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Kinematic Analysis of POSTECH Hand I with New Symbolic Notation

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Abstract

Recently, dexterous mechanical hands have become of interest in the field of robotics. In this paper, a new symbolic C–Y notation is proposed for the kinematic modeling, and we solve the kinematics of a simplified model of POSTECH Hand I, which is a 5–fingered, 20 degrees of freedom anthropomorphic hand. POSTECH Hand I is designed to have distinctive kinematic structure and the kinematic analysis of the hand is carried out using C–Y notation. To prove the feasibility of C–Y notation, D–H notation is also applied to the POSTECH Hand I. In the inverse kinematic analysis, we neglect the fingertip geometry and assume the point contact with 3 degrees of freedom constraints. The configurations which optimize manipulability index [2] was obtained based on the simulation experiments on the SUN–4 graphic workstation using **SUNPhigs** graphic software.

1. Introduction

Articulated hands can grasp and handle objects of various shape flexibly. These hands promise to offer an adaptability and flexibility which are useful for applying robot to perform complicated tasks with a suitable posture for a required task. There have been a lot of research works reported for the articulated hands. Okada [1] succeeded to handle balls and small boards with the hand which has three fingers and eleven joints. Salisbury [3] investigated the basic properties of articulated hands. He used the concept of the mobility and connectivity to obtain the conditions for grasping and handling of objects, and designed a hand with three fingers which has nine joints using a tendon control system. Hanafusa [4] proposed an algebraic solutions of basic kinematic problems for articulated hands. Jacobson [5] designed four-fingered fully anthropomorphic UTAH–MIT hand, actuated by an electro–mechanical system. For a grasping posture, Yoshikawa [2] suggested to use the posture for which the determinant of the Jacobian matrix is maximum, while Salisbury [3] recommended the posture for the minimum condition number of the matrix.

Also, many methods are available for the description of the geometry of robots [8, 9, 10, 11, 12]. Among them, the most common use is the Denavit–Hartenberg notation (D–H) [8]. The D–H notation can be applied to only serial links, and the description of a joint with respect to the preceding one is carried out by 4 parameters. The use of D–H notation in robotics has facilitated greatly all the modeling problems although it is almost impossible to be applied to robots with complicated kinematic structures. Sheth and Uicker [9] developed modified D–H notation which describes each link by 7 parameters. The S–U notation can be used to described any mechanism but the definition of the link coordinates are not unique, and owing to its complexity it has been applied only for closed–loop robots. A method known as the Spherical–Euler Transformation (SET) was developed by Yih [10] ,which employed the homogeneous spherical coordinates and Euler angles transformation matrices. For robots with rather complicated structures, like human hands, D–H notation can not be applied , and thus a new method is needed. In this paper, we propose a new symbolic notation that can be generally applied to any kind of robots. This notation, called Chung–Youm (C–Y) notation, uses five parameters and has advantages over D–H, S–U, and SET notations. This will be described in the next section.

2. New Symbolic Notation (Chung–Youm Notation)

To easily describe the relation between joints, the C–Y notation is proposed which can be applied to general kinematic structures without ambiguity. The proposed notation can be used also for simple structures which have been dealt with D–H notation. Whereas, D–H notation causes some trouble for the structures of the non–perpendicular rotating axis. The C–Y notation defines the transformation matrix with 5 parameters (θ, ϕ, α, $\vec{r}$, β). The shape matrix for a spatial binary linkage element is shown in Fig. 1 and its joint reference frame and principal joint parameters are defined as follows:

Numbering schemes

- Each (X_i, Y_i, Z_i) coordinate frame corresponds to joint $i+1$ and is fixed in link i and it moves together with the link i.

Reference frame

The origin of the reference frame $X_iY_iZ_i$ is chosen at the center of the kinematic joint, J_{i+1}.

- Z_i: along the $(i+1)$-th joint axis.
- X_i: in the plane of $\vec{r}_i$-$\hat{Z}_i$ and normal to $\hat{Z}_i$.
- Y_i: keeps the right-handed rule.

Principal joint parameters

- θ_i: angle of rotation of the i-th joint, measured counterclockwise about $\hat{Z}_{i-1}$ from $\hat{X}_{i-1}$ to plane $\vec{r}_i$-$\hat{Z}_{i-1}$.
- ϕ_i: angle from $\hat{X}'_{i-1}$ to $\vec{r}_i$, measured counterclockwise about $\hat{Y}'_{i-1}$.
- α_i: twist angle between $\hat{Z}''_{i-1}$ and $\hat{Z}'''_{i-1}$, measured counterclockwise about $\vec{r}_i$.
- $\vec{r}_i$: vector $\vec{r}_i$ is defined from origin J_i to J_{i+1}.
- β_i: angle from $\hat{Z}^{iv}_{i-1}$ to $\hat{Z}_i$, measured counterclockwise about $\hat{Y}^{iv}_{i-1}$.

Auxiliary angles

The following angles can be easily measurable and play an important role in defining the angles ϕ_i and β_i.

- ψ_i: angle from $\hat{Z}_{i-1}$ to $\vec{r}_i$, measured counterclockwise about $\hat{Y}'_{i-1}$.
- γ_i: angle from $\hat{Z}_i$ to $\vec{r}_i$, measured counterclockwise about $\hat{Y}_i$.

The auxiliary angles ψ_i and γ_i are related to the C-Y parameters by

$$\phi_i = \psi_i - 90^\circ \tag{1}$$

$$\beta_i = 90^\circ - \gamma_i \tag{2}$$

To transform the joint reference frame $X_{i-1}Y_{i-1}Z_{i-1}$ to $X_iY_iZ_i$ first, transform $X_{i-1}Y_{i-1}Z_{i-1}$ to $X'''_{i-1}Y'''_{i-1}Z'''_{i-1}$ through angles θ_i, ϕ_i, and α_i. The sequences, $\boldsymbol{T}(\hat{Z}, \theta_i)$, $\boldsymbol{T}(\hat{Y}', \phi_i)$, $\boldsymbol{T}(\hat{X}'', \alpha_i)$ can be formulated in the form of homogeneous transformation matrix:

$$\begin{aligned}\boldsymbol{T}_{i1} &= \boldsymbol{T}_{r_1}(\hat{Z}_{i-1}, \theta_i)\boldsymbol{T}_{r_2}(\hat{Y}'_{i-1}, \phi_i)\boldsymbol{T}_{r_3}(\hat{X}''_{i-1}, \alpha_i) \\ &= \begin{bmatrix} c_{\theta_i}c_{\phi_i} & c_{\theta_i}s_{\phi_i}s_{\alpha_i} - s_{\theta_i}c_{\alpha_i} & c_{\theta_i}s_{\phi_i}c_{\alpha_i} + s_{\theta_i}s_{\alpha_i} & 0 \\ s_{\theta_i}c_{\phi_i} & s_{\theta_i}s_{\phi_i}s_{\alpha_i} + c_{\theta_i}c_{\alpha_i} & s_{\theta_i}s_{\phi_i}c_{\alpha_i} - c_{\theta_i}s_{\alpha_i} & 0 \\ -s_{\phi_i} & c_{\phi_i}s_{\alpha_i} & c_{\phi_i}c_{\alpha_i} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\end{aligned} \tag{3}$$

Secondly, the transformation $X'''_{i-1}Y'''_{i-1}Z'''_{i-1}$ to the resultant $X_iY_iZ_i$ through parameters r_i and β_i can be derived as:

$$\begin{aligned}\boldsymbol{T}_{i2} &= T_{t_1}(r_i, 0, 0)T_{r_4}(\hat{Y}^{iv}_{i-1}, \beta_i) \\ &= \begin{bmatrix} c_{\beta_i} & 0 & s_{\beta_i} & r_i \\ 0 & 1 & 0 & 0 \\ -s_{\beta_i} & 0 & c_{\beta_i} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\end{aligned} \tag{4}$$

Then the shape matrix of the i-th link described by the C-Y notation can be derived as

$$\begin{aligned}\boldsymbol{T}_i &= \boldsymbol{T}_{i1}(\theta_i, \phi_i, \alpha_i)\boldsymbol{T}_{i2}(r_i, \beta_i) \\ &= \left[\begin{array}{ccc:c} & & & \\ & \boldsymbol{D}_i & & \boldsymbol{P}_i \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 1 \end{array}\right]\end{aligned} \tag{5}$$

where $\boldsymbol{D}_i$ is the 3×3 *direction cosine matrix* of the $X_iY_iZ_i$ joint coordinates with respect to the reference frame $X_{i-1}Y_{i-1}Z_{i-1}$, and $\boldsymbol{P}_i$ is the position vector of origin O_i with respect to O_{i-1}. The elements of $\boldsymbol{D}$ and $\boldsymbol{P}$ are:

$$D_{i11} = c_{\theta_i}c_{\phi_i}c_{\beta_i} - (s_{\theta_i}s_{\alpha_i} + c_{\theta_i}s_{\phi_i}c_{\alpha_i})s_{\beta_i}$$

$$D_{i12} = c_{\theta_i}s_{\phi_i}s_{\alpha_i} - s_{\theta_i}c_{\alpha_i}$$

$$D_{i13} = (s_{\theta_i} s_{\alpha_i} + c_{\theta_i} s_{\phi_i} c_{\alpha_i}) c_{\beta_i} + c_{\theta_i} c_{\phi_i} s_{\beta_i}$$

$$D_{i21} = s_{\theta_i} c_{\phi_i} c_{\beta_i} - (s_{\theta_i} s_{\phi_i} c_{\alpha_i} - c_{\theta_i} s_{\alpha_i}) s_{\beta_i}$$

$$D_{i22} = s_{\theta_i} s_{\phi_i} s_{\alpha_i} + c_{\theta_i} c_{\alpha_i}$$

$$D_{i23} = (s_{\theta_i} s_{\phi_i} c_{\alpha_i} - c_{\theta_i} s_{\alpha_i}) c_{\beta_i} + s_{\theta_i} c_{\phi_i} s_{\beta_i}$$

$$D_{i31} = -s_{\phi_i} c_{\beta_i} - c_{\phi_i} c_{\alpha_i} c_{\beta_i}$$

$$D_{i32} = C_{\phi_i} s_{\alpha_i}$$

$$D_{i33} = C_{\phi_i} c_{\alpha_i} c_{\beta_i} - c_{\phi_i} s_{\beta_i}$$

$$P_{i1} = r_i c_{\theta_i} c_{\phi_i}$$

$$P_{i2} = r_i s_{\theta_i} c_{\phi_i}$$

$$P_{i3} = -r_i s_{\phi_i}$$

where $s_{\phi_i} \triangleq \sin\phi_i$, $c_1 \triangleq \cos\phi_i \cdots$. Using the above notation the parameters of POSTECH Hand I is described in the next section.

3. C–Y Parameters of POSTECH Hand I

The schematics of the anthropomorphic five–fingered hand and coordinates defined by C–Y notation are shown in Fig. 2. For convenience, we call the joint of each finger as proximal (P), proximal interphalangeal (PI), distal interphalangeal (DI), distal joints (D) from the wrist to the finger tip. Fingers 1–4 have four joints and the PI, DI, and D joints for the fingers 1–4 are parallel to each other, and P, PI joints comprise a universal joint. The finger 5 is composed of four links and three of them are parallel and the other one is orthogonal to the others. This structure is adequate to generate folding motion. According to the C–Y notation, the parameters of each fingers are:

Table. 1 C–Y parameters of fingers 1–4

links	θ	ϕ	α	γ	β
P	θ_1	0	90°	0	0
PI	θ_2	0	0	ℓ_2	0
DI	θ_3	0	0	ℓ_3	0
D	θ_4	0	0	ℓ_4	0

Table. 2 C–Y parameters of finger 5

links	θ	ϕ	α	γ	β
P	θ_1	$-(90° - \psi)$	90°	ℓ_1	0
PI	θ_2	0	0	ℓ_2	0
DI	θ_3	0	0	ℓ_3	0
D	θ_4	0	0	ℓ_4	0

where $\phi = 90° - \psi$, According to the D–H notation, the principal parameters of each fingers are:

Table. 3 D–H parameters of fingers 1–4

links	θ	α	a	d
P	θ_1	90°	0	0
PI	θ_2	0	ℓ_2	0
DI	θ_3	0	ℓ_3	0
D	θ_4	0	ℓ_4	0

Table. 4 D–H parameters of finger 5

links	θ	α	a	d
P	θ_1	90°	$\ell_1 \sin\psi$	$\ell_1 \cos\psi$
PI	θ_2	0	ℓ_2	0
DI	θ_3	0	ℓ_3	0
D	θ_4	0	ℓ_4	0

In the case of fingers 1–4, if the shape matrix is formulated, the overall transformations of each finger $T_4^0 = T_1^0 T_2^1 T_3^2 T_4^3$ described by C-Y notation are the same as the ones of D–H notation. However, it is not the case for the finger 5, because the definitions of coordinate $X_1 Y_1 Z_1$ are different from the case of fingerl 1–4. Fig 3 shows the coordinate system of C–Y's and D–H's for the thumb. Although θ_2 is defined as the rotation angle about the Z_1 axis in both coordinates systems, there is offset angle resulting from the differences of starting positions. For the thumb of POSTECH Hand I, the relationship is:

$$\theta_{2DH} = \theta_{2CY} + (90° - \psi) \tag{6}$$

Therefore, if the D–H coordinate is rotated as much as the offset angle, the description of D–H notation becomes the same as that of C–Y notation.

4. Inverse Kinematic Solution

Fig. 4 illustrates the homogeneous transformations that specify the locations of the hand, finger base frames and contact point locations of the four distal links. The T_1, T_2, T_3, T_4 and T_5 give their absolute locations with respect to the reference coordinate system. Q, R, S, W, U give their locations with respect to the hand's base coordinate system. T_0 is the hand's base coordinate system with respect to the reference frame. $Q^{-1}T_0^{-1}T_1$, $R^{-1}T_0^{-1}T_2$, $S^{-1}T_0^{-1}T_3$, $W^{-1}T_0^{-1}T_4$ and $U^{-1}T_0^{-1}T_5$ specify the relative locations of distal links of the fingers with respect to finger base frame. The inverse kinematic problem for the hand can be stated as follows: *Given the compound transformation vectors $Q^{-1}T_0^{-1}T_1$, $R^{-1}T_0^{-1}T_2$, $S^{-1}T_0^{-1}T_3$, $W^{-1}T_0^{-1}T_4$ and $U^{-1}T_0^{-1}T_5$, find hand's twenty joint angles.* A systematic approach for obtaining the inverse kinematic solution is described by Paul [7]. Typically, we select one of the transformations whose components are given by $Q_{-1}T_0^{-1}T_1$, then according to the C–Y notations the transformation matrix from finger base to finger tip is given as:

$$Q^{-1}T_0^{-1}T_1 = \begin{bmatrix} c_1c_{234} & -c_1s_{234} & s_1 & c_1(\ell_4c_{234}+\ell_3c_{23}+\ell_2c_2+\ell_1) \\ s_1c_{234} & -s_1s_{234} & c_1 & s_1(\ell_4c_{234}+\ell_3c_{23}+\ell_2c_2+\ell_1) \\ s_{234} & c_{234} & 0 & \ell_4s_{234}+\ell_3s_{23}+\ell_2s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (7)$$

where $c_1 \triangleq \cos\theta_1$, $s_1 \triangleq \sin\theta_1$. $c_{23} \triangleq \cos(\theta_2+\theta_3)$, $\cdots$ and $s_{234} \triangleq \sin(\theta_2+\theta_3+\theta_4)$.

We do not concern with the orientations of finger tips and the dummy variables u, v are defined as:

$$u = \ell_4c_{234}+\ell_3c_{23}+\ell_2c_2 , \quad (8)$$

$$v = \ell_4c_{234}+\ell_3c_{23}+\ell_2c_2 . \quad (9)$$

Then, the position can be denoted by

$$\begin{aligned} p_x^1 &= c_1(\ell_1+v) , \quad (10) \\ p_y^1 &= s_1(\ell_1+v) , \\ p_z^1 &= u . \end{aligned}$$

To solve the inverse kinematic problem, we first determine the proximal joint and the proximal angle θ_1 is formulated as

$$\theta_1 = \mathrm{atan2}(p_y^1/p_x^1) \quad (11)$$

The PI, DI, and D angles should satisfy the following equations, based on the determined proximal angles.

$$u = p_z , \quad (12)$$

$$v = p_x/c_1 - \ell_1,\ c_1 \neq 0 . \quad (13)$$

if $c_1 = 0$,

$$u = p_z , \quad (14)$$

$$v = p_x/s_1 - \ell_1 . \quad (15)$$

The other 3 fingers can be considered in the same way.

As for the thumb, the projection matrix to the finger base frame is:

$$U^{-1}T_0^{-1}T_5 = \quad (16)$$

$$\begin{bmatrix} c_1c_{234}c_\phi - c_1s_{234}s_\phi & -c_1s_{234}c_\phi - c_1c_{234}s_\phi & s_1 & c_1vc_\phi - c_1us_\phi + \ell_1c_1c_\phi \\ s_1c_{234}c_\phi - s_1s_{234}s_\phi & -s_1s_{234}c_\phi - s_1c_{234}s_\phi & -c_1 & s_1vc_\phi - s_1us_\phi + \ell_1s_1c_\phi \\ c_{234}s_\phi + s_{234}c_\phi & -s_{234}s_\phi + c_{234}c_\phi & 0 & vs_\phi + uc_\phi + \ell_1s_\phi \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Similarly,

$$\begin{aligned} p_x^5 &= c_1(vc_\phi - us_\phi + \ell_1c_\phi) , \quad (17) \\ p_y^5 &= s_1(vc_\phi - us_\phi + \ell_1c_\phi) , \\ p_z^5 &= vs_\phi + uc_\phi + \ell_1s_\phi . \end{aligned}$$

First, the proximal angle is obtained as:

$$\theta_1 = atan2(p_y^5, p_x^5) \quad (18)$$

To get the dummy variable u, v as

$$u = -s_\phi(p_x^5/c_1) + p_z^5c_\phi , \quad (19)$$

$$v = c_\phi(p_x^5/c_1) + p_z^5s_\phi ,\ c_1 \neq 0. \quad (20)$$

If $c_1 = 0$,

$$u = -\sin\phi(p_y^5/s_1) + p_z^5\cos\phi , \quad (21)$$

$$v = \cos\phi(p_y^5/s_1) + p_z^5\sin\phi . \quad (22)$$

After determining the P angles, the inverse kinematics of multi-fingered hands becomes the problem of determining PI, DI, and D angles in the 2 dimensional space. As Yoshikawa stated [2], the motions of remaining links will be similar to human hands, if guided according to the manipulability index. The manipulability indexes of Eq. (8) and (9) is given by

$$H(\boldsymbol{\theta}) = \sqrt{\det(\boldsymbol{J}\boldsymbol{J}^T)} \quad (23)$$

where $\boldsymbol{x}$ is defined as $\begin{bmatrix} u & v \end{bmatrix}^T$ and $\boldsymbol{\theta}$ is defined as $\begin{bmatrix} \theta_2 & \theta_3 & \theta_4 \end{bmatrix}^T$ and $\boldsymbol{J}$ represents the Jacobian of the vector $\boldsymbol{x}$.

The system of kinematic equations which optimize the function $H(\boldsymbol{\theta})$ at the position level [6] is

$$\boldsymbol{x} = f(\boldsymbol{\theta}) , \quad (24)$$

$$\boldsymbol{Zh} = \boldsymbol{O} . \quad (25)$$

where $\boldsymbol{Z}$ is the vector which spans the null space of $\boldsymbol{J}$, and h is the gradient of a optimizing function $\boldsymbol{H}(\boldsymbol{\theta})$. The joint angles are obtained by solving the nonlinear Eqs. (24) and (25) using Quasi-Newton algorithm. Fig. 5 shows the simulated motion of POSTECH Hand I. As shown in this figure, the inverse kinematic solution of the POSTECH HAND I was obtained easily using the proposed C-Y notation.

5. Conclusion

A new symbolic notation called Chung-Youm notation is proposed, and applied to the kinematic analysis of POSTECH Hand I. Also, it is shown the transformation matrices derived by C-Y notation are the same as that derived by D-H notation. In the robot analyses C-Y notation has the following advantages in comparison with other notations.

- The absolute joint positions for a spatial open-chain robot or close-chain mechanism with non-concurrent adjacent joint axis can be determined.
- It has unique definition of a reference frame.
- It has clear definition of α_i angle.

Although the C-Y notation has the disadvantage that it is computationally inefficient, it can handle robots with general kinematic structure and consequently, the D-H notation can be considered as one of the special cases of C-Y notation. POSTECH Hand I has several distinctive features over the other reported hands. The design concepts are based on the motion of human hand and especially, the thumb design issue occupies the major feature of the hand. As we can find in the human hand, the joints of the thumb are not parallel nor perpendicular, and thus it generates more dexterous motion in cooperation with the other four fingers. For this kind of structure, the C-Y notation can be the most adequate tool to model the kinematic structure. The kinematic analysis was performed in the graphic workstation using the developed hand simulator and it was shown that the proposed C-Y notation and inverse kinematic work well for the developed POSTECH Hand I.

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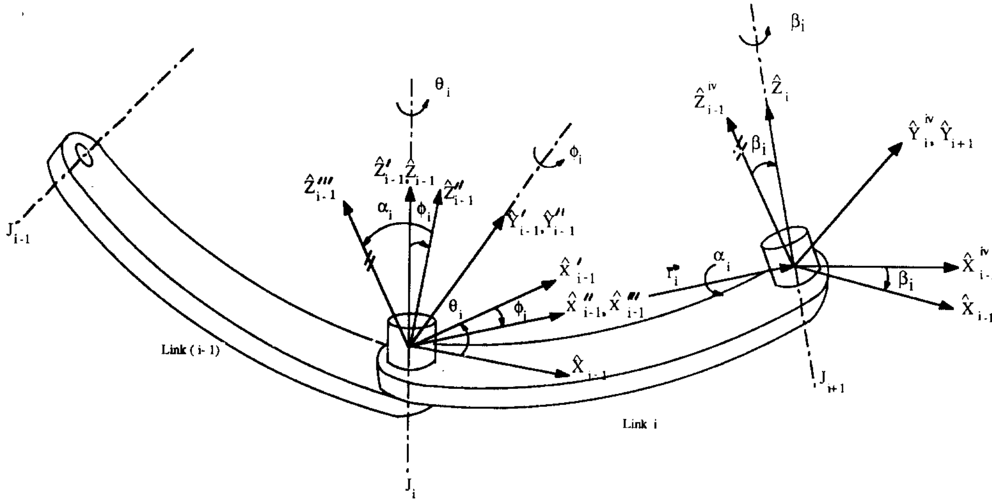

Fig. 1 A spatial binary linkage element

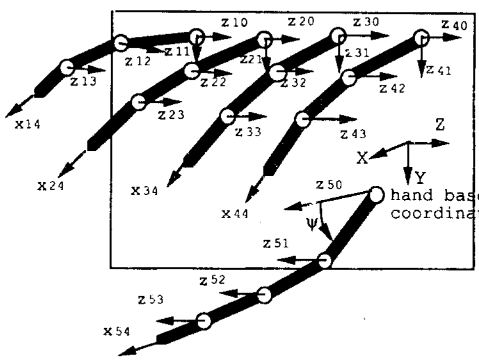

Fig. 2 The coordinate defined by C–Y notation.

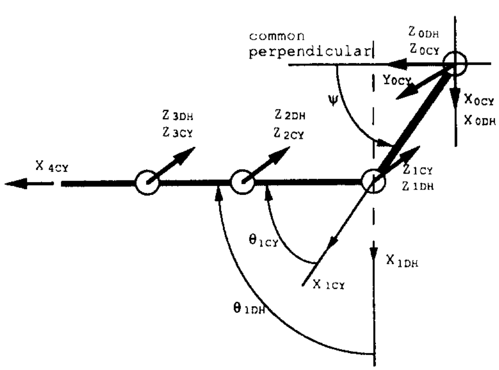

Fig. 3 The coordinate system of C–Y and D–H for the thumb

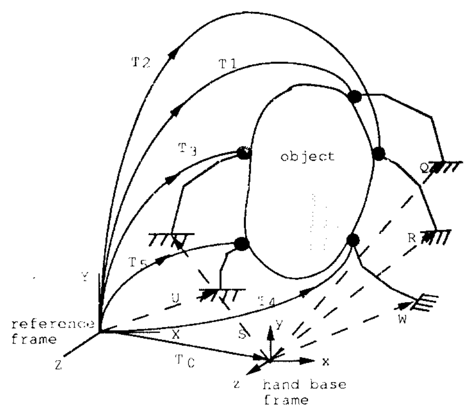

Fig. 4 The locations of the hand

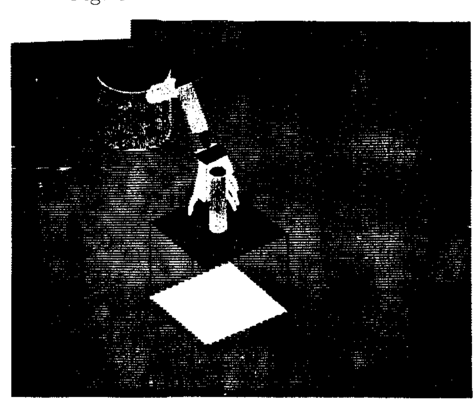

Fig. 5 The graphic simulation of the hand motion