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AN INVESTIGATION OF THE DYNAMIC ERRORS OF THE REMOTE-INSTANTANEOUS FLOWRATE MEASUREMENT DUE TO PARAMETER CHANGES

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ABSTRACT

The paper describes estimation errors of unsteady flowrate measurements due to parameter changes in *a quasi-remote instantaneous flowrate measurement method* (abbreviate as **QIFM**) and *an instantaneous flowrate measurement method using two points pressure measurements* (abbreviate as **TPFM**). By introducing error performance index, the influence of parameter changes on the accuracy and dynamic response of the estimated unsteady flowrate are evaluated. Of four parameters, the variation of the length of the pipeline and speed of sound produce large errors in the estimated unsteady flowrate during transient periods. The effect of kinematic viscosity of the working fluid(oil) is relatively insensitive in unsteady flowrate estimation.

NOMENCLATURE

a =inner radius of the pipeline, c =speed of sound, $J_0(x)$,$J_1(x)$ =Bessel functions of the first kind, L =the length of the pipeline, N =number of convolution terms, p =pressure, q =flowrate, $\hat{q}(t)$ = estimated flowrate $\Delta q(t)$ = estimation error in the unsteady flowrate, s = Laplace operator, t =time, ΔT =sampling time interval, $Z_0(s)$ =characteristic impedance of the pipeline, $\Gamma(s)$ =propagation operator, ρ =density, ν =kinematic viscosity

subscripts

u =upstream section of the pipeline, d =downstream section of the pipeline

1. INTRODUCTION

Recently we have been proposed three approaches, that is, *a remote and quasi-remote instantaneous flowrate measurement methods and two points pressure measurement method* for estimating unsteady flowrate accurately passing through an arbitrary cross section of the pipeline in hydraulic control systems[1,2,3].

In these approaches, an unsteady flowrate can be remotely estimated by using the distributed parameter models of a hydraulic pipeline with considering unsteady velocity distribution over a cross section of the pipeline. Using these approaches, it is possible to measure unsteady flowrate difficult to install an instantaneous flowmeter at the desired location to obtain flowrate information. Under unsteady laminar oil flow conditions, the excellent agreement of the estimated and measured unsteady flowrate waveforms by a cylindrical choke type instantaneous flowmeter(abbreviate as CCFM)[4] is obtained.

Because these approaches make use of the distributed parameter models with considering frequency dependent viscous friction, the spatial resolution along the axial direction of the pipeline has an outstanding ability. The accuracy and dynamic response of the estimated unsteady flowrate depend on the values of parameters representing the hydraulic pipeline dynamics such as the length of the pipeline, kinematic viscosity, radius, speed of sound within oil filled the pipeline, therefore, it is possible to bring about estimation errors of unsteady flowrate due to parameter changes accompanied by varying an operating conditions or environmental changes. In order to scheme a high - accuracy, fast - response and improvements of functions in these approaches, it is necessary to investigate the effect of estimation errors due to the variation of geometric configuration of the pipeline and physical properties of the working fluid.

The paper describes estimation errors of unsteady flowrate due to parameter changes in a quasi-remote instantaneous flowrate measurement method (**QIFM**) and an instantaneous flowrate measurement method using two points pressure measurements(**TPFM**). By introducing error performance index, the influence of parameter changes on the accuracy and dynamic response in the estimated unsteady flowrate are evaluated.

2. EXPERIMENT

Fig.1 shows a schematic of overall experimental apparatus used in the experiment. In **Fig.1**, the experimental apparatus consists of a hydraulic pipeline systems, which is contained a straight long copper pipeline installed CCFM at the end of the pipeline, data acquisition and numerical operating systems.

In the experiment, upstream pressure $p_u(t)$ and downstream pressure $p_d(t)$ are directly measured by semiconductor type pressure transducers (the natural frequency is about 100kHz). In order to compare with the estimated unsteady flowrate, an unsteady flowrate through upstream and downstream section of the pipeline are measured by CCFM1 and CCFM2.

For a **QIFM**, CCFM1 is utilized to compare the estimated unsteady flowrate with the measured one $q_u(t)$ in case of the upstream flowrate estimation. The upstream pressure $p_u(t)$ as input data is measured by a pressure transducer installed the downstream portion of differential amplifier, and flowrate information at the remote location of the unsteady flowrate estimating point, that is, downstream flowrate $q_d(t)$ is obtained by CCFM2.

The upstream portion of the pipeline is connected a constant pressure source and an electro-hydraulic servo valve is set at downstream manifold to generate a change of flowrate in hydraulic pipeline systems. Oil temperature is measured by a thermistor type thermometer.

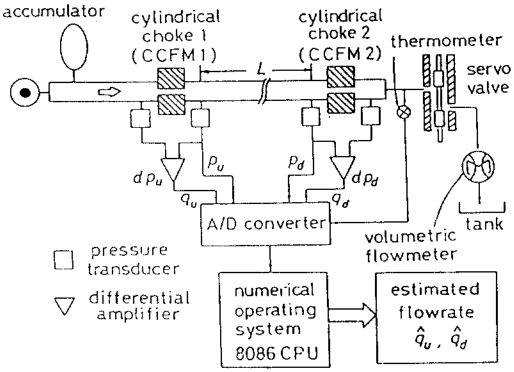

Fig.1 Schematic of experimental apparatus

3. AN INVESTIGATION OF ESTIMATION ERRORS DUE TO PARAMETER CHANGES

3.1 THE PRINCIPLE OF UNSTEADY FLOWRATE MEASUREMENT

3.1.1 The Quasi-Remote Instantaneous Flowrate Measurement Method (QIFM)

For a hydraulic pipeline system as shown in **Fig.1**, the dynamic characteristics of pressures and flowrates between upstream and downstream section along the pipeline are described in the forms of transfer matrix (four terminal networks representations)[5].

$$\begin{pmatrix} P_u(s) \\ Q_u(s) \end{pmatrix} = \begin{bmatrix} \cosh\Gamma(s) & Z_0(s)\cdot\sinh\Gamma(s) \\ \frac{1}{Z_0(s)}\cdot\sinh\Gamma(s) & \cosh\Gamma(s) \end{bmatrix} \cdot \begin{pmatrix} P_d(s) \\ Q_d(s) \end{pmatrix} \tag{1}$$

where $P(s)$ and $Q(s)$ denote Laplace transforms of the pressure and flowrate derivation, and subscripts u and d refer to upstream and downstream section of the pipeline, respectively.

With consideration of frequency dependent viscous friction, propagation operator $\Gamma(s)$ and characteristic impedance of the pipeline $Z_0(s)$ is given as follows.

$$\Gamma(s) = \bar{s}_c \cdot \{1 - \frac{2J_1(j\sqrt{\bar{s}_\nu})}{j\sqrt{\bar{s}_\nu}J_0(j\sqrt{\bar{s}_\nu})}\}^{-1/2} \tag{2}$$

$$Z_0(s) = Z_{0c} \cdot \{1 - \frac{2J_1(j\sqrt{\bar{s}_\nu})}{j\sqrt{\bar{s}_\nu}J_0(j\sqrt{\bar{s}_\nu})}\}^{-1/2} \tag{3}$$

where $\bar{s}_c = s \cdot L/c$, $\bar{s}_\nu = s \cdot a^2/\nu$ and $Z_{0c} = \pi a^2/\rho c$.

The principle of a **QIFM** describes briefly[2]. In a **QIFM**, for example, upstream flowrate $q_u(t)$ is estimated by measured upstream pressure $p_u(t)$ and measured downstream flowrate $q_d(t)$ and by use of the dynamic characteristics of pressures and flowrates represented by equation (1). By using this method, an instantaneous flowrate is estimated in real time without inserting a flowmeter at the desired location, but pressure information is needed at that point.

Based on the transfer matrix equation (1), upstream flowrate is given as follows.

$$Q_u(s) = G_p(s) \cdot P_u(s) + G_q(s) \cdot Q_d(s) \tag{4}$$

where

$$G_p(s) = \tanh\{\lambda(s) \cdot L\}/Z_0(s) \tag{5}$$

$$G_q(s) = 1/\cosh\{\lambda(s) \cdot L\} \tag{6}$$

In equation (5),(6), $G_p(s)$ represents transfer function relating upstream flowrate $Q_u(s)$ to upstream pressure input $P_u(s)$, $G_q(s)$ is transfer function relating upstream flowrate $Q_u(s)$ to downstream flowrate input $Q_d(s)$, which are derived from the transfer matrix equation (1). By taking inverse Laplace transform of equation(4), upstream flowrate $\hat{q}_u(t)$ in the time domain is determined by the following convolution integrals.

$$\hat{q}_u(t) = \int_0^t g_p(t-\tau)\cdot p_u(\tau)d\tau + \int_0^t g_q(t-\tau)\cdot q_d(\tau)d\tau \tag{7}$$

$$g_p(t) = 0 \qquad \text{for} \quad t < 0$$

$$g_q(t) = 0 \qquad \text{for} \quad t < 0$$

where $g_p(t)$ and $g_q(t)$ are the weighting functions in the time domain of transfer functions $G_p(s)$ and $G_q(s)$.

By discretizing the convolution integrals with respect to time, equation (7) then can be written as difference equation as follows.

$$\hat{q}_u(n\Delta T) = \sum_{i=0}^{N-1} g_p(n\Delta T - i\Delta T)\cdot p_u(i\Delta T)\cdot\Delta T$$

$$+ \sum_{i=0}^{N-1} g_q(n\Delta T - i\Delta T)\cdot q_d(i\Delta T)\cdot\Delta T \tag{8}$$

where ΔT is sampling time interval, N is the number of convolution terms.

3.1.2 AN INSTANTANEOUS FLOWRATE MEASUREMENT METHOD USING TWO POINTS PRESSURE MEASUREMENTS (TPFM)

In the present situations, it is difficult to procure an instantaneous flowmeter with high - accuracy and fast - response. By utilizing this method, there is no necessity for installing an instantaneous flowmeter at the desired location to obtain flowrate information.

The principle for estimating unsteady flowrate is similar to a **QIFM** . From equation (1), an unsteady flowrate passing through at the upstream or downstream section of the pipeline in **Fig.1**, can be estimated in real time from measurements of the upstream and downstream pressure $p_u(t)$, $p_d(t)$. In case of an unsteady flowrate estimation at the upstream section in **Fig.1**, the transfer function relating upstream flowrate $Q_u(s)$ to upstream pressure input $P_u(s)$ is $G_u(s) = 1/[Z_0(s) \cdot \tanh\Gamma(s)]$, the transfer function relating upstream flowrate $Q_u(s)$ to downstream pressure input $P_d(s)$ is $G_d(s) = 1/[Z_0 \cdot \sinh\Gamma(s)]$ and these transfer functions are derived from equation (1).

An unsteady flowrate in the time domain is determined by convolution integrals between the weighting functions $g_u(t)$, $g_d(t)$ and the measured values of the upstream and downstream pressure $p_u(t)$, $p_d(t)$, respectively. The weighting functions $g_u(t)$, $g_d(t)$ corresponding to the transfer functions $G_u(s)$, $G_d(s)$ are obtained by the application of inverse fast Fourier transform.

3.2 A REMARK ON THE ESTIMATION ERRORS OF UNSTEADY FLOWRATE

Since **QIFM** and **TPFM** make use of the hydraulic pipeline dynamics between two cross sections along the pipeline, it is possible to cause errors of unsteady flowrate measurements due to changes of parameters according as an operating conditions are varied or environmental variations. The parameters of interest are the length of the pipeline L, kinematic viscosity of the working fluid ν, speed of sound within the working fluid c and radius of the pipeline a. If the values of parameters x_0 at a given operating conditions are changed by an amount $x_0 + \Delta x$, the frequency characteristics of transfer functions will change by an amount of $G(s) + \Delta G_(s)$. If the weighting functions corresponding to variations of the frequency characteristics $\Delta G(s)$, estimation errors in the estimated unsteady flowrate due to parameter changes can be quantitatively investigated by using equation (4), respectively.

(1) QUASI–REMOTE INSTANTANEOUS FLOWRATE MEASUREMENT METHOD (QIFM)

The variation of the frequency characteristics for the change of speed of sound c_0, by 10 percentage of its mean value of 1.35×10^3m/s is shown in **Fig.2**.

Fig.3 shows an example of the weighting function $\Delta g_{qc}(t)$ corresponding to variation of the frequency characteristics $\Delta G_{qc}(j\omega)$ in **Fig.2**. In this study, the weighting functions are obtained by the application of inverse fast Fourier transform. This method is relatively simple and the validity of the the weighting functions obtained here are confirmed in the previous author's research works[2,3].

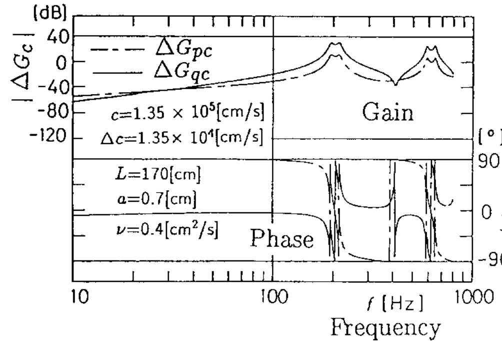

Fig.2 Variation of the frequency characteristics for the change of speed of sound $\Delta G_{pc}(j\omega)$, $\Delta G_{qc}(j\omega)$

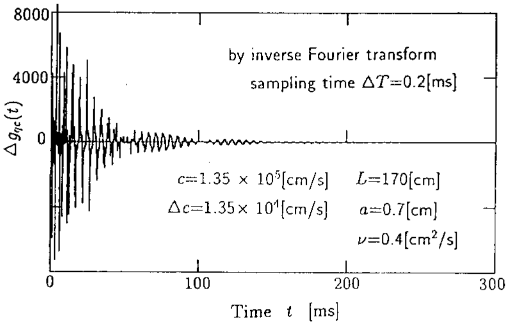

Fig.3 The weighting function for the change of speed of sound $\Delta g_{qc}(t)$

In experiment, the reference values of parameters at an operating point are as follows; the length of the pipeline L_0=170 cm, radius of the pipeline a_0=0.7 cm, speed of sound $c_0 = 1.35 \times 10^5$ cm/s, kinematic viscosity of the oil ν_0=0.4 cm^2/s, mean pressure is 3.8MPa, mean flowrate is 36 cm^3/s and oil temperature is maintained 35 °C.

Fig.4 shows comparison of estimation errors in the estimated unsteady flowrate due to parameter changes 10 % with respect to reference values for applying square wave input 10 Hz.

The top and 2nd waveforms indicate the measured upstream pressure $p_{uv}(t)$ and downstream flowrate $q_{dv}(t)$, which are corresponded to input data for estimating unsteady flowrate. The 3rd is a directly measured upstream flowrate waveforms $q_{uv}(t)$ by a CCFM1 in order to compare with estimation errors . The estimation errors due to variation of values of parameter L, c, a, ν, by an amount equal to 10 % of the reference values are shown in 4th to 7th in **Fig.4**.

The error performance index is here introduced to investigate the effect of parameter changes on the accuracy and dynamic response in the estimated unsteady flowrate. The estimation errors presented here provide some informations about how the estimation errors varied with time, and it is important to point out the variation of the estimation errors with time.

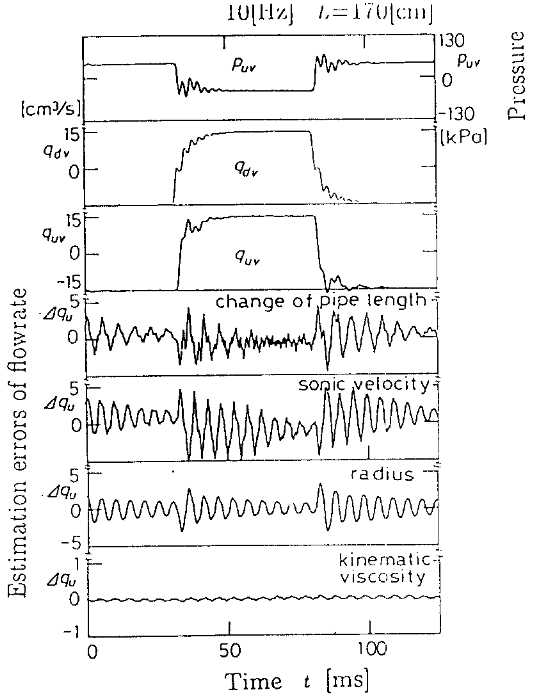

Fig.4 Comparison of estimation errors due to parameter changes 10 %

Various error performance indexes have been proposed in the literature. In this study, the error error performance index ISE(t) propose:

$$\mathrm{ISE}(t) = \frac{1}{t}\int_0^t \Delta q_u(t)^2 dt \tag{9}$$

This criterion has a characteristics that in the step response a large initial errors during transient periods is weighted heavily and small errors lightly.

Fig.5 shows the result of the error performance index ISE(t) as function of the time $0 \leq t < T_p/2(T_p$ is period of the square wave).

Fig.6 shows error performance index due to parameter changes during one period of a square wave input. It is known that parameter changes have an important effect on transient periods rather than steady state in the estimated error waveforms. In comparing estimation errors and the values of ISE(t), estimation errors are affected in proper sequence of changes of speed of sound, the length of the pipeline, radius of the pipeline, kinematic viscosity of the oil. In equation (2), (3), $\bar{s}_c$ is normalized Laplace operator defined by $s \cdot L/c$ or $\bar{s}_\nu$ is a normalized Laplace operator defined by $s \cdot a^2/\nu$ in the propagation operator $\Gamma(s)$ and characteristic impedance $Z_0(s)$. Therefore, it is expected that L and c, a^2 and ν are approximately same effect in the estimation errors. Because the propagation operator $\Gamma(s)$ is equal to $\bar{s}_c(= s \cdot L/c)$ with exception of the Bessel function terms relating frequency dependent viscous friction in equation (2), estimation errors are largely affected by the changes of L and c. Moreover, speed of sound c is included the first term $Z_{0c}(= \rho \cdot c/\pi a^2)$ in characteristic impedance of the pipeline in equation (3), it is deduced that speed of sound is largely affected estimation errors rather than the length of the pipeline L. The radius of the pipeline a affected larger than kinematic viscosity of the oil.

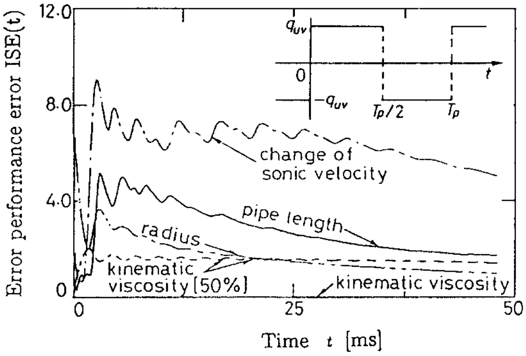

Fig.5 Error performance index ISE(t) versus time

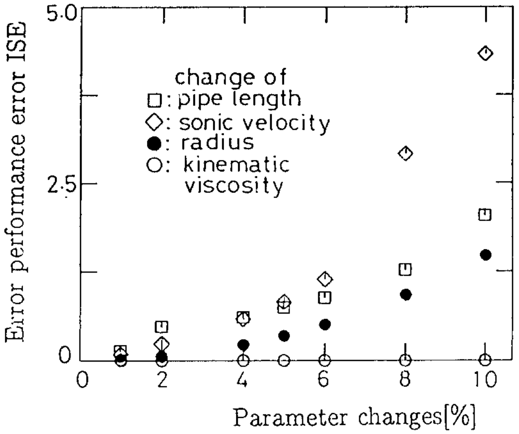

Fig.6 Error performance index for parameter changes

(2) AN INSTANTANEOUS FLOWRATE MEASUREMENT METHOD USING TWO POINTS PRESSURE MEASUREMENTS (TPFM)

In this approach, estimation errors are investigated in similar manner of a QIFM.

Fig.7 shows variation of the frequency characteristics of varying the length of the pipeline L, by 10 percentage of its reference value of 170 cm. By using inverse fast Fourier transform, the weighting function $\Delta g_{uL}(t)$ referred to variation of the frequency characteristics of transfer function $\Delta G_{uL}(j\omega)$ is shown in **Fig. 8**.

Fig.9 shows estimation error waveforms of varying parameters, by 10 percentage of those reference values. In

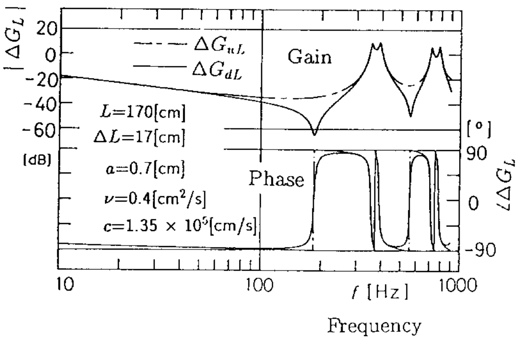

Fig.7 Variation of the frequency characteristics for change of the length of the pipeline $G_{uL}(j\omega)$, $G_{dL}(j\omega)$

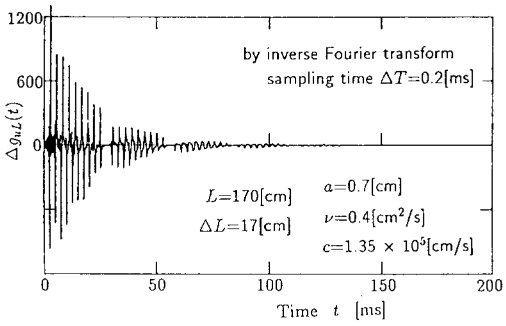

Fig.8 The weighting function $\Delta g_{uL}(t)$ for variation of the frequency characteristics $G_{uL}(j\omega)$

this case, the operating condition and reference values of parameters are similar to the case of **Fig.4**. The top and 2nd are upstream and downstream pressure waveforms $p_{uv}(t)$, $p_{dv}(t)$ which are input data for estimating upstream unsteady flowrate. The 3rd is upstream flowrate waveform $q_{uv}(t)$ measured by a CCFM1. The estimation errors due to parameter changes are plotted the 4th to 7th in **Fig.9**.

Fig.10 shows curves of error performance index ISE(t) of estimation errors as function of the time $0 \leq t < T_p/2$ as shown in **Fig.9**. Also, **Fig.11** shows ISE(t) during one period of a square wave input 10 Hz indicated in **Fig.9**. It can be seen that in this approach the variation of the length of the pipeline L and speed of sound c have largely influence on estimation errors. In comparing estimation errors in a **QIFM** with a **TPFM**, it is clear that radius of the pipeline a, kinematic viscosity of the oil ν and the length of the pipeline L are affected the steady state as well as transient state in the estimated error waveforms. As time elapse, radius of the pipeline is largely affected the estimation errors.

In these approach, the variation of kinematic viscosity rather than other parameters is relatively insensitive. For many hydraulic working fluids, the values of kinematic viscosity are considerably affected by changes of temperature. For example, the kinematic viscosity is changed about 50 % due to changes of temperature at 10 °C. For this reason, the effect of variation of kinematic viscosity investigate in detail. **Fig.12** shows comparison of the error performance index for changes of kinematic viscosity. In **Fig.12**, the variation of kinematic viscosity for a **TPFM** produce a large estimation errors than a **QIFM**.

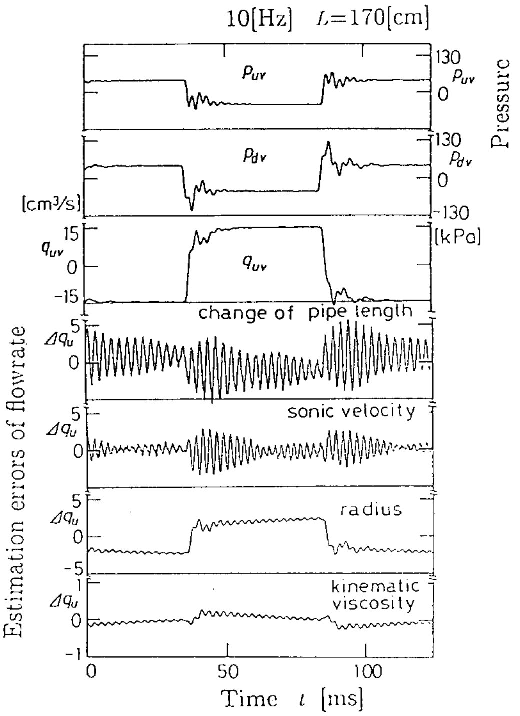

Fig.9 Comparison of estimation errors due to parameter changes 10 %

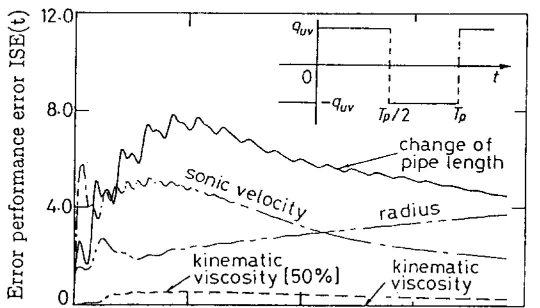

Fig.10 Error performance index ISE(t) versus time

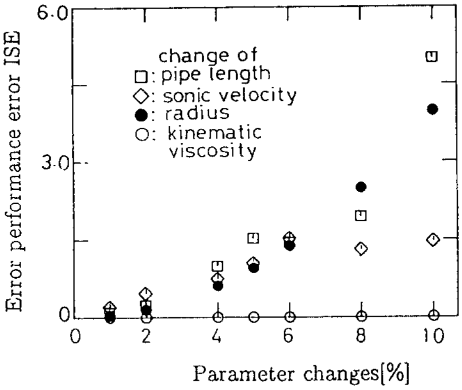

Fig.11 Error performance index for parameter changes

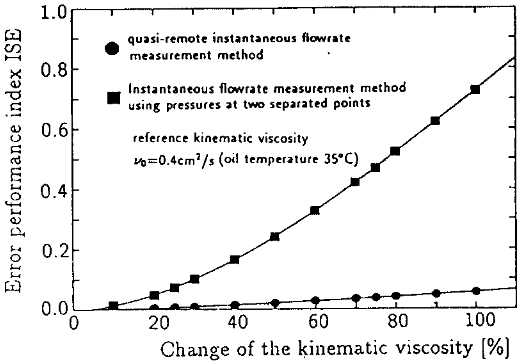

Fig.12 Comparison of error performance index for the change of kinematics viscosity

4. CONCLUSION

The paper describes the estimation errors of unsteady flowrate measurement due to variation of parameters such as the length of the pipeline, kinematic viscosity of the oil, radius, speed of sound in a **QIFM** and a **TPFM**. By introducing error performance index, the influence of parameter changes on the accuracy and dynamic response in the estimated unsteady flowrate are evaluated. The variation of the length of the pipeline and speed of sound have largely influence on the estimated unsteady flowrate during transient periods. The changes of kinematic viscosity is relatively insensitive in the estimated unsteady flowrate.

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