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ON DISCRETE NONLINEAR SELF-TUNING CONTROL

R.R. Mohler, V. Rajkumar, and R.R. Zakrzewski

Oregon State University
Department of Electrical and Computer Engineering
Corvallis, OR 97331
U.S.A.

ABSTRACT

A new control design methodology is presented here which is based on a nonlinear time-series reference model. It is indicated by highly nonlinear simulations that such designs successfully stabilize troublesome aircraft maneuvers undergoing large changes in angle of attack as well as large electric power transients due to line faults. In both applications, the nonlinear controller was significantly better than the corresponding linear adaptive controller. For the electric power network, a flexible a.c. transmission system (FACTS) with series capacitor power feedback control is studied. A bilinear auto-regressive moving average (BARMA) reference model is identified from system data and the feedback control manipulated according to a desired reference state. The control is optimized according to a predictive one-step quadratic performance index (J). A similar algorithm is derived for control of rapid changes in aircraft angle of attack over a normally unstable flight regime. In the latter case, however, a generalization of a bilinear time-series model reference includes quadratic and cubic terms in angle of attack.

These applications are typical of the numerous plants for which nonlinear adaptive control has the potential to provide significant performance improvements. For aircraft control, significant maneuverability gains can provide safer transportation under large wind-shear disturbances as well as tactical advantages. For FACTS, there is the potential for significant increase in admissible electric power transmission over available transmission lines along with energy conservation.

Electric power systems are inherently nonlinear for significant transient variations from synchronism such as may result for large fault disturbances. In such cases, traditional linear controllers may not stabilize the swing (in rotor angle) without inefficient energy wasting strategies to shed loads, etc. Fortunately, the advent of power electronics (e.g., high-speed thyristors) admits the possibility of adaptive control by means of FACTS. Line admittance manipulation seems to be an effective means to achieve stabilization and high efficiency for such FACTS. This results in parametric (or multiplicative) control of a highly nonlinear plant.

1. INTRODUCTION

The purpose of this paper is to study the effectiveness of nonlinear self-tuning control (NSTC) to stabilize such highly nonlinear complex systems as high-performance aircraft and electric power systems. In particular, nonlinear time series (NTS) models are identified and used to determine the control policy from output measurements. The power system application considers a single-machine, infinite-bus (SMIB) model with controlled series capacitance (SC) for a flexible a.c. system (FACTS). High-performance aircraft control considers large angle of attack variations over a short time domain such that linear perturbation models have questionable validity.

Conventional adaptive control generally is based on linear theory, and involves the adjustment of controller gains to more or less maintain good transient response. In some cases, a linear model is identified according to input-output measurements, and the controller gains (e.g., proportional plus integral, PI) appropriately adjusted. In many cases the method is quite successful, but in others it fails.

Limited numbers of publications are available on nonlinear adaptive control. Even though controlled parameter adjustments of linear feedback configuration results in nonlinear systems (bilinear system for linear plants), such adaptive schemes generally are analyzed according to linear system theory (e.g., pole placement). A nonlinear model algorithmic control (MAC) is studied in [1] which solves an approximating nonlinear model for the required control according to a measured or estimated output. The output typically is generated from a desired response or command model. Since the Volterra series and kernels of nonlinear systems is a natural generalization of linear systems and impulse responses, they would seem to represent a natural extension of present adaptive control theory and methodology. Such continuous time controller design is proposed in [2] where a linearizing controller is derived along with formulae for controller kernels in terms of plant kernels and desired closed-loop system. Unfortunately, the problem is complicated by the fact that even for a finite-kernel open-loop configuration, the closed-loop system is characterized by an infinite number of kernels. Consequently, the controller theoretically requires an infinite number even for a quadratic system. The entire concept of realistic approximations, especially in conjunction with "linearizing" controls must be carefully studied for such designs. Discrete time Volterra kernels and multidimensional Z transfer functions are utilized to derive a controller in [3]. Again, formulae are derived for theoretical linearizing control, but here an elegant transformation is provided so that the controller only requires as many kernels as the assumed open-loop plant.

Other investigations use the Volterra series as a mechanism to formulate the basis for the adaptive control algorithm but not its actual implementation. Such studies are made for biomedical and aircraft applications in [4,5].

2. METHODOLOGY SUMMARY

The self-tuning control proposed here includes an on-line, recursive least-squares (RLS) model identification in conjunction with either a quadratic optimal or an inverse model control computation based on a nonlinear time series approximating model and a reference command. In case of a linear reference command model, the method results in a linearizing control approximation.

3. ELECTRIC POWER SYSTEM CONTROL

The scheme summarized in 2 is developed in this section for the series capacitor (SC) transient control of a single-machine, infinite bus (SMIB) power system undergoing destabilizing faults. The thyristor-controlled series capacitor represents a typical mechanism for cost effective operation of flexible a.c. transmission systems (FACTS).

3.1 Model Formulation

A simple model of a SMIB system with series capacitor control can be written as:

$$\begin{aligned} \dot{w} &= (T_m - EV\sin(\delta)B)/M \\ \dot{\delta} &= w_b(w - 1) \end{aligned} \tag{1}$$

where w is the rotor speed, δ the rotor angle, T_m the prime mover torque assumed constant, E the excitation voltage of the machine, V the infinite-bus voltage, and $B = 1/(X_d+X_e-X_c)$ the assumed controlled variable, with X_e = line reactance and X_c = series capacitor reactance, and M the machine inertia.

We introduce

$$K(\delta) = \sin(\delta)/\delta \tag{2}$$

so that $\sin(\delta) = K(\delta)\delta$ for mathematical convenience in representation. This choice will not be restrictive in the design of the controller. Then,

$$\begin{aligned} \dot{w} &= (T_m - EVBK(\delta)\delta)/M \\ \dot{\delta} &= w_b(w-1) \end{aligned} \tag{3}$$

Using Euler's approximation for the differentials with appropriately "small" time steps, we get

$$\begin{aligned} w(k) &= w(k-1) + T(T_m - EVB(k-1)K(\delta(k-1))\delta(k-1))/M \\ \delta(k) &= \delta(k-1) + Tw(w(k-1) - 1) \end{aligned} \tag{4}$$

where T is the sampling time chosen, and the subscript k is the discrete-time step.

If $\delta(\cdot)$ is small $\sin\delta(\cdot) \sim \delta(\cdot)$ and $K(\cdot) \sim 1$, and as $\delta(\cdot)$ increases it is assumed that an effective adaptive feedback controller $B(\cdot)$ will compensate for the variation $\sin\delta(\cdot)/\delta(\cdot)$ by the nonlinear nature of the derived feedback policy. As demonstrated by simulation below, such bilinearizing control is very effective while even an adaptive linearizing control fails. Consequently, it seems reasonable to assume a bilinear approximating system for the generation of an adaptive nonlinear controller with $u(\cdot) = K(\cdot)B(\cdot)$ in (3) and (4). The restriction posed by the requirement of "smooth" variation of u(k) may be accommodated in the control design.

With real power as the measured output, one obtains a nonlinear time-series structure with time-varying (updated) coefficients to represent system (1). The power systems purist will recognize that Eq. (1) at best grossly approximates the true behavior of a power system. Accordingly, one might expect more nonlinear terms to be added to Eq. (4) when considering higher model orders. However, since the nonlinear structure of the multimachine system, as well as higher-order SMIB models, has this form, it is more likely that the bilinear model (of higher-order) will suffice. In a number of simulation experiments conducted with the self-tuning control in the next section, it was found that a 4th-order, bilinear-model based policy conveniently stabilizes an unstable high-order power system, whereas such a control based on 2nd- or 3rd-order bilinear models and any so prescribed linear model fails.

Here, the 4th-order bilinear model has the following time-series form:

$$y(k) = \sum_{i=1}^{4} a(i)y(k-i) + \sum_{i=1}^{4} b(i)y(k-i)u(k-i) + \sum_{i=1}^{4} c(i)u(k-1) \tag{5}$$

where a(i), b(i), c(i) are appropriate (possibly time-dependent) parameters, y(k) = plant output, and u(k) is the additional series capacitor reactance added/removed. Equation (5) normally can be put in standard bilinear discrete state-space form. while the additive control term in (5) does not follow directly from (4) it is added for generalization of the results.

3.2 Self-Tuning Controller Design

Equation (5) can be expressed in compact form as

$$y(k+1) = \phi^T\theta \qquad (6)$$

where

$$\theta^T = [a(1), a(2), a(3), a(4), b(1), b(2), b(3), b(4), c(1), c(2), c(3), c(4)]$$

and

$$\phi^T = [y(k), y(k-1), y(k-2), y(k-3), y(k)u(k), y(k-1)u(k-1), y(k-2)u(k-2), y(k-3)u(k-3), u(k), u(k-1), u(k-2), u(k-3)]$$

Since they appear linearly, the parameters of the model in (5) can be estimated using recursive least squares (RLS) estimation with

$$\begin{aligned} \theta_{k+1} &= \theta_k + L_k\left[y_{k+1} - \phi_k^T\theta_k\right] \\ L_{k+1} &= \frac{\boldsymbol{P}_k\,\phi_{k+1}}{1 + \phi_{k+1}^T\,\boldsymbol{P}_k\phi_{k+1}} \\ \boldsymbol{P}_{k+1} &= \boldsymbol{P}_k - \frac{\boldsymbol{P}_k\,\phi_{k+1}\,\phi_{k+1}^T\,\boldsymbol{P}_k}{1 + \phi_{k+1}^T\,\boldsymbol{P}_k\,\phi_{k+1}} \end{aligned} \qquad (7)$$

where $\boldsymbol{P}_k$ is a covariance matrix [6]. Here, y_{k+1} refers to actual 11th-order nonlinear system measured power at k+1 th instant and the subsequent subscript sample-time notation designates estimates based on this information.

To check out the validity of the BARMA model (5), an 11th-order nonlinear model of a power system [7] equipped with a series capacitor, and an IEEE type-1 excitation system, was subjected to a random, binary switching of the capacitor, and the proposed model was identified on-line from the measurements of power and capacitance.

The performance of the bilinear model was compared with an equivalent linear ARMA model, i.e., (5) with b(i) = 0.

It was observed that the bilinear model shows insignificant growth of the loss function, compared with the linear model [6]. The comparisons are significantly less pronounced when the loss function is computed during the identification of lower order linear and bilinear models. However, the bilinear model is always more accurate. This would be expected since the linear model is a special case of the bilinear model.

The bilinear time-series model (5) now is used to design a bilinear self-tuning controller (BSTC) for the power system. For this purpose, a one-step-ahead cost function can be defined for (5) as:

$$J(k) = [y(k) - y^*(k)]^2 + \rho\,[u(k-1)]^2 \qquad (8)$$

where $y^*(k)$ is the desired output at the next sampling instant, a constant, in the case of regulation, and ρ is a weight imposed to keep the control "smooth."

The minimization of this function with respect to u(k-1) with dJ(k)/du(k-1) = 0 yields the self-tuning control

$$\begin{aligned} u(k-1) = -\,[&a(1)y(k-1) + a(2)y(k-2) + a(3)y(k-3) + \\ &a(4)y(k-4) + b(2)y(k-2)u(k-2) + \\ &b(3)y(k-3)u(k-3) + b(4)y(k-4)u(k-4) + \\ &c(2)u(k-2) + c(3)u(k-3) + \\ &c(4)u(k-4) - y^*(k)]\,/\,[\gamma + b(0)] \end{aligned} \qquad (9)$$

where

$$b(0) = c(1) + b(1)\,y(k-1)\,, \quad \gamma = \rho/b(0)$$

The parameters a(i), b(i), and c(i) are estimated recursively by the RLS identifier (7). Of course, measured real power y_k from the complex 11th-order model replaces y(k) in (9).

3.3 Simulation

A single machine on an infinite bus equipped with an IEEE type-1 excitation system and series capacitor was modeled with 11 nonlinear differential equations using Park's equations with currents chosen as the state variables. The system is stable for a 1-cycle short-circuit at the generator terminals, but unstable for a 2-cycle fault. It was assumed that the lines remain intact after the fault. The power system is assumed to operate at a loading of P + jQ = 0.6 - j0.1. The tie-line reactance is 0.35 p.u and the series capacitor has a nominal value of 0.175 p.u (50% compensation).

The self-tuning controller was updated every 0.005 seconds, with the real power chosen as the output and the additional capacitor-reactance required chosen as the input. The identifier was initialized with all the parameter estimates zero, except for c(1), arbitrarily selected 1.0, which helps in "cold start" of the algorithm. The covariance matrix was initialized as $\boldsymbol{P}(0) = 10\,\mathbf{I}$. The selected model order was 4 and a control weight of unity was assumed.

Figure 1 shows the response of the system to a 2-cycle short-circuit at the generator terminals, with and without the BSTC. It can be observed that without the BSTC, the system is unstable. It can be observed that the BSTC quickly stabilizes the unstable system and returns it to the original equilibrium. Figure 2 shows the smooth modulation of the series capacitor by the BSTC.

For comparison with BSTC, the linear self-tuning controller (LSTC) was derived to manipulate the series capacitor according to measured power. Figures 3 and 4 show the inadequacy of the linear controller to stabilize the system. Seemingly, the controller attempts to linearize the system according to an unrealizable model reference in the latter case. The bilinear reference, on the other hand, offers a more compatible reference to accomplish this mission. While this unstable result is not realizable, note that in Fig. 3 the LSTC is attempting to stabilize the rotor angle variation at different new equilibria.

It should be noted that in either case, the response can be further improved if it is assumed that swing angle δ is available to the controller.

4. LONGITUDINAL AIRCRAFT CONTROL

Control of the angle of attack (α), with motion constrained to the vertical plane, is considered for high-α aircraft such as the F18. The approximating model for on-line identification and control is more highly nonlinear than the above bilinear one due to the complex nonlinearity of α in the aircraft stability derivatives (i.e., coefficients of the systems differential equations of motion). Among many possible variants tried, the following model for α and pitch rate q was found to fit simulation data very well:

$$\begin{aligned} \alpha(k+1) &= p_{1\alpha}\alpha(k) + p_{2\alpha}\alpha^2(k) + p_{3\alpha}\alpha^3(k) + \\ &\quad p_{4\alpha}q(k) + p_{5\alpha}q(k)\alpha(k) + p_{6\alpha}q(k)\alpha^2(k) + \\ &\quad p_{7\alpha}q(k)\alpha^3(k) + p_{8\alpha}u(k) + p_{9\alpha}u(k)\alpha(k) + \\ &\quad p_{10\alpha}u(k)\alpha^2(k) + p_{11\alpha}u(k)\alpha^3(k) + p_{12\alpha} \qquad (10) \\ q(k+1) &= p_{1q}\alpha(k) + p_{2q}\alpha^2(k) + p_{3q}\alpha^3(k) + \\ &\quad p_{4q}q(k) + p_{5q}q(k)\alpha(k) + p_{6q}q(k)\alpha^2(k) + \\ &\quad p_{7q}q(k)\alpha^3(k) + p_{8q}u(k) + p_{9q}u(k)\alpha(k) + \\ &\quad p_{10q}u(k)\alpha^2(k) + p_{11q}u(k)\alpha^3(k) + p_{12q} \end{aligned}$$

Several other changes were made in the above NSTC algorithm for the aircraft control. In order to keep the elevator control smooth, the controller can be designed to minimize the one-step-ahead cost function

$$J = (\alpha_m(k+1) - \alpha_r(k+1))^2 + \rho(u(k) - u(k-1))^2 , \quad (11)$$

where α_r is a desired reference output, and α_m is the prediction of the output based on the model of the plant. The correction term $(\alpha(k)-\alpha_m(k))$ takes into consideration the error of the model and introduces integral action into control. In a situation when $\alpha(k)$ is not yet available at the time when the control u(k) is computed, as is often the case due to time delays and/or time needed for the system state, the correction term may be taken as $(\alpha(k-1)-\alpha_m(k-1))$, and $\alpha_m(k+1)$ must be based on the measurements from moment k-1 which means that the algorithm becomes two-step-ahead. The reference trajectory α_r was chosen according to $1/(z^2-1.6z+0.65)$. Minimization of (11) with respect to u(k) yields

$$u(k) = \frac{(\alpha_r - a)b + \rho u(k-1)}{b^2 + \rho} \qquad (12)$$

where

$$\begin{aligned} a &= p_{1\alpha}\alpha + p_{2\alpha}\alpha^2 + p_{3\alpha}\alpha^3 + p_{4\alpha}q + p_{5\alpha}q\alpha + \\ &\quad p_{6\alpha}qa^2 + p_{7\alpha}q\alpha^3 + p_{12} \\ b &= p_{8\alpha} + p_{9\alpha}\alpha + p_{10\alpha}\alpha^2 + p_{11\alpha}\alpha^3 \end{aligned}$$

For $\rho = 0.05$ the control contains no one-pulse spikes, and the accuracy of reference following deteriorates only slightly from that without control weighting.

Figures 5 and 6 show the smooth stable response in angle of attack and in elevator angle (control) for the NLSTC. LSTC was computed in the same manner and was not as satisfactory. If a control weighted performance index is not considered in the derivation, it was found that the LSTC was considerably more underdamped than the response shown. The NSTC in that case is somewhat faster but requires more erratic control variation.

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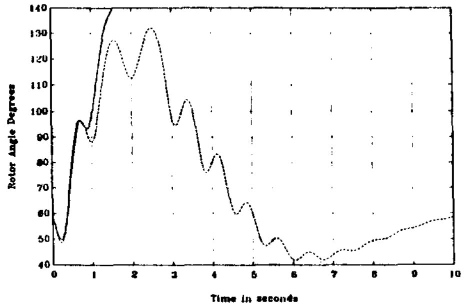

Fig. 1. Response of SMIB rotor angle to two-cycle short-circuit. (Solid line - without control. Dashed line - with BSTC)

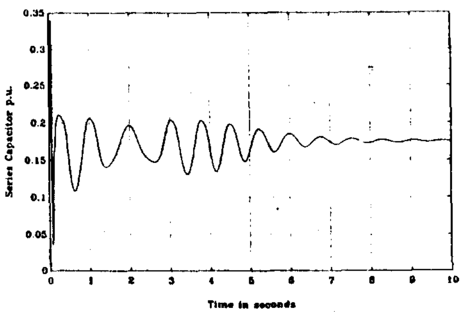

Fig. 2. Demand of series capacitor by the BSTC following a two-cycle short-circuit on a SMIB.

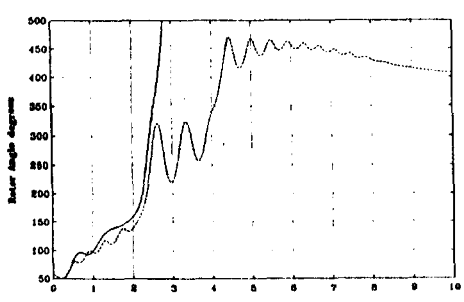

Fig. 3. Response of SMIB rotor angle to two-cycle with linear STC. (Solid line - without control. Dashed line - with LSTC)

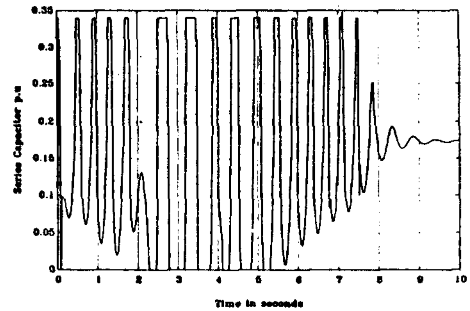

Fig. 4. Demand on series capacitor by the linear STC following a two-cycle short-circuit on a SMIB.

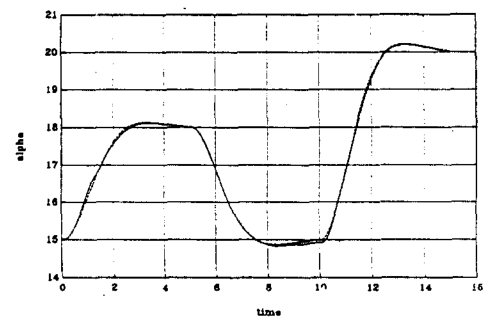

Fig. 5. NLSTC with control weighting (ρ = 0.02): α(t).

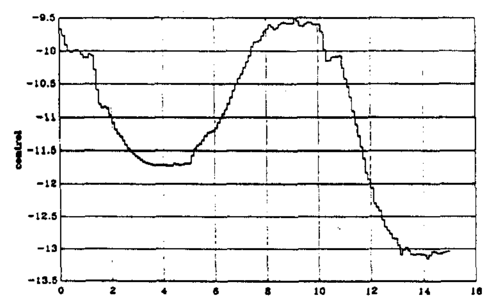

Fig. 6. NLSTC with control weighting (ρ = 0.02): δ(t).