자코비안 행렬의 마이너(Minor)에 기초한 여유자유도 로봇의 동력학적 제어

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Dynamic Control of Redundant Manipulators based on the Minors of Jacobian Matrix

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Abstract

For the control of redundant manipulators, conventional dynamic control methods of local torque optimization showed the instability which resulted in physically unrealizable torque requirements. In this paper, a new dynamic control method which is based on the concept of *aspects* is proposed. The proposed method starts with the basic understanding of the minors in the Jacobian matrix. It was shown by computer simulations that the proposed method demonstrates a drastic reduction of torque loadings at the joints in the tracking motion of a long trajectory, and thus guarantees the stability of joint torque.

1 Introduction

Kinematically redundant manipulators have more joint degrees-of-freedoms (DOF) than are required to complete a specified task. The majority of researches on utilizing redundancy have been focused on the resolution of redundancy at the kinematic level. This paper presents a new real-time control law which is based on the *full row rank* minors of a Jacobian matrix, and the effectiveness of the new approach is evaluated by computer simulations in comparison with other conventional methods.

2 Preceding Studies on Control of Redundant Manipulators

The dynamic resolution of redundancy, which is taking account of the manipulator dynamics, has been mainly focused on minimizing torque loadings at the joints. The command torque $\boldsymbol{\tau}$ of model-based control is in general generated by the measured values of joint angle and velocity vectors, namely $\boldsymbol{\theta}$ and $\dot{\boldsymbol{\theta}}$, and the command joint acceleration $\ddot{\boldsymbol{\theta}}_d$. That is,

$$\boldsymbol{\tau} = \boldsymbol{M}(\boldsymbol{\theta})\,\ddot{\boldsymbol{\theta}}_d + \boldsymbol{N}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}}) \tag{1}$$

where $\boldsymbol{M}(\boldsymbol{\theta}) \in \Re^{n\times n}$ is a symmetric, positive definite inertia matrix, and $\boldsymbol{N}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}}) \in \Re^n$ is a vector containing terms such as Coriolis, centripetal, and gravitational torques.

The dynamic resolution of redundancy was at first presented by Khatib [1] who proposed the inertia-weighted pseudoinverse $\boldsymbol{J}_M^+(\boldsymbol{\theta})$ which instantaneously minimizes the kinetic energy $1/2\,\dot{\boldsymbol{\theta}}^T\boldsymbol{M}(\boldsymbol{\theta})\,\dot{\boldsymbol{\theta}}$. In this case, the inertia-weighted pseudoinverse matrix is given by

$$\boldsymbol{J}_M^+ = \boldsymbol{M}^{-1}\boldsymbol{J}^T\left(\boldsymbol{J}\,\boldsymbol{M}^{-1}\,\boldsymbol{J}^T\right)^{-1} \tag{2}$$

when $\boldsymbol{J}$ has full rank. Thus, the command joint acceleration $\ddot{\boldsymbol{\theta}}_d$ is expressed in the form

$$\ddot{\boldsymbol{\theta}}_d = \boldsymbol{J}_M^+\,(\ddot{\boldsymbol{x}}_d + \boldsymbol{K}_v\,\dot{\boldsymbol{e}} + \boldsymbol{K}_p\,\boldsymbol{e} - \dot{\boldsymbol{J}}\,\dot{\boldsymbol{\theta}}) \tag{3}$$

where $\boldsymbol{e} \triangleq \boldsymbol{x}_d - \boldsymbol{x}$ is a tracking error vector, $\boldsymbol{K}_v$ and $\boldsymbol{K}_p$ are constant velocity and position feedback gain matrices, respectively.

While Khatib's approach is indirectly related to torque optimization, Hollerbach and Suh [2] resolved redundancy at the acceleration level to instantaneously minimize joint torque. In this case, the command torque is obtained in the form

$$\boldsymbol{\tau} = \tilde{\boldsymbol{\tau}} - \boldsymbol{M}\left[\boldsymbol{M}\,(\boldsymbol{I} - \boldsymbol{J}^+\boldsymbol{J})\right]^+\tilde{\boldsymbol{\tau}}, \tag{4}$$

where $\tilde{\boldsymbol{\tau}}$ is given by

$$\tilde{\boldsymbol{\tau}} = \boldsymbol{M}\,\boldsymbol{J}^+(\ddot{\boldsymbol{x}}_d + \boldsymbol{K}_v\,\dot{\boldsymbol{e}} + \boldsymbol{K}_p\,\boldsymbol{e} - \dot{\boldsymbol{J}}\,\dot{\boldsymbol{\theta}}) + \boldsymbol{N}. \tag{5}$$

Unfortunately, it has been shown in [2] that both Khatib's approach and the method of Hollerbach and Suh require remarkable large joint torques in the tracking motion of a long trajectory. To overcome this instability of local torque optimization, Hirose and Ma [3] proposed a control method named "Redundancy Decomposition Control", which decomposes the degrees-of-freedom of a redundant manipulator into a subset of non-redundant combinations. Only the decomposed non-redundant manipulator with the selected m joint DOF is presumed to be accelerated/decelerated during the time interval from t to $t+\Delta t$, while the other $(n-m)$ joints are kept at constant velocity during the time interval Δt. Comparing the magnitudes of command torques for ${}_nC_m$ combinations, the optimal combination which has the minimum magnitude is adopted for generating the command torque during the time interval from t to $t+\Delta t$. The redundancy decomposition control method also fails to hold down the joint torques at reasonable values, which will be shown in the computer simulations performed later, and thus does not guarantee the stability of torque optimization.

3 Proposal of a New Control Law

In this section, we propose a new dynamic control law based on the concept of an *aspect* which are a function of a manipulator's configuration. Borrel and Liégeois [4] found that there exist various classes of configurations called "aspects". The admissible domain in the joint space is divided into ${}_nC_m$ aspects for m task variables and n joint variables. One of the separating surfaces between aspects is the locus of joint coordinates corresponding to one of the m-th order minors of the manipulator Jacobian matrix $J \in \Re^{m\times n}$ equal to zero.

Based on the definition of an aspect, we can suggest the hypothesis that *the necessary condition to guarantee the stability of joint torque is to preserve the aspect to which an initial configuration at the beginning of a task belongs.* Thus the proposed method for the dynamic control of redundant manipulators aims at preserving this preferred aspect by using the performance function $H(\theta)$ which has a direct control over each m-th order minor, and generating the command joint velocity $\dot{\theta}_d$ from the resolved motion method [5] expressed by

$$\dot{\theta}_d^{i+1} = J^+(\theta^i)\,\dot{x}_d^i + \kappa\left\{I - J^+(\theta^i)\,J(\theta^i)\right\}\nabla H(\theta^i) \quad (6)$$

where i denotes the current state at time $t = i\cdot\Delta t$ (Δt: sampling time); θ^i and $\dot{\theta}^i$ are the measured values of joint angle and velocity at time t, respectively. Especially, the performance function $H(\theta)$ in the above equation plays an important role in the proposed method because the aspect can be preserved by maximizing this function.

In order to preserve an aspect or prevent the switching of aspects, Chang [6] proposed the minor measure as the product of minors of the Jacobian matrix, which is expressed in the form

$$H = \left|\prod_{i=1}^{p}\Delta_i\right|^{1/p}, \quad (7)$$

where Δ_i's for $i = 1,2,\ldots,p$ are the minors of rank m of the Jacobian matrix $J \in \Re^{m\times n}$ and $p = {}_nC_m$. Each minor is maximized as well as kept at nonzero by maximizing through the gradient vector of this measure in the second term of Eq. (6), so the aspect can be preserved.

The desired joint angle (or displacement for a prismatic joint) θ_d is calculated by numerical integration:

$$\theta_d^{i+1} = \theta_d^i + \dot{\theta}_d^i\cdot\Delta t \quad (8)$$

where $\dot{\theta}_d^0$ is assumed to be equal to $\dot{\theta}^0$ for $i=0$. The command joint acceleration $\ddot{\theta}_d$ is generated by numerical differentiation:

$$\ddot{\theta}_d^{i+1} = \frac{\dot{\theta}_d^{i+1} - \dot{\theta}_d^i}{\Delta t} \cong \frac{\dot{\theta}_d^i - \dot{\theta}_d^{i-1}}{\Delta t} \quad (9)$$

where $(\dot{\theta}_d^{i+1} - \dot{\theta}_d^i)$ is assumed to be approximately equal to $(\dot{\theta}_d^i - \dot{\theta}_d^{i-1})$, and $\ddot{\theta}_d^0$ is assumed to be zero. The above numerical differentiation can generate relatively large values of the command joint accelerations, which may in turn induce large joint torques, especially at the beginning stage of motion when $\dot{\theta}_d^0 = 0$ and a large value of $\dot{\theta}_d^1$ due to the improper choice of κ in Eq. (6) are assigned to Eq. (9). This difficulty can be fixed by selecting a small value of κ which is enough to induce smooth self-motion. It is noted that the command joint acceleration is not influenced by any noise signal because the measured joint velocities are not involved in Eq. (9).

By using the computed-torque control law [7] which is well known for non-redundant manipulators, the command torque can be easily generated in the form

$$\tau = M(\theta^i)\left\{\ddot{\theta}_d^i + K_v'(\dot{\theta}_d^i - \dot{\theta}^i) + K_p'(\theta_d^i - \theta^i)\right\} + N(\theta^i,\dot{\theta}^i), \quad (10)$$

where $K_v' \in \Re^{n\times n}$ and $K_p' \in \Re^{n\times n}$ are position and velocity feedback gain matrices, respectively. It is worthwhile noticing that the proposed method does not have any torque optimization scheme explicitly, but focuses on suppressing the switching of aspects at the kinematic level.

4 Computer Simulations

The comparative evaluation of the proposed method against the conventional methods summarized in Section 2 has been conducted by computer simulations. The simulated model is a planar 3-DOF manipulator depicted in

Fig. 1. The links are all identical and modeled as the uniform thin rods with lengths of 1 m and masses of 10 kg. The joints are labelled 1, 2, 3 from the base of the manipulator. The presumed movements of the end-effector are straight-line Cartesian trajectories starting and ending with zero velocity, with constant tangential acceleration and deceleration over first and last halves of the movements, respectively. The fourth-order Runge-Kutta algorithm is used to find the next joint velocities and angles with the integration time of $\Delta t = 2$ ms or the sampling rate of 500 Hz.

To generate a long trajectory of the end-effector, the command acceleration/deceleration of the end-effector and the total duration time are given by $\ddot{x}_d = [\pm 1.4 \ \mp 1.1]^T$ m/s^2 and $T = 3$ s, respectively, where the signs $\pm$ and $\mp$ denote acceleration/deceleration and deceleration/acceleration over first and last halves of the movement. The simulation for the proposed method was performed for this long trajectory. The initial configuration for the given trajectory is chosen as $\theta = [140° \ -50° \ -10°]^T$ which is close to a singular configuration. The minor measure was used for the proposed method as the performance function which plays a key role in suppressing the switching of aspects, and is expressed for the simulated manipulator by

$$\begin{aligned} H &= \left\{\det[J^1 \ J^2]\cdot\det[J^2 \ J^3]\cdot\det[J^3 \ J^1]\right\}^{1/3} \\ &= \{(\ell_1\ell_2 S_2 + \ell_1\ell_3 S_{23})\,\ell_2\ell_3 S_3\,(-\ell_2\ell_3 S_3 \\ &\qquad - \ell_1\ell_3 S_{23})\}^{1/3}, \qquad (11) \end{aligned}$$

where J^i is the i-th column vector of the matrix $\boldsymbol{J}$; $S_2 = \sin\theta_2$, $S_3 = \sin\theta_3$, and $S_{23} = \sin(\theta_2+\theta_3)$. In the proposed method, a small value of κ in Eq. (6) is desirable for suppressing large self-motion at the beginning stage of the task, and thus the constant of κ was selected as 0.001.

The arm motion illustrated in Fig. 2 implies that the manipulator's configurations stay within one kind of aspects, namely the aspect to which the initial configuration belongs. This is verified in Fig. 3 which demonstrates that any minor among the three ones does not become zero. Fig. 4 illustrates that the proposed method has extremely small values of joint torque norms when compared with the other three methods including the method of Khatib, the method of Hollerbach and Suh, and the method of Hirose and Ma which are simulated for the same trajectory. This figure implies that the proposed method guarantees the stability of joint torque.

5 Conclusion

This paper presented a new real-time dynamic control law which is based on the observation that the conventional methods of local torque optimization have shown the instability of joint torques in the tracking motion of a long trajectory due to the frequent switching of aspects. By maximizing the minor measure which has a direct control over each minor of full row rank in a manipulator Jacobian matrix, the proposed method can preserve an aspect which is a function of a manipulator's configuration.

It was shown by computer simulations that the proposed method generates in the global sense reasonably low values of joint torques which are within physically achievable limits. The proposed method is simple from the viewpoint of real-time control, and efficient in torque minimization when compared with the other conventional methods.

References

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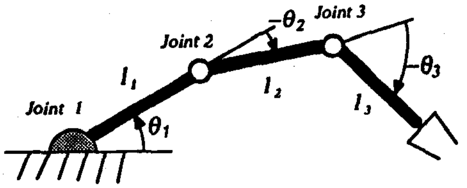

Fig. 1 Geometry of the planar 3-DOF manipulator.

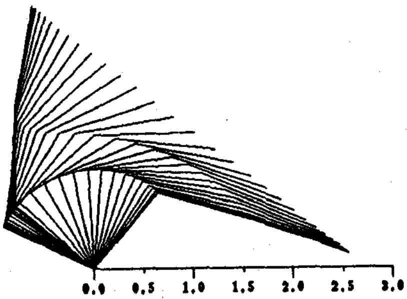

Fig. 2 Arm motion.

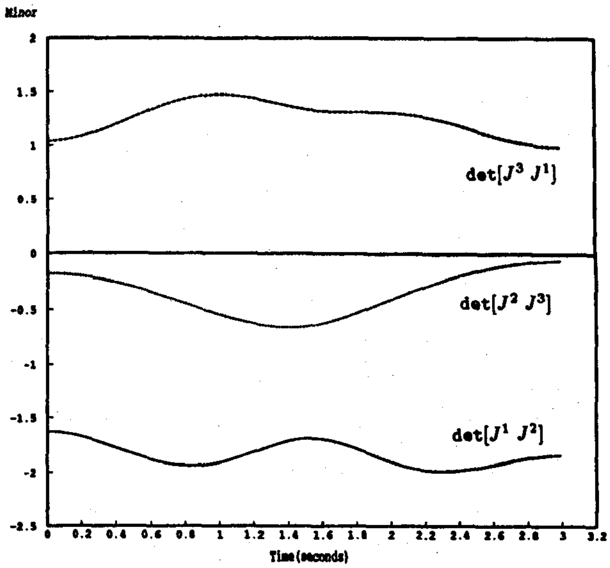

Fig. 3 Minor profiles.

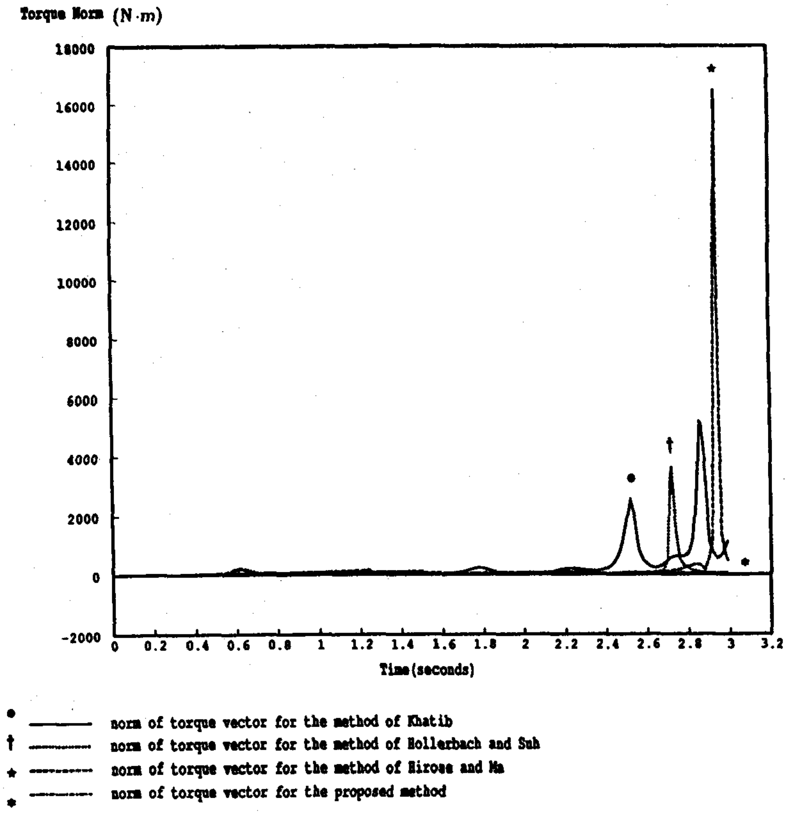

Fig. 4 Profiles of torque magnitude for the four methods.