'91 KACC 1991. 10. 22~24

OPTIMAL LOAD DISTRIBUTION FOR TWO COOPERATING ROBOT ARMS USING FORCE ELLIPSOID

Myoung Hwan Choi, Hye Kyung Cho, Bum Hee Lee and Myoung Sam Ko

Automation and Systems Research Institute and
Dept. of Control and Instrumentation Eng.
Seoul National University
Shinrim-dong, Kwanak-ku, Seoul, 151-742, Korea

Abstract

The optimal load distribution for two cooperating robots is studied in this paper, and a new solution approach utilizing force ellipsoid is proposed. The load distribution problem is formulated as a nonlinear optimization problem with a quadratic cost function. The limit on instantaneous power is considered in the problem formulation as the joint torque constraints. The optimal solution minimizing energy consumption is obtained using the concept of force ellipsoid and the nonlinear optimization theory. The force ellipsoid provides a useful geometrical insight into the load distribution problem. Despite the presence of the joint torque constraints, the optimal solution is obtained almost as a closed form, in which the joint torques are given in terms of a single scalar parameter that can be obtained numerically by solving a scalar equation.

I. INTRODUCTION

In recent years, growing research efforts have been focused on the subject of cooperative multiple robot systems. Multiple robot arms in cooperation can perform many tasks that would be impossible to perform for a single robot arm. Examples of these tasks are the manipulation of objects without auxiliary equipments such as jigs or fixtures, and the handling of heavy or large objects which preclude single arm grip on the object. Potential application areas include industrial automation where high level of flexibility is required, and space station construction works where the environment is unstructured.

When multiple robot arms grasp a common object, they form a closed kinematic chain. For multiple robot arms forming a closed kinematic chain, the degrees of freedom are less than the total number of joints, and the linear mapping from joint torque vector space to the space of resultant force vector on the object has a null space. As a result, there are infinite number of joint torque solutions that can be applied for a particular motion of the object. A suitable performance index and constraints need to be introduced, so that the optimal joint torques can be obtained.

A performance index commonly used in the literature is the minimum energy consumption [1]-[4]. Other performance indices used are the maximum safety margin on the friction and joint torque constraints [6], the minimum norm of contact forces [7] [8], the balance of normal components of contact forces [1], the temporal continuity of the applied forces [12], and the minimum total absolute sum of normal components of contact forces [12]. Combinations of some of these functions have been also studied [1] [12].

Constraints introduced in the problem formulation is a factor governing the solution approaches. Closed form solutions are obtained without considering the physical constraints [2] [4] [7] [9] [10]. Closed form solutions show clearly the relationship between the input variables and the solution and require short computation time. However, the closed form solutions are difficult to obtain when constraints are imposed on the joint torques. When the constraints are considered, the solutions are obtained by numerical methods. The physical constraints treated in the literature are friction constraints only [5] [8], joint torque constraints only [3], or friction constraints in addition to the joint torque constraints [1] [12] [13]. Numerical methods generally require intensive computation. An efficient computation algorithm has been developed in [13], which can be used when the performance index and the constraints are formulated as linear functions.

The object grasping methods considered in the model of the robot system can be divided into two cases. In one case, it is assumed that end effectors of the robot arms maintain a firm grip on the object and no motion between the end effectors and the object is allowed [2]-[4] [7] [9]-[11]. In the other case, frictional contacts between the object and the end effectors are assumed, and frictional forces prevent the object from slipping away [1] [5] [8].

Force ellipsoid and manipulability ellipsoid [22] concerns the manipulating ability of the robot mechanism in positioning and orienting the object. The manipulability ellipsoid (or sometimes called the velocity ellipsoid) represents the characteristics of the end effector velocities that can be generated by all unit norm joint velocities, while the force ellipsoid represents the characteristics of the end effector forces that correspond to all unit norm joint torques. The manipulability ellipsoid and the force ellipsoid have a dual relationship. They have been used frequently as indicators of the dexterity measure of the redundant manipulators, and many researchers worked on the control algorithms of redundant manipulators to maximize the dexterity measure and avoid the singular positions in the workspace [20]-[29].

The objective of this paper is to apply the concept of the force ellipsoid to the optimal load distribution problem for two cooperating robot arms. Literature survey reveals that the possibility of using the force ellipsoid in the load distribution has not received proper attention, and the problem of the optimal load distribution and the concept of force ellipsoid have been unrelated subjects. The only work relating the two concepts is found in Tao and Luh [4], where the solution of the optimal load distribution problem is used in deriving the dual arm force ellipsoid equation. However, the force ellipsoid, by definition, shows the characteristics of the end effector forces that correspond to all joint torque vectors of unit instantaneous power, and as a consequence, its concept is well suited for the use in the load distribution problem when the optimality criterion is the minimum energy.

This paper is organized as follows. In section II, the mathematical models of robots and object are given, and the optimal load distribution problem is formulated. In section III and IV, the optimal load distribution is solved with and without joint torque constraints, followed by conclusions in section V.

II. PROBLEM FORMULATION

The model of the two robot arms and the object is shown in Fig. 1, where the end effectors of the two robots are grasping an object. The object is held rigidly so that no relative motion is allowed between the object and the end effectors. The two robots are working in the undistinguished mode, so that no distinction is made regarding the master or slave status. For the purpose of convenience, the superscripts i = 1, 2 are used to indicate the two robots. Let

- ${}^{i}x$ = Position and orientation vector of the end effector of robot i in Cartesian space, ${}^{i}x \in R^m$
- ${}^{i}q$ = Joint position vector of robot i, ${}^{i}q \in R^n$
- ${}^{i}J$ = Manipulator Jacobian matrix, ${}^{i}J \in R^{m \times n}$, $m \le n$
- ${}^{i}T$ = Joint torque vector of robot i, ${}^{i}T \in R^n$
- ${}^{i}F$ = Cartesian force vector applied by robot i at the object reference point, ${}^{i}F \in R^m$
- F = Resultant force applied by the two robots at the object

reference point , $F \in R^m$

The end effectors of the robots are imaginarily extended and the reference position of the object is viewed as the end effector positions of the two robots. It is assumed that two robots are non-redundant, that is, n = m, have the same number of degrees of freedom, and do not pass through singular positions so that the Jacobians always have full ranks. The dynamic equations of motion for the two robots are given by the following equations.

$$ {}^iD({}^iq)\,{}^i\ddot{q} + {}^iH({}^iq, {}^i\dot{q}) + {}^iG({}^iq) = {}^iT + ({}^iJ)^T\,{}^iF \ , \ i = 1,\ 2 \qquad (1)$$

In general, the motion of the object is completely determined by the three dimensional position and orientation vectors. If the object reference coordinate is located at the center of mass, and the reference coordinate axes coincide with the principal axes of the object, the equation of motion of the object is described as follows.

$$\begin{bmatrix} M & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} \ddot{p} \\ \dot{\omega} \end{bmatrix} + \begin{bmatrix} 0 \\ \omega \times I\omega \end{bmatrix} + \begin{bmatrix} MG \\ 0 \end{bmatrix} = -({}^1F + {}^2F) \qquad (2)$$

where M is a 3 x 3 diagonal matrix whose diagonal elements represent the mass of the object, I is the 3 x 3 diagonal inertia matrix of the object, p is the position vector of the object in the world coordinates, ω is the angular velocity of the object, and $G = [\,0\ 0\ g\,]^T$ is the gravitational acceleration vector.

Given the trajectory of the robots and the object, $\ddot{p}$, ω, $\dot{\omega}$, ${}^iq$, ${}^i\dot{q}$ and ${}^i\ddot{q}$ can be found , since there is no relative motion between the end effectors and the object. Using ${}^iB$ and -F to denote the left sides of (1) and (2) respectively, these equations can then be rewritten [2].

$$\begin{aligned} ({}^1J)^T\,{}^1F + {}^1T &= {}^1B \\ ({}^2J)^T\,{}^2F + {}^2T &= {}^2B \\ F &= {}^1F + {}^2F \end{aligned} \qquad (3)$$

Since the robots are assumed to be non-redundant, i. e. n = m and the Jacobians are nonsingular, (3) can be reformed as below.

$$\begin{aligned} {}^1f &= ({}^1J)^{-T}\,{}^1B - {}^1F \\ {}^2f &= ({}^2J)^{-T}\,{}^2B - {}^2F \\ F &= {}^1F + {}^2F \qquad , \text{ where } {}^if = ({}^iJ)^{-T}\,{}^iT \end{aligned} \qquad (4)$$

Combining three equations and using ${}^cf$ to denote ($({}^1J)^{-T}\,{}^1B + ({}^2J)^{-T}\,{}^2B - F$), we get

$$ {}^1f + {}^2f = {}^cf \qquad (5)$$

The joint torques of the robots must satisfy the constraint (5) to follow the specified trajectory. Among the joint torques that satisfy this constraint, the optimal joint torque that minimize energy consumption is to be obtained. Let T denote the composite vector $[\,{}^1T^T\ {}^2T^T\,]^T$ where ${}^iT$ is the joint torque vector of robot i, and let E denote the instantaneous power consumption of the two robots. If the joint torque of a motor is proportional to the motor current, the power consumption is proportional to the square of the joint torque. Without loss of generality, it can be assumed that the joint torque variables in (1) have been scaled such that the power consumption of the two robots is equal to $E = T^T T = \sum_{j=1}^{n} ({}^1T_j)^2 + ({}^2T_j)^2$ where ${}^iT_j$ denotes the joint torque of the j-th actuator of robot i. Since the minimization of power consumption at every instant results in the minimization of energy, the objective function is formulated as the power consumption $E = T^T T$. Hence ,

$$\begin{aligned} E &= T^T T \\ &= {}^1T^T\,{}^1T + {}^2T^T\,{}^2T = {}^1f^T\,{}^1J\,{}^1J^T\,{}^1f + {}^2f^T\,{}^2J\,{}^2J^T\,{}^2f \\ &= {}^1f^T\,{}^1A\,{}^1f + {}^2f^T\,{}^2A\,{}^2f \end{aligned} \qquad (6)$$

, where ${}^iA$ denotes ${}^iJ\ {}^iJ^T$ and is symmetric positive definite. The optimal load distribution problem to solve can be expressed as below.

$$\begin{aligned} \text{Minimize} \quad & E = {}^1f^T\,{}^1A\,{}^1f + {}^2f^T\,{}^2A\,{}^2f \\ \text{subject to} \quad & {}^1f + {}^2f = {}^cf \end{aligned} \qquad (7)$$

III. OPTIMAL LOAD DISTRIBUTION WITH NO JOINT TORQUE CONSTRAINTS

Force ellipsoid indicates the characteristics of the Cartesian forces that correspond to all unit norm joint torques. The force ellipsoid is mathematically defined as follows [22]. Assuming that an n degree of freedom robot arm is working in an m dimensional task space, where $m \le n$, we have,

$$\tau = J(q)^T f \qquad (8)$$

τ and f denotes the joint torque and the end effector force respectively. The unit sphere in R^n defined by $\|\tau\|^2 = 1$ can be mapped into an ellipsoid in R^m through J.

$$\begin{aligned} \|\tau\|^2 &= \tau^T \tau \\ &= f^T (JJ^T) f = 1 \end{aligned} \qquad (9)$$

The ellipsoid defined in (9) is called the force ellipsoid. The principal axes of the force ellipsoid are aligned with the directions of the eigenvectors of (JJ^T), and the length of each axis is the reciprocal of the square root of corresponding eigenvalues. The force ellipsoid provides a new view point of the load distribution problem, as will be shown below.

Let $f = [\,{}^1f^T\ {}^2f^T\,]^T$, $Q = \text{diag}\{\,{}^1A\,,\ {}^2A\,\}$, $W = [\,I\ \ I\,]$, where I is the m x m identity matrix. Then, the optimization problem in (7) can be reformulated into *Problem 1* using simplified notations.

Problem 1 **:**

$$\begin{aligned} \text{Minimize} \quad & \frac{1}{2} f^T Q f \\ \text{subject to} \quad & \\ & W f = {}^cf \end{aligned}$$

Solution :
The optimal solution f^* satisfies the first order necessary condition for this problem [31].

$$\begin{aligned} Q f^* + W^T \lambda^* &= 0 \\ W f^* - {}^cf &= 0 \end{aligned} \qquad (10)$$

where λ^* is a m x 1 Lagrange multiplier. Using the previous notations, the necessary conditions can be rewritten.

$$ {}^1A\,{}^1f^* = -{}^2A\,(\,{}^1f^* - {}^cf\,) \qquad (11)$$

The objective function can be interpreted using the equation of the force ellipsoid. Let ${}^iE = {}^iT^T\,{}^iT = {}^if^T\,{}^iA\,{}^if$. Then ${}^iE$ is the power consumption of robot i that is required to exert ${}^if$. Since ${}^iE = {}^if^T\,{}^iA\,{}^if$ is the equation of an ellipsoid, ${}^iE$ is also equivalent to the size of the ellipsoid. The shape of the ellipsoid is determined by ${}^iA$. ${}^1E$ and ${}^2E$ can be written using the constraint in (7) as ${}^1E = {}^1f^T\,{}^1A\,{}^1f$ and ${}^2E = (\,{}^1f - {}^cf\,)^T\,{}^2A\,(\,{}^1f - {}^cf\,)$. Now, the necessary condition (11) means that the normal vectors of the two ellipsoids at ${}^1f^*$ must be in the opposite directions and their magnitude be equal. Hence the optimization problem is equivalent to finding the powers of two robot arms, ${}^1E$ and ${}^2E$, such that the two ellipsoids meet sharing the same tangent plane and , at the same time, the sum of the two powers ${}^1E + {}^2E$ is minimized. ${}^1f^*$ is then equal to the meeting point of the two ellipsoids. Fig. 2 shows the two ellipsoids when the dimension m is equal to 2. This geometric interpretation provides a useful intuition in this load distribution problem .

Since Q is positive definite and W has full ranks, this problem is known to have a unique solution [31]. Using the matrix identity $(\,{}^1H^{-1} + {}^2H^{-1}\,)^{-1} = {}^1H\,(\,{}^1H + {}^2H\,)^{-1}\,{}^2H$, where ${}^1H$, ${}^2H$ and ${}^1H + {}^2H$ are nonsingular matrices, the solution is obtained from (10).

$$
\begin{aligned}
f^{*} &= [\ {}^{1}f^{*T}\ \ {}^{2}f^{*T}\ ]^{T} \\
&= Q^{-1} W^{T} [\ W Q^{-1} W^{T}\ ]^{-1}\ {}^{c}f \\
&= \begin{bmatrix} ({}^{1}J{}^{1}J^{T} + {}^{2}J{}^{2}J^{T})^{-1}\ {}^{2}J{}^{2}J^{T} (({}^{1}J)^{-T}\ {}^{1}B + ({}^{2}J)^{-T}\ {}^{2}B - F) \\ ({}^{1}J{}^{1}J^{T} + {}^{2}J{}^{2}J^{T})^{-1}\ {}^{1}J{}^{1}J^{T} (({}^{1}J)^{-T}\ {}^{1}B + ({}^{2}J)^{-T}\ {}^{2}B - F) \end{bmatrix} \square
\end{aligned} \tag{12}
$$

A similar method was used for multi-arm cooperating robots by Hayati [9]. The optimization problem has a unique optimal solution. The solution in (12) is the same as that obtained by the conventional pseudo-inverse method in Zheng and Luh [2].

IV. OPTIMAL LOAD DISTRIBUTION WITH POWER CONSTRAINTS ON JOINT TORQUES

A. Formulation of constrained optimal load distribution problem

The load distribution problem is studied when the joint torque constraints are included in the problem formulation. The joint torque constraints commonly used in the literature [1] [3] [12] [13] are the constant bounds on the individual joint torques, i. e.,

$$|{}^{i}T_{j}| \le {}^{i}T_{max} \quad j = 1, \dots, n \tag{13}$$

, where ${}^{i}T_{j}$ denotes the joint driving torque of the j-th actuator of robot i. This constraint is an approximation of the limit of the joint torque, which, in reality, can be a function of the joint speed. Expression (13) represents a rectangle in n-dimensional torque space.

In this paper, we treat the case where the power supplied by each robot is constrained by a maximum value. The constraint on power consumption is well suited for the force ellipsoid approach, and it can be handled with ease. As mentioned in section II, the power consumed by the joint actuators of robot i is equal to ${}^{i}T^{T}\,{}^{i}T$. Let the power limit be given by ${}^{i}E_{max}$, i. e.,

$$
\begin{aligned}
{}^{i}T^{T}\,{}^{i}T &= ({}^{i}T_{1}, \dots, {}^{i}T_{n}) ({}^{i}T_{1}, \dots, {}^{i}T_{n})^{T} \\
&= ({}^{i}T_{1})^{2} + ({}^{i}T_{2})^{2} + \dots + ({}^{i}T_{n})^{2} \\
&\le {}^{i}E_{max}
\end{aligned} \tag{14}
$$

This constraint represents a sphere in n-dimensional torque space. Also , from the definition of ${}^{i}f$ in (4),

$$ {}^{i}E = {}^{i}T^{T}\,{}^{i}T = {}^{i}f^{T}\,{}^{i}J{}^{i}J^{T}\,{}^{i}f = {}^{i}f^{T}\,{}^{i}A\,{}^{i}f \le {}^{i}E_{max} \quad , \quad i = 1, 2 \tag{15}$$

The constrained optimal load distribution problem can be formulated as below.

***Problem 2*:**

Minimize $\quad \dfrac{1}{2} f^{T} Q f$

subject to

$$W f = {}^{c}f$$
$$f^{T} R f \le {}^{1}E_{max} \tag{16}$$
$$f^{T} S f \le {}^{2}E_{max} \tag{17}$$

where $R = \mathrm{diag}\{ {}^{1}A , 0 \}$ and $S = \mathrm{diag}\{ 0 , {}^{2}A \}$. *Problem 2* is the same as *Problem 1* except that the feasible region of f is further constrained by the two additional inequality constraints. Since the objective function and the feasible region of f are strictly convex, the optimal solution, if it exists, occurs at a unique point. The solution of *Problem 2* belongs to one of the following cases.

[Case I] The unique solution of *Problem 1* satisfies the two inequality constraints. In this case, the two inequality constraints do not affect the solution and the solution of *Problem 2* is the same as that of *Problem 1*.

[Case II] The solution of *Problem 1* violates both inequality constraints. In this case, *Problem 2* has no solution.

[Case III] The solution of *Problem 1* violates one of the two inequality constraints. Then, the global optimal solution of *Problem 1* lies outside the feasible region given by the three constraints of *Problem 2* . Let this feasible region be denoted by Ω. Since there is no local optimal solution of *Problem 1* inside the feasible region Ω, the solution of *Problem 2*, if it exists, must lie on the boundary of the feasible region Ω, which means that at least one of the two inequality constraints is active. Let f^{*} denote the solution of *Problem 2* . Then, the Case III can be subdivided into three cases that can possibly occur.

[Case III-a] f^{*} satisfies $f^{*T} R f^{*} = {}^{1}E_{max}$ and $f^{*T} S f^{*} < {}^{2}E_{max}$.

[Case III-b] f^{*} satisfies $f^{*T} R f^{*} < {}^{1}E_{max}$ and $f^{*T} S f^{*} = {}^{2}E_{max}$.

[Case III-c] f^{*} satisfies $f^{*T} R f^{*} = {}^{1}E_{max}$ and $f^{*T} S f^{*} = {}^{2}E_{max}$.

B. Geometric Interpretation

The geometric interpretation of *Problem 2* is similar to that of *Problem 1* given in section III. The optimization problem is equivalent to finding the powers corresponding to the two meeting ellipsoids such that the sum of the two powers is minimized. However, in *Problem 2*, there are limits on the sizes of the ellipsoids as given by (15). Fig. 3a shows the Case I, where the powers of the two robots, or equivalently, the sizes of the two ellipsoids corresponding to the optimal solution of the unconstrained problem (*Problem 1*) are within their limits. The ellipsoids in real line correspond to the solution of *Problem 1* and the ellipsoids in broken line correspond to the limits of the ellipsoids. Fig. 3b shows the Case II, where the limits on the two ellipsoids prevent them from overlapping, and no solution exists. Fig. 3c and 3d shows the Case III-a and III-b respectively, where the power corresponding to one of the two ellipsoids is fixed to its maximum value, and the other ellipsoid that meets the fixed ellipsoid with the least power is to be determined. Fig. 3e shows the Case III-c where the two limit ellipsoids meet at a point. This situation is a special case of Case III-a and III-b.

C. Solution Steps

From the above observations, the solution of *Problem 2* is obtained by the following steps.

[Step 1] *Problem 1* is solved first. If the unique solution of *Problem 1* satisfies the two inequality constraints of *Problem 2*, then, the unique solution of *Problem 2* is the same as that of *Problem 1*. If the unique solution of *Problem 1* violates both inequality constraints, then, *Problem 2* has no solution, and the trajectory of the robots and the object should be replanned.

[Step 2] If the solution of *Problem 1* violates only one of the two inequality constraints, the candidates for the solution of *Problem 2* are found by solving *Problem 3* and *Problem 4* below.

***Problem 3*:**

Minimize $\dfrac{1}{2} f^{T} Q f$

subject to

$$W f - {}^{c}f = 0$$
$$f^{T} R f - {}^{1}E_{max} = 0$$

***Problem 4*:**

Minimize $\dfrac{1}{2} f^{T} Q f$

subject to

$$W f - {}^{c}f = 0$$
$$f^{T} S f - {}^{2}E_{max} = 0$$

[Step 3] The candidate solutions obtained in step 2 are examined and the candidates violating any of the two inequality constraints are discarded. From the remaining candidates, the power $E = f^{T} Q f$ is calculated and the candidate with the smallest power is the final solution. If all solutions of *Problem 3* and *Problem 4* violate the inequality constraints, then, the *Problem 2* has no solution. The joint torques of the two robots are given by the relation ${}^{i}T = {}^{i}J^{T}\,{}^{i}f$.

[End of Step]

The solutions of the *Problem 3* and *Problem 4* are examined next.

D. Solution of *Problem 3*

***Problem 3*:**

Minimize $\quad \dfrac{1}{2} f^{T} Q f$

subject to

$$W f - {}^{c}f = 0$$
$$f^{T} R f - {}^{1}E_{max} = 0$$

This problem can be subdivided into three cases depending on the magnitude of ${}^{1}E_{max}$.

Assume ${}^1E_{max} > {}^cf^T\, {}^1A\, {}^cf$. Then, since Q = R + S, it is clear that,

$$\text{minimum cost of } \textit{Problem 3} \geq {}^1E_{max} > {}^cf^T\, {}^1A\, {}^cf$$

However if we choose $f = [\, {}^1f \;\; {}^2f \,] = [\, {}^cf \;\; 0 \,]$, then, this selection of f satisfies the constraints in *Problem 2* while its cost is ${}^cf^T\, {}^1A\, {}^cf$ which is smaller than the minimum cost of *Problem 3*. Hence, the solution of *Problem 3* when ${}^1E_{max} > {}^cf^T\, {}^1A\, {}^cf$ can not be the optimal solution of *Problem 2*, and need not be considered. Assume ${}^1E_{max} = {}^cf^T\, {}^1A\, {}^cf$. Then, the minimum cost is equal to ${}^1E_{max} = {}^cf^T\, {}^1A\, {}^cf$ and the optimal solution of *Problem 3* is equal to $f^* = [\, {}^cf \;\; 0 \,]$.

Finally, assume ${}^1E_{max} < {}^cf^T\, {}^1A\, {}^cf$. Then, the uniqueness of the optimal solution of *Problem 3* can be shown.

Lemma 1. If ${}^1E_{max} < {}^cf^T\, {}^1A\, {}^cf$, the solution of *Problem 3* exists and is unique.

(proof)

Problem 3 is equivalent to minimizing the power ${}^2E$ such that ${}^1f^T\, {}^1A\, {}^1f = {}^1E_{max}$ and ${}^2E = (\, {}^1f - {}^cf \,)^T\, {}^2A\, (\, {}^1f - {}^cf \,)$. These two equations are the equations of ellipsoids. The geometric interpretation of this problem is that we want to find the smallest size of the ellipsoid given by ${}^2E = (\, {}^1f - {}^cf \,)^T\, {}^2A\, (\, {}^1f - {}^cf \,)$ such that the intersection of the two ellipsoids is non-empty. Fig. 4 illustrates the problem.

Let ${}^1L = \{\, {}^1f \mid {}^1f^T\, {}^1A\, {}^1f \leq {}^1E_{max} \,\}$ and ${}^2L(x) = \{\, {}^1f \mid (\, {}^1f - {}^cf \,)^T\, {}^2A\, (\, {}^1f - {}^cf \,) \leq x \,\}$. Then, since ${}^1E_{max} < {}^cf^T\, {}^1A\, {}^cf$, there exist $\epsilon > 0$, such that ${}^1L \cap {}^2L(\epsilon) = \emptyset$. Also, there exist α such that ${}^1L \cap {}^2L(x) = \emptyset, \forall x < \alpha$, and ${}^1L \cap {}^2L(x) \neq \emptyset, \forall x \geq \alpha$. α is the minimum value of ${}^2E$, and two ellipsoids ${}^1L$ and ${}^2L(\alpha)$ meet at a point sharing a common tangent plane. Hence, the solution is unique. ■

Let the constraints be denoted by h(f) = 0. Then ,

$$h(f) = \begin{bmatrix} Wf - {}^cf \\ f^T R f - {}^1E_{max} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (18)$$

$$\nabla h(f)^T = [\; W^T \;\; 2Rf \;] \quad (19)$$

Hence, $\nabla h(f^*)$ has full ranks and the optimal solution is a regular point. The first order necessary conditions for this problem are

$$Q f^* + W^T \lambda^* + \mu^* R f^* = 0 \quad (20)$$

$$W f^* - {}^cf = 0 \quad (21)$$

$$f^{*T} R f^* - {}^1E_{max} = 0 \quad (22)$$

where λ^* and μ^* are Lagrange multipliers of dimension (m x 1) and (1 x 1) respectively. From (20),

$$(\, Q + \mu^* R \,) f^* = - W^T \lambda^* \quad (23)$$

We show that $(\, Q + \mu^* R \,)$ is nonsingular. We note

$$(\, Q + \mu^* R \,) = \text{diag}\{\, (1+\mu^*)\, {}^1A ,\; {}^2A \,\} \quad (24)$$

$$\det(\, Q + \mu^* R \,) = (\, 1 + \mu^* \,)^m \det {}^1A \; \det {}^2A \quad (25)$$

Since ${}^iA = {}^iJ\, {}^iJ^T$ is positive definite, $\det(\, Q + \mu^* R \,)$ is zero if and only if $\mu^* = -1$. However, if we assume that $\mu^* = -1$, then, from (23),

$$\begin{bmatrix} 0 & 0 \\ 0 & {}^2A \end{bmatrix} \begin{bmatrix} {}^1f^* \\ {}^2f^* \end{bmatrix} = -1 \begin{bmatrix} \lambda^* \\ \lambda^* \end{bmatrix} \quad (26)$$

Solving the equation,

$$\lambda^* = 0 ,\; {}^2f^* = 0 ,\; {}^1f^* = {}^cf \quad (27)$$

and by (22), we get ${}^cf^T\, {}^1A\, {}^cf = {}^1E_{max}$ which contradicts the assumption. Hence $\mu^* \neq -1$ and $(\, Q + \mu^* R \,)$ is nonsingular. Thus, from (23),

$$f^* = - (\, Q + \mu^* R \,)^{-1} W^T \lambda^* \quad (28)$$

From (28) and (21),

$$W (\, Q + \mu^* R \,)^{-1} W^T \lambda^* = - {}^cf \quad (29)$$

The nonsingularity of $[\, W (\, Q + \mu^* R \,)^{-1} W^T \,]$ is shown next. Using the definitions of f, Q, W, R and S, the necessary conditions (20), (21) and (22) can be rewritten as

$$-(\, 1 + \mu^* \,)\, {}^1A\, {}^1f^* = {}^2A\, (\, {}^1f^* - {}^cf \,) \quad (30)$$

$${}^1f^{*T}\, {}^1A\, {}^1f^* = {}^1E_{max} \quad (31)$$

The normal vectors of the ellipsoids ${}^1f^T\, {}^1A\, {}^1f = c_1$ and $(\, {}^1f - {}^cf \,)^T\, {}^2A\, (\, {}^1f - {}^cf \,) = c_2$, where c_1 and c_2 are constants, are given by $2\, {}^1A\, {}^1f$ and $2\, {}^2A\, (\, {}^1f - {}^cf \,)$ respectively. Further, as shown in Fig. 4, the optimal solution occurs when the two ellipsoids make a tangential contact with each other, and the meeting point is the optimal solution ${}^1f^*$. Hence, the normal vectors of the two ellipsoids at ${}^1f^*$ must be in the opposite direction, and we get from (30) ,

$$-(\, 1 + \mu^* \,) < 0 \quad (32)$$

$$\mu^* > -1 \quad (33)$$

In (29), the matrix $[\, W (\, Q + \mu R \,)^{-1} W^T \,]$ is equal to $[\, (\, 1 + \mu^* \,)^{-1}\, {}^1A^{-1} + {}^2A^{-1} \,]$, and from (33), it is nonsingular. Thus,

$$\lambda^* = - [\, W (\, Q + \mu^* R \,)^{-1} W \,]^{-1}\, {}^cf \quad (34)$$

Combining (34) and (28),

$$f^*(\mu^*) = (\, Q + \mu^* R \,)^{-1} W^T [\, W (\, Q + \mu^* R \,)^{-1} W^T \,]^{-1}\, {}^cf = \begin{bmatrix} {}^1A^{-1}\, {}^2A\, [\, (\, 1 + \mu^* \,)\, {}^1A + {}^2A \,]^{-1}\, {}^1A\, {}^cf \\ (\, 1 + \mu^* \,)\, [\, (\, 1 + \mu^* \,)\, {}^1A + {}^2A \,]^{-1}\, {}^1A\, {}^cf \end{bmatrix} \quad (35)$$

Furthermore, $f^*(\mu^*)$ must satisfy the constraint (22),

$$f^{*T}(\mu^*)\, R\, f^*(\mu^*) - {}^1E_{max} = 0 \quad (36)$$

which is a function of the scalar variable μ^*. μ^* can be obtained by solving (36) numerically. Once μ^* is known, the optimal solution of *Problem 3* when ${}^1E_{max} < {}^cf^T\, {}^1A\, {}^cf$ is given by (35).

Lemma 2. Some properties of (36) are examined. Let ${}^1E_{max} < {}^cf^T\, {}^1A\, {}^cf$ and $g(\, f(\mu)\,) = f(\mu)^T R\, f(\mu) - {}^1E_{max}$, where $f(\mu)$ is given by (35) . Then ,

i) $g(\, f(-1)\,) > 0$

ii) $d\, g(\, f(\mu)\,) / d\mu < 0$ for all $\mu > -1$

From Lemma 2 and (33), μ^* is obtained by simply searching for the zero crossing point of the function $g(\mu)$ for $\mu > -1$, and requires a short computation time.

E. Solution of *Problem 4*

***Problem 4*:**

$$\text{Minimize} \quad \frac{1}{2} f^T Q f$$

subject to

$$W f - {}^cf = 0$$

$$f^T S f - {}^2E_{max} = 0$$

The *Problem 4* has the same structure as the *Problem 3*, and the solution is obtained by the same procedure as in *Problem 3*.

Assume ${}^2E_{max} > {}^cf^T\, {}^2A\, {}^cf$. Then, since Q = R + S, it is clear that

$$\text{minimum cost of } \textit{Problem 4} \geq {}^2E_{max} > {}^cf^T\, {}^2A\, {}^cf$$

However if we choose $f = [\, {}^1f \;\; {}^2f \,] = [\, 0 \;\; {}^cf \,]$, then, this selection of f satisfies the constraints in *Problem 2* while its cost is ${}^cf^T\, {}^2A\, {}^cf$ which is smaller than the minimum cost of *Problem 4*. Hence, the solution of *Problem 4* when ${}^2E_{max} > {}^cf^T\, {}^2A\, {}^cf$ can not be the optimal solution of *Problem 2*, and need not be considered.

Assume ${}^2E_{max} = {}^c f^T\, {}^2A\, {}^c f$. Then, the minimum cost is equal to ${}^2E_{max} = {}^c f^T\, {}^2A\, {}^c f$ and the optimal solution of *Problem 4* is $f^* = [\, 0 \;\; {}^c f\,]$.

Finally, assume ${}^2E_{max} < {}^c f^T\, {}^2A\, {}^c f$. Then, the optimal solution of *Problem 4* is found in the exactly symmetrical way as in the Case III-a.

$$
\begin{aligned}
f^* &= (\, Q + \mu^* S\,)^{-1} W^T [\, W (\, Q + \mu^* S\,)^{-1} W^T\,]^{-1}\, {}^c f \\
&= \begin{bmatrix} (\, 1 + \mu^*\,)\, [\, {}^1A + (\, 1 + \mu^*\,)\, {}^2A\,]^{-1}\, {}^2A\, {}^c f \\ {}^2A^{-1}\, {}^1A\, [\, {}^1A + (\, 1 + \mu^*\,)\, {}^2A\,]^{-1}\, {}^2A\, {}^c f \end{bmatrix}
\end{aligned}
\tag{37}
$$

Furthermore , f^* must satisfy the constraint,

$$
f^*(\mu^*)^T\, S\, f^*(\mu^*) - {}^2E_{max} = 0 \tag{38}
$$

which is a function of the scalar variable μ^* only. μ^* can be obtained by solving (38) numerically. The results symmetrical to those of (33) and Lemma 2 in section III.D can be derived, and the numerical search for μ^* can be simplified to finding the zero crossing point of the function $g(\mu)$ for $\mu > -1$. Once μ^* is known, the optimal solution of *Problem 4* when ${}^2E_{max} < {}^c f^T\, {}^2A\, {}^c f$ is given by (37).

V. CONCLUSIONS

A new geometric solution approach to the optimal load distribution utilizing force ellipsoid is proposed. It is shown that the optimal load distribution problem for two cooperating robots can be solved using the force ellipsoid and the nonlinear optimization theory. The concept of the force ellipsoid gives a useful geometrical insight into the problem. The load distribution problem is formulated as a nonlinear optimization problem with a quadratic cost function and constraints, and the optimal joint torque solution minimizing the energy consumption is obtained.

In the unconstrained optimal load distribution, the optimal solution is obtained in a closed form, and it agrees with the solution obtained by the conventional pseudo-inverse method. In the constrained optimal load distribution, the torques are constrained such that the maximum value of the instantaneous power is specified. It is shown that, despite the presence of the joint torque constraints, the optimal solution can be obtained almost as a closed form, in which the joint torques are given in terms of a single scalar parameter, and the parameter itself can be obtained by solving a scalar equation numerically.

REFERENCES

[1] D. E. Orin and S.Y. Oh, "Control of Force Distribution in Robotic Mechanisms Containing Closed Kinematic Chains, " ASME J. of Dyn. Sys. Meas. Control, Vol. 102, pp. 134-141, 1981.

[2] Y. F. Zheng and J. Y. S. Luh, "Optimal Load Distribution for Two Industrial Robots Handling a Single Object, " Proc. IEEE Int. Conf. on Robotics and Automation, pp. 344-349, Philadelphia, PA, 1988.

[3] J. Y. S. Luh and Y. F. Zheng, "Load Distribution Between Two Coordinating Robots by Nonlinear Programming, " 1988 American Control Conference, pp. 479-482, 1988.

[4] J. M. Tao and J. Y. S. Luh, " Coordination of Two Redundant Robots, " Proc. IEEE Int. Conf. on Robotics and Automation, pp. 425-430, Scottsdale, AZ, 1989.

[5] C. A. Klein and S. Kittivatcharapong, "Optimal Force Distribution for the Legs of a Walking Machine with Friction Cone Constraints, " IEEE Trans. Robotics and Automation, Vol. 6, No. 1, pp. 73-85, Feb. , 1990.

[6] J. Kerr and B. Roth, "Analysis of Multifingered hands, "The Int. J. Robotics Research, Vol. 4, No. 4, pp. 3-17, Winter, 1985.

[7] T. E. Alberts and D. I. Soloway, " Force Control of a Multi-Arm Robot System, " Proc. IEEE Int. Conf. on Robotics and Automation, pp. 1490-1496, Philadelphia, PA, 1988.

[8] Y. Nakamura, K. Nagai and T. Yoshikawa, "Mechanics of Coordinative Manipulation by Multiple Robotic Mechanisms, " Proc. IEEE Int. Conf. on Robotics and Automation, pp. 991-998, Raleigh, NC, 1987.

[9] S. Hayati,"Hybrid Position/Force Control of Multi-Arm Cooperating Robots, " Proc. IEEE Int. Conf. on Robotics and Automation, pp. 82-90, San Francisco, CA, 1986.

[10] V. Kumar and K. J. Waldron, "Sub-Optimal Algorithms for Force Distribution in Multi-fingered Grippers, " Proc. IEEE Int. Conf. on Robotics and Automation, pp. 252-257, Raleigh, NC, 1987.

[11] M. Uchiyama, "A Unified Approach to Load sharing, Motion Decomposing, and Force Sensing of Dual Arm Robots, " Robotics Research: The Fifth Int. Symp. , pp. 225-232, Tokyo, August, 1989, MIT Press.

[12] F. T. Cheng and D. E. Orin, "Optimal Force Distribution in Multiple Chain Robotic Systems, " IEEE Trans. Systems, Man and Cybernetics , Vol. 21, No. 1, pp. 13-24, Jan/Feb, 1991

[13] F. T. Cheng and D. E. Orin, "Efficient Algorithm for Optimal Force Distribution in Multiple-Chain Robotic Systems - The Compact-Dual LP Method, " Proc. IEEE Int. Conf. on Robotics and Automation, pp. 943-950, Scottsdale, AZ, 1989.

[14] I. D. Walker, R. A. Freeman, S. I. Marcus, "Dynamic Task Distribution for Multiple Cooperating Robot Manipulators, " Proc. IEEE Int. Conf. on Robotics and Automation, pp. 1288-1290, Philadelphia, PA, 1988.

[15] J. Lim and D. H. Chyung, "On a Control Scheme for Two Cooperating Robot Arms, " Proc. of 24th Conf. on Decision and Control, pp. 334-337, 1985.

[16] Y. F. Zheng and J. Y. S. Luh,"Constrained Relations Between Two Coordinated Industrial Robots, " Proc. of 1985 Conf. of Intelligent Systems and Machines, Rochester, Michigan, 1985.

[17] Y. F. Zheng and J. Y. S. Luh, "Joint Torques for Control of Two Coordinated Moving Robots, " Proc. IEEE Int. Conf. on Robotics and Automation, pp. 1375-1380, San Francisco, CA, 1986.

[18] B. H. Lee and C. S. G. Lee,"Collision Free Motion Planning of Two Robots," IEEE Trans. on Systems, Man, and Cybernetics, Vol. 17, No. 1, pp. 21-32, 1987.

[19] B. H. Lee, "Constraints Identification in Time Varying Obstacle Avoidance for Robot Manipulators, " IEEE Trans. on Systems, Man and Cybernetics, Vol. 19, No. 1, pp. 140-143, 1989.

[20] D. Whitney, " The Mathematical Coordinated Control of Manipulators and Human Prostheses, " ASME J. Dyn. Sys. Meas. Cont. pp. 303-309, 1972.

[21] T. Yoshikawa," Analysis and Control of Robot Manipulators with Redundancy, " Proc. 1st Int. Symp. of Robotics Research, MIT Press, pp. 735-748, 1984.

[22] T. Yoshikawa, " Manipulability and Redundancy Control of Robotic Mechanisms, " Proc. IEEE Int. Conf. on Robotics and Automation, pp. 1004-1009, St. Louis, MO, 1985.

[23] C. A. Klein, "Use of Redundancy in the Design of Robotic Systems, " 2nd Int. Symp. on Robotics Research, MIT Press, pp. 207-214, 1985.

[24] M. Uchiyama, K. Shimizu, and K. Hakomori, " Performance Evaluation of Manipulators Using the Jacobian and Its Application to Trajectory Planning, " Proc. 2nd Int. Symp. of Robotics Research, MIT Press pp. 447-454, 1985.

[25] K. C. Suh and J. Hollerbach, " Local Verses Global Torque Optimization of Redundant Manipulators, " Proc. IEEE Int. Conf. on Robotics and Automation, pp. 619-624, Raleigh, NC, 1987.

[26] S. Lee, " Dual Redundant Arm Configuration Optimization with Task-Oriented Dual Arm Manipulability, " IEEE Trans. on Robotics and Automation, Vol. 5, No. 1, pp. 78-97, Feb. , 1989.

[27] S. Lee and J. Lee, " Multiple Task Point Control of a Redundant Manipulator, " Proc. IEEE Int. Conf. on Robotics and Automation, pp. 988-993, Cincinnati, Ohio, 1990.

[28] W. Homsup and J. N. Anderson, "Performance Evaluation of Robotic Mechanisms, " 1986 American Control Conference, pp. 130-136, 1986.

[29] S. L. Chiu,"Control of Redundant Manipulators for Task Compatibility, " Proc. IEEE Int. Conf. on Robotics and Automation, pp. 1718-1724, Raleigh, NC, 1987.

[30] B. Noble and J. W. Daniel, " Applied Linear Algebra, " 3rd Edition, Prentice-Hall, 1988.

[31] Luenberger, " Linear and Nonlinear Programming, " Second Edition, Addison Wesley, 1984.

[32] M. Avriel, " Nonlinear Programming : Analysis and Methods," Prentice-Hall, 1976

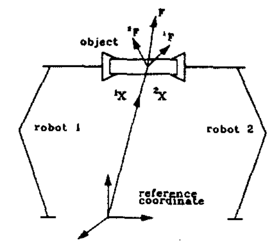

Fig. 1 Two robot arms forming a closed kinematic chain

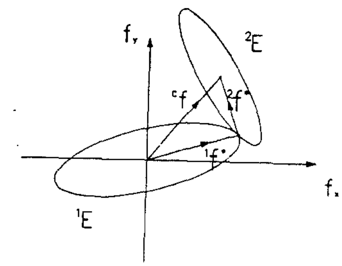

Fig. 2 Geometric interpretation of the load distribution

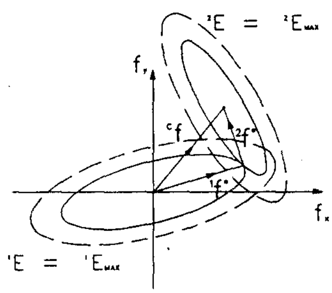

Fig. 3 Constrained load distribution

Fig. 3a. Case I

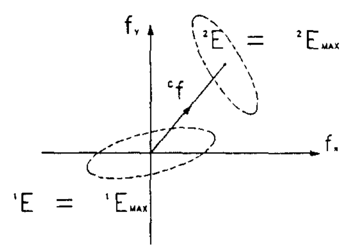

Fig. 3b. Case II

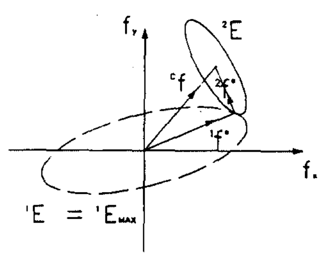

Fig. 3c. Case III-a

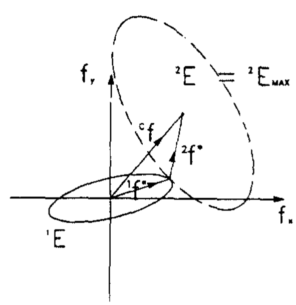

Fig. 3d. Case III-b

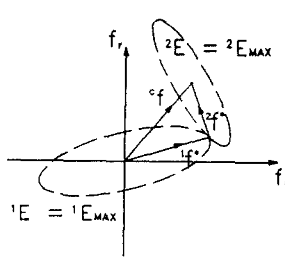

Fig. 3e. Case III-c

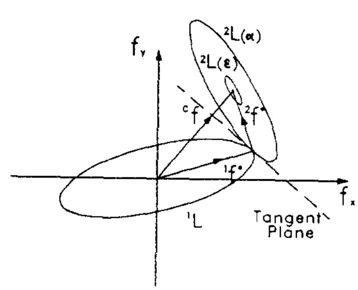

Fig. 4 Illustration of Lemma 1