설계민감도 해석을 이용한 정전소자의 형상최적화

고 창 섭* 정 현 교 한 송 엽
서울 대학교 전기공학과 강원 대학교 전기공학과 서울 대학교 전기공학과

OPTIMAL SHAPE DESIGN OF ELECTROSTATIC DEVICES USING DESIGN SENSITIVITY ANALYSIS

Chang-Seop Koh* Hyun-Kyo Jung Song-Yop Hahn
Seoul National University Kangwon National University Seoul National University

ABSTRACT : This paper describes a new algorithm based on design sensitivity analysis for optimal shape design of electrostatic devices. The design sensitivity, the variation of the object function with respect to the design variables, is derived by using implicit differentiation and direct boundary element methods.

The proposed algorithm is applied to the optimal shape design of a concentric cable and the rod electrode enclosed by earthed case. It is shown, from the numerical results, that the algorithm is very usefull for the optimal shape design of the electrostatic devices.

1. INTRODUCTION

The inverse problems in electrostatic field theory have been receiving a considerable attention. Among the inverse problems, a popular and difficult one is shape optimization problems[1,2,3].

To date, the finite element method has been extensively used for the analysis of the electromagnetic devices and successfully applied to some shape optimization problems[4,5]. However, From the view point of design sensitivity analysis, the method is not appropriate because it often requires to redefine the finite element meshes at each iteration step and can not provide accurate solutions on the boundary enough to obtain the accurate design sensitivity[6,7].

Very recently, the boundary element method was recognized as a more attractive technique and good alternative to finite element method in design sensitivity analysis because it can give more accurate solutions on the boundary, which are shown to yield more accurate design sensitivity.[6,7] Moreover the method requires much less effort in regriding due to shape change than the finite element method and reduces dimensionality by one[8].

In this paper, a shape optimization method with design sensitivity analysis is formulated employing the boundary element method and applied to the shape optimal design of some electrostatic devices to demonstrate its effectiveness and usefullness.

2. BOUNDARY ELEMENT FORMULATION

The essence of the boundary element method is the transformation of the governing differential equations into equivalent sets of integral equations. The basic integral equations for the electrostatic field problems are derived as the following form by using the saclar Green's theorem and point-matching method[8].

$$C(r)\,\phi(r) + \int_{\Gamma} \left\{ \phi(r')\,G_n(r,r') - \varepsilon G(r,r')\,\phi_n(r') \right\} ds = 0 \qquad (1)$$

where $C(r)$ is constant determined by the interior angle at point r, ϕ and ϕ_n are electric potential and its normal derivative, G and G_n are fundamental solution and its normal derivative, r and r' are field and source points respectively, n is outward unit normal vector, and ε becomes $\varepsilon_1/\varepsilon_2$ on interface between two different regions and 1 on other boundaries.

Eq.(1) can be reduced to the following numerical equation by discretizing the boundary contour into a series of constant elements and substituting the field variables of each element[8], i.e.,

$$\frac{1}{2}\phi_i + \sum_{j=1}^{M} \int_{-1}^{1} G_n(r_i, r'(\xi))\cdot|J|\,d\xi\ \phi_j - \sum_{j=1}^{M} \varepsilon \int_{-1}^{1} G(r_i, r'(\xi))\cdot|J|\,d\xi\ \phi_{nj} = 0,\ i=1,2,\ldots,M \qquad (2)$$

where J is Jacobian matrix and M is the number of elements. Since r_i refers to the mid-point of i-th element, the collocation is carried out for all the elements on the boundary, which allows eq.(2) to be expressed in the form of matrix equation. Applying the known boundary conditions to the obtained matrix equation, we obtain

$$[K][X] = [F] \qquad (3)$$

where [K] represents the system matrix, [X] and [F] are the unknown state variable and the forcing vectors respectively.

3. DESIGN SENSITIVITY

The most important part of the design sensitivity analysis is the evaluation of the design sensitivity defined as the implicit differentiation of the object function with respect to the design variables. The object function, which has its minimum value at optimal shape, can be defined as[4]

$$\Psi = \Psi\,(\,[P],[X([P])]) \tag{4}$$

where [P] and [X] are design and state variable vectors respectively. Hence, the design sebsitivity subject to i-th design variable P_i, can be expressed as follows:

$$\frac{d\Psi}{dP_i} = \frac{\partial\Psi}{\partial P_i} + \frac{\partial\Psi}{\partial[X]^T}\cdot\frac{d[X]}{dP_i} \tag{5}$$

The first term of the right hand side of eq.(5) shows the direct variation of Ψ under constant [X] and the second one is the indirect variation due to the variation of state variable with respect to P_i.

By differentiating both sides of eq.(3) with respect to P_i, we can obtain the derivative of [X] with respect to P_i in eq.(5) as follows:

$$[K]\frac{d[X]}{dP_i} = -\frac{d}{dP_i}\Big\{[K]\cdot[\tilde{X}] - [F]\Big\} \tag{6}$$

where $[\tilde{X}]$ represents the operating values of state variables previously obtained from eq.(3). The derivative of forcing vector in eq.(6) can be neglected because it contains only known state variables. If [L][U] decomposition method is employed for the solution of eq.(3), lots of computational effort can be saved in solving eq.(6) because only the back substitution is needed. This is a significant advantage of the implicit differentiation method.

The forcing vector of eq.(6) includes the differentiation of the first and second integrations of eq.(2) with respect to design variable P_i. These are calculated as follows:

$$\frac{d}{dP_i}\int_{-1}^{1}\nabla G\cdot n\,|J|\,d\xi\cdot\phi_j$$
$$= \int_{-1}^{1}\Big(\frac{d}{dP_i}(\nabla G)\Big)\cdot n\,|J|\,d\xi\cdot\phi_j + \int_{-1}^{1}\nabla G\cdot\frac{dn}{dP_i}\,|J|\,d\xi\cdot\phi_j$$
$$+ \int_{-1}^{1}\nabla G\cdot n\,\frac{d|J|}{dP_i}\,d\xi\cdot\phi_j \tag{7}$$

$$\frac{d}{dP_i}\int_{-1}^{1}G\,|J|\,d\xi\cdot\phi_{nj}$$
$$= \int_{-1}^{1}\frac{dG}{dP_i}\,|J|\,d\xi\cdot\phi_{nj} + \int_{-1}^{1}G\,\frac{d|J|}{dP_i}\,d\xi\cdot\phi_{nj} \tag{8}$$

The derivatives with respect to design variable P_i in eqs.(7) and (8) can be evaluated by the chain rule, i.e.,

$$\frac{df(r,r')}{d\,P_i} = \nabla f\cdot\frac{\partial r}{\partial P_i} + \nabla' f\cdot\frac{\partial r'}{\partial P_i} \tag{9}$$

where ∇ and ∇' operate on the field and source points respectively.

4. NUMERICAL EXAMPLES

Two examples from electrostatic problems are considered to illustrate numerical implementation of the developed algorithm.

4.1 Shape design of insulators in power cable

Firstly, a power cable which has two-layer of different dielectric constants is considered. The initial shape discretized into the boundary elements is shown in Fig.1 where the numbers of elements and nodes are 71 and 70 respectively. Our task is to find the optimal shape of the interface between insulators, where the tangential components of electric field intensity are negligible and the normal components become constant value along the interface. Hence, the coordinates of the 21 nodes on the interface become the design variables and are allowed to move along the normal direction as shown in Fig.1.

The composit object function Ψ is also defined along the interface as follows:

$$\Psi = \omega_1\int_\gamma E_t^2\,dl + \omega_2\int_\gamma (D_n - D_{no})^2\,dl \tag{10}$$

where E_t is the tangential component of electric field intensity, D_n the normal component of electric flux density, D_{no} its target value given previously, γ the interface between insulators and ω_1, ω_2 the weighting coefficients.

If the constant elements are used in boundary element analysis, eq.(10) can be transformed into as follows because the uniform potential along the interface results the zero tangential components of electric field intensity.

$$\Psi = \sum_{j=1}^{N}\Big\{\omega_1(\phi_j - \phi_o)^2 + \omega_2(\varepsilon_1\phi_{nj} - D_{no})^2\Big\}\,l_j \tag{11}$$

where ϕ_j is calculated potential at j-th element, ϕ_{nj} its normal derivative, ϕ_o average value of potentials over the interface, l_j length of j-th element and N the number of elements on the interface. The weighting coefficients ω_1 and ω_2 have the value of 1 and $(l_j/\varepsilon_1)^2$ in order to adjust the units of first and second terms of eq.(11) into that of potential. The design sensitivity, then, is obtained as

$$\frac{d\Psi}{dP_i} = \sum_{j=1}^{N}\Big\{\omega_1(\phi_j - \phi_o)\cdot\frac{d\phi_j}{dP_i} + \omega_2\varepsilon_1(\varepsilon_1\,\phi_{nj} - D_{no})\,\frac{d\phi_{nj}}{dP_i}\Big\}\cdot 2l_j$$
$$+ \sum_{j=1}^{N}\Big\{\omega_1(\phi_j - \phi_o)^2 + \omega_2(\varepsilon_1\phi_{nj} - D_{no})^2\Big\}\frac{d\,l_j}{d\,P_i} \tag{12}$$

From eq.(5), the derivatives of state variables(ϕ_j, ϕ_{nj}) with respect to design variable P_i of eq.(12) become

$$\frac{d\phi_j}{dP_i} = \frac{\partial\phi_j}{\partial P_i} + \frac{\partial\phi_j}{\partial[X]^T}\cdot\frac{d[X]}{dP_i} \tag{13}$$

$$\frac{d\phi_{nj}}{dP_i} = \frac{\partial\phi_{nj}}{\partial Pi} + \frac{\partial\phi_{nj}}{\partial[X]^T}\cdot\frac{d[X]}{dP_i} \tag{14}$$

The first terms ($\partial\phi_j/\partial P_i$, $\partial\phi_{nj}/\partial P_i$) of eqs.(13),(14) have no contributions bacause ϕ_j and ϕ_{nj} belong to the state variables.

The variation of object function value versus iteration number is shown in Fig.2 and the optimal shape is obtained after 9 times of iteration using the steepest decent method. The initial and final optimized shapes of cable are shown in Fig.3 with equi-potential lines. The distributions of potential and normal component of electric flux density along the interface of insulators at final optimized shape are shown in Fig.4 and Fig.5. It is shown that the uniform potential and constant electric flux density are achieved.

4.2 Shape design of case of electrode

The earthed case enclosing the rod electrode is considered. The initial shape discretized into the boundary elements is shown in Fig.6 where the numbers of elements and nodes are 47 and 46 respectively. Out task , in this model, is to find an optimal shape of the case where the normal components of electric field intensity become constant value along the surface of the electrode.

The object function Ψ is defined as

$$\Psi = \sum_{j=1}^{N} (-\phi_{nj} - E_{no})^2 \cdot l_j \tag{15}$$

where N is the number of elements on the surface of electrode, l_j the length of j-th element and E_{no} the constant target value given as the value of the cylinder part of electrode at initial shape. The coordinates of 15 nodes on the case become the design variables and are allowed to move along the normal direction as shown in Fig.6. The design sensitivity can be derived similarly to eqs.(12) and (14).

The final optimized shape is obtained after 13 times of iteration and is shown in Fig.7 with initial shape and equi-potential lines. The distribution of electric field intensity along the surface of electrode is shown in Fig.8 where the constant electric field intensity is obtained. It is noted that almost of the design variables are expanded but some are reduced. This result is consistent well with that of previous result obtained by different algorithm[9].

5.CONCLUSION

A shape optimization method with design sensitivity analysis employing the boundary element method is presented. The design densitivity is based on the implicit differentiation of the discretized boundary integral equation with respect to the design variables. The integration of the new kernels that involve the derivative of Jacobian, normal vector, fundamental solution and its normal derivative with respect to the design variables is performed numerically by using the Gauss-Legendre and Stroud-Secret formula. The effectivess of this formulation is demonstrated through some numerical examples.

Finally it is found that the boundary element method is more usefull for the shape optimal design than the finite element method, because it enables us to advoid the difficulties in regenerating of mesh at each iteration step and affords better solutions on the boundary which are most important for the evaluation of the design sensitivity.

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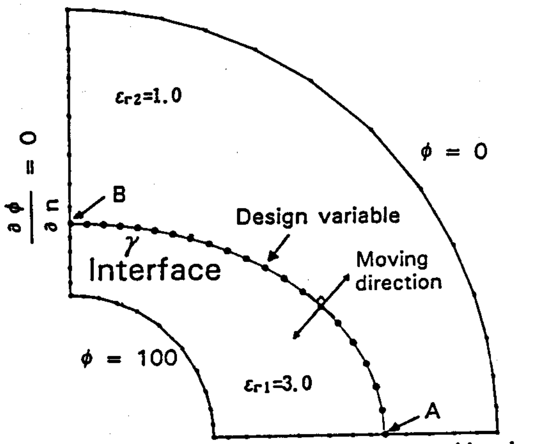

Fig. 1 Power cable composed of two dielectrics and boundary elements discretization.

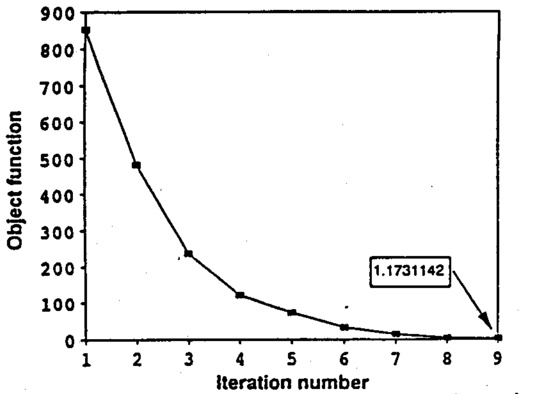

Fig. 2 Variation of object function versus iteration number.

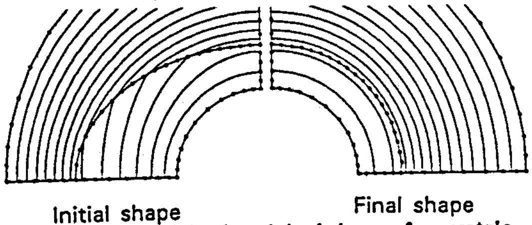

Fig. 3 Initial and final optimized shapes of concentric cable with equi-potential lines.

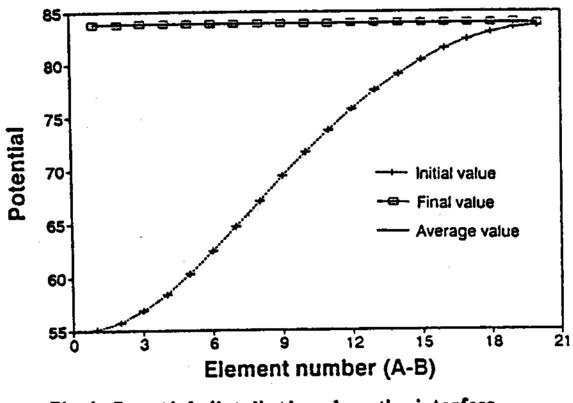

Fig. 4 Potential distribution along the interface.

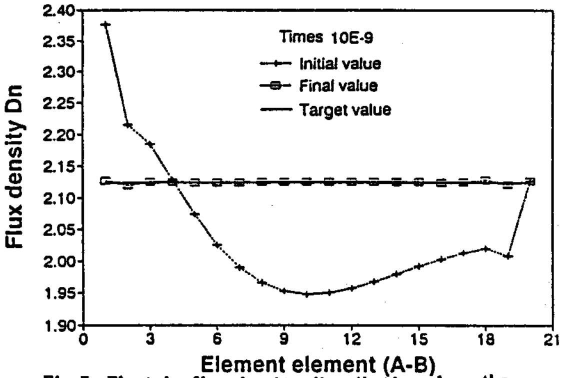

Fig. 5 Electric flux density distribution along the interface.

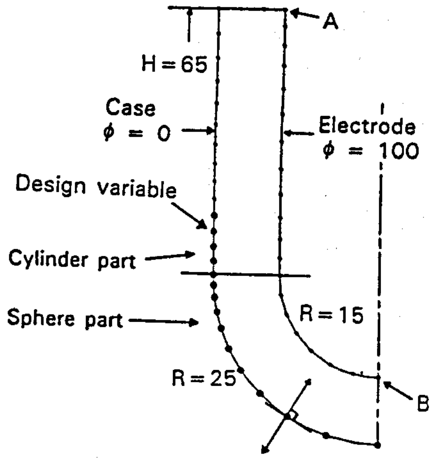

Fig. 6 Initial shape of cylineric case.

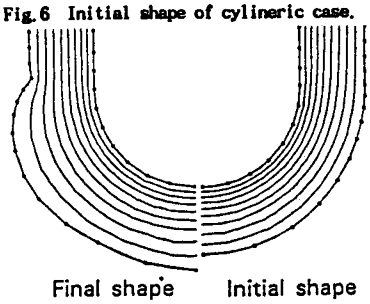

Fig. 7 Initial and final optimized shapes of cylindric case with equi-potential lines.

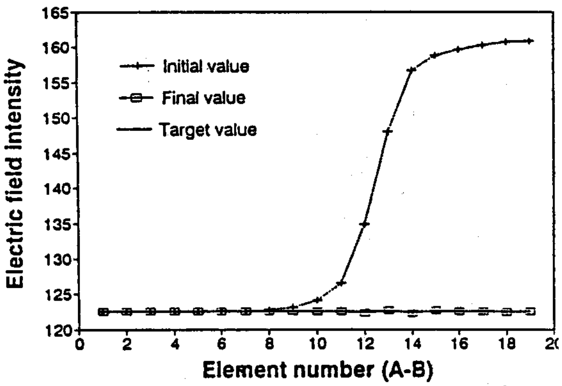

Fig. 8 Electric field intensity distribution along the interface.