference model matrices of a PSS from (2) are given by

$$A_{m} = \begin{bmatrix} -0.1590 & -5.7165 & -0.0566 & 0.1125 & 0 \\ 0 & 0 & -0.1080 & 0 & 0 \\ -0.1033 & 157.5822 & -0.0368 & 0.0732 & 0 \\ -1000 & 0 & 0 & -20 & -1000 \\ -30 & 0 & 0 & -0.6 & -31 \end{bmatrix}$$

$$B_{m} = \begin{bmatrix} 0 & 0 & 0 & 1000 & 30 \end{bmatrix}^{T}$$

Then, the switching surface vectors are ... 'v

$$s = \begin{bmatrix} 340 & -0.0002 & 5.7952 & 0.03 & -0.0042 \end{bmatrix}^T$$

The switched gain values are chosen in such a way that the control effort required is moderate and the values are given by

# 5. Simulations

Fig. 1 shows the dynamic performance of a PSS when a step change in the terminal voltage of 0.003 p.u is applied. Fig. 2 shows the corresponding error waveform between the model and the plant. Fig. 3 shows the dynamic performance of a PSS when a step change in the mechanical torque of 0.003 p.u is applied. Fig. 4 shows the corresponding error waveform between the model and the plant. And, the simulations are carried out for a time interval of  $\Delta t = 0.00014[\text{sec}]$ .

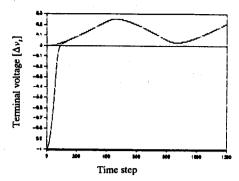


Fig. 1 Terminal voltage  $(\Delta v_t)$  waveforms.

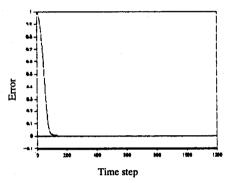


Fig. 2 Asymptotic error waveform for terminal voltage.

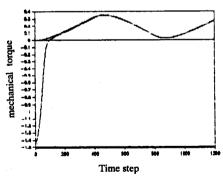


Fig. 3 Mechanical torque waveforms.

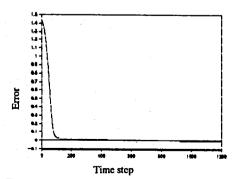


Fig. 4 Asymptotic error waveform for mechanical torque.

# 6. Conclusions

When a step change in the terminal voltage and in the mechanical torque is applied, the proposed VSS-MFAC exhibits insensitivity to the unmodeled plant parameter variations and the external disturbances when operated in the sliding mode. The simulation waveforms for a PSS are clearly shown in the results that the asymptotic error waveform behaviour of the mechanical torque and the terminal voltage between the model and the plant for the uncertain turbo-generator system is achieved with a model following adaptation law.

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