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Application of a TDOF controller to Chaotic Dynamical Systems

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Abstract

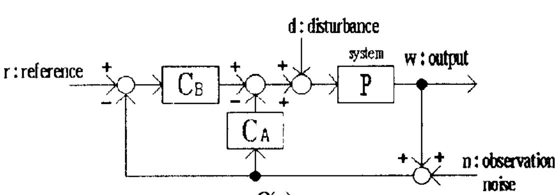
We apply a TDOF (Two Degrees of Freedom) robust controller to chaotic systems. We show that the TDOF robust controller is effective not only for rejection of chaotic disturbance but also for control of a chaotic plant.

1.Introduction

Recently deterministic chaos has been receiving much attention in various fields of science and technology. In particular, since it has been shown that deterministic chaos can be observed even in many engineering systems such as strong vibrations of elastic structures and articulated moving towers, how to control chaos is becoming an important problem in engineering. The aim of this study is to examine possibility of controlling chaotic dynamical systems with a two degrees of freedom (TDOF) robust controller [1-3].

First we investigate the case where disturbance to systems is produced by chaotic dynamics. Chaos we used here is generated by the Lorenz equations [4].

Next we investigate the case where the plant itself is the Lorenz system. We examine properties of three control systems with the chaotic plant.



$$C_A(s)=\frac{Q(s)}{Pn(S)(1-Q(s))}$$

$$C_B(s)=\frac{Gry(s)}{1-Gry(s)}\frac{1}{Pn(S)(1-Q(s))}$$

Fig.1. The Configuration of the TDOF controller [2] where *P* is the actual plant, *Pn(s)* is the nominal model of the plant, *Gry(s)* is the reference model of the command input response and *Q(s)* is a strictly proper low-pass filter.

2.TDOF robust controller

In this paper, we use a TDOF robust controller[3] both for rejection of chaotic disturbance and for control of a chaotic plant. The TDOF robust controller has an important feature that both of the command input response and the disturbance response can be independently designed. The efficacy of the TDOF robust controller has been confirmed in the case of controlling robotic manipulators [3].

Fig.1 shows the configuration of the TDOF controller [3]. We can nominalize the plant to a nominal system with the robust controller even if we cannot get the exact model of the plant. It is also shown that this controller suppresses stochastic disturbance to the system and parameter variations of the plant.

3.The Lorenz Equations

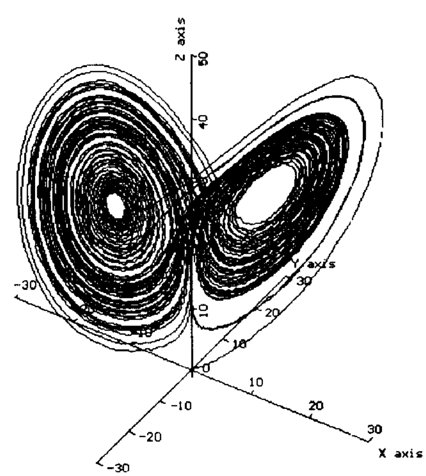


Fig.2. The Lorenz attractor.

We use the Lorenz equations[4] as the chaotic system in this report. The Lorenz equations are described by the following ordinary differential equations with three state variables.

$$\frac{dx}{dt}=\sigma(-x+y) \quad (1)$$
$$\frac{dy}{dt}=-xz+rx-y \quad (2)$$
$$\frac{dz}{dt}=xy-bz \quad (3)$$

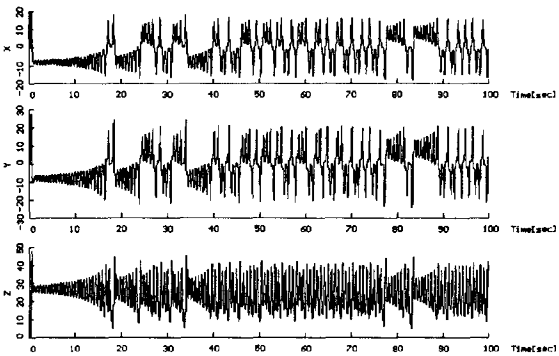


Fig.3. Waveforms of the state variables x,y and z

The parameter values are follows : σ=10, r=28, and b=8/3. The Lorenz equations are numerically calculated by 4th order Runge-Kutta algorithm. We set the initial condition at x=0.0, y=0.1, z=0.0 and the step time dt=0.01. Fig.2 shows the Lorenz attractor of the equations. We also show time series of three state variables in Fig.3.

4.Rejection of chaotic noise

Fig.4 is the equivalent block diagram of Fig.1. We use the time series of the state variable x as chaotic disturbance.

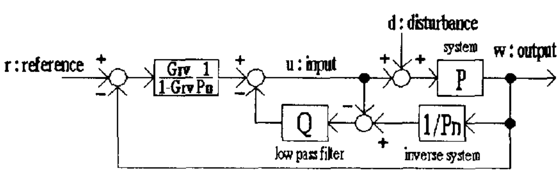


Fig.4. The equivalent block diagram of Fig.1.

Fig.5 is the essential part of the TDOF robust system which performs the nominalization of the plant. The dynamics of the control is analyzed on this system of Fig.5 in the following.

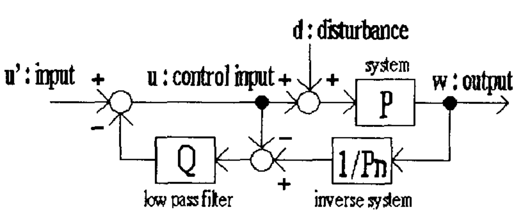
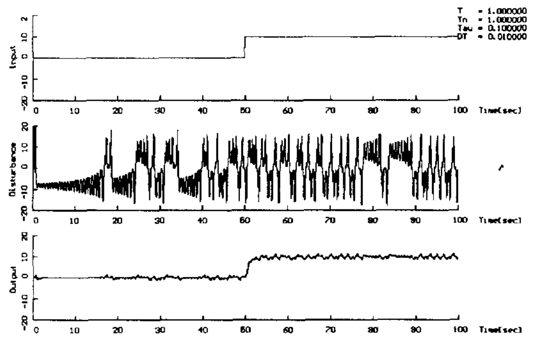


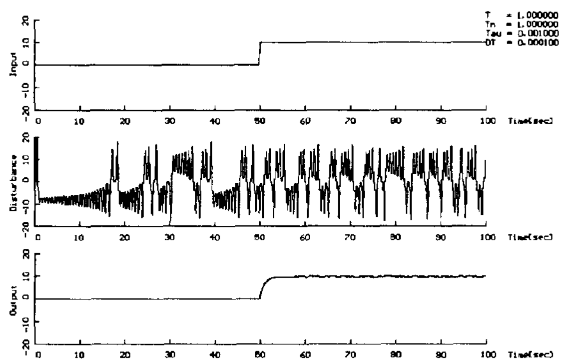
Fig.5. The essential part of the TDOF robust system which performs the nominalization of the plant.

For the sake of simplicity, We represent the plant $P(s)$, the inverse nominal system $Pn^{-1}(s)$ and the low pass filter $Q(s)$ as follows, where T=1.0 and Tn=1.0.

$$P=\frac{1}{1+Ts} \quad (4)$$
$$Pn^{-1}=1+Tns \quad (5)$$
$$Q=\frac{1}{1+\tau s} \quad (6)$$



(a) The case τ = 0.1



(b) The case τ = 0.001

Fig.6. Response characteristics of the system with chaotic disturbance.

Figs.6(a) and (b) show the results in which the values of the time constant τ of the lowpass filter are 0.1 and 0.001, respectively.

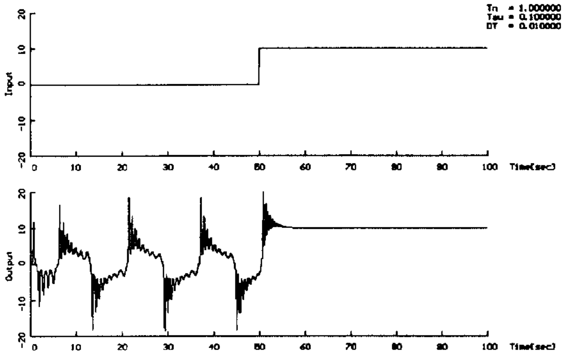
These results show that the TDOF robust controller can nominalize the plant and that the chaotic disturbance is effectively suppressed.

5.Control of a chaotic system

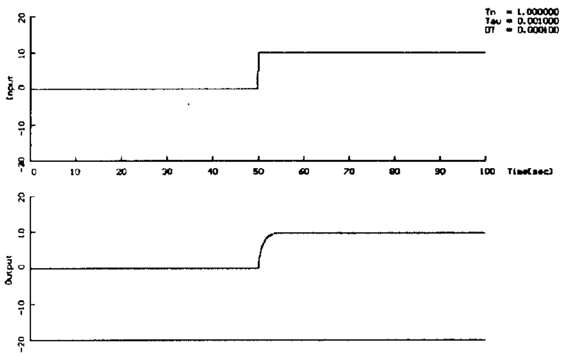
We apply the TDOF robust controller to a chaotic plant. We assume that the dynamics of the plant is represented with Lorenz equations.

First, we deal with the case where the output of the plant is the variable x of the Lorenz equations and the control input is added on the right-hand side of eq.(1) (Model 1).

$$\frac{dx}{dt}=\sigma(-x+y)+u \quad (7)$$
$$\frac{dy}{dt}=-xz+rx-y \quad (8)$$
$$\frac{dz}{dt}=xy-bz \quad (9)$$



(a) The case τ = 0.1



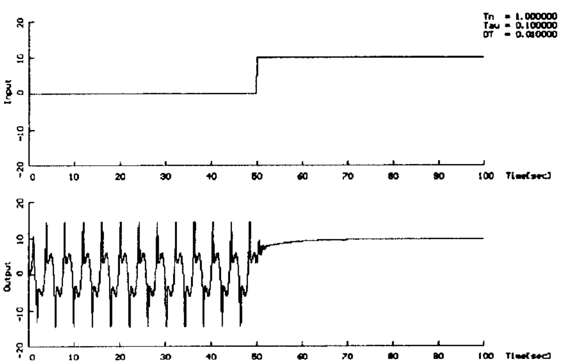
(b) The case τ = 0.001

Fig.7. Simulation results of Model 1.

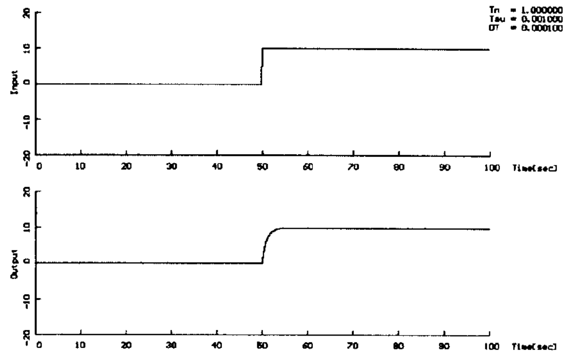
The results of Model 1 are shown in Fig.7(a) and 7(b). Fig.7 shows that the chaotic plant is nominalized to the linear system to some extent.

Next, we deal with the case where the output of the plant is the variable y of the Lorenz equations and the control input is added on the right-hand side of eq.(2) (Model 2). These results are shown in Fig.8(a) and 8(b). These results are similar to those of Model 1.

Last, we deal with the case where the output of the plant is the variable z of Lorenz equations and the control input is added on the right-hand side of eq.(3) (Model 3). These results are different from those of Model 1 and Model 2. In this case, we cannot control the chaotic plant.

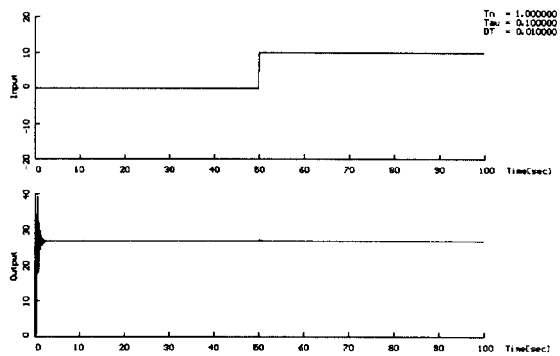


(a) The case τ = 0.1

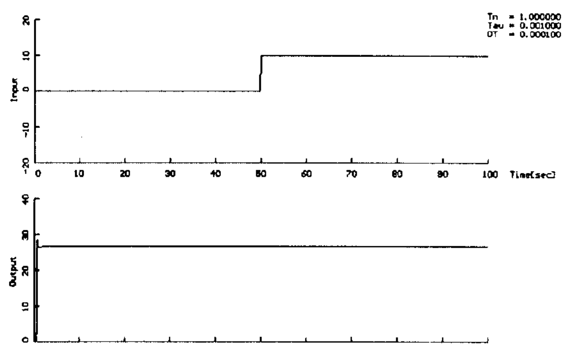


(b) The case τ = 0.001

Fig.8. Simulation results of Model 2.



(a) The case τ = 0.1



(b) The case τ = 0.001

Fig.9. Simulation results of Model 3.

Although Model 1 and Model 2 clearly demonstrate that the chaotic plant can be controlled with the TDOF controller, this controller doesn't always succeed to control the chaotic dynamical system. Model 3 shows an example of the failure. This failure is consistent with the fact that right and left wings peculiar to the Lorenz attractor can not be distinguished by observing only the variable z.

6.Discussion

Through the simulations, we have examined the ability to reject chaotic noise and to control chaotic system using TDOF robust controller.

It is concluded that (1) disturbance of deterministic chaos can be suppressed with the TDOF controller and that (2) chaotic plan can be controller with the TDOF controller under some conditions depending upon control and observation variables. It is a future problem to discuss the problem of controlling chaotic dynamical systems from the viewpoint of controllability and observability of nonlinear dynamical systems.

References

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