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Observers for Nonautonomous Discrete-time Nonlinear Systems

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Abstract

We study the observer design problem for nonautonomous discrete-time nonlinear systems. We investigate the structure of nonautonomous discrete-time systems which are state equivalent to the nonlinear observer form and characterize their class. Necessary and sufficient conditions for the existence of an input independent (local) diffeomorphism are derived which transforms multi-input, multi-output nonlinear systems into the nonlinear observer form.

1. Introduction

The design of a nonlinear observer with linearizable error dynamics can be viewed as a dual concept to the feedback linearization problem. A nonlinear observer form can be defined as a canonical structure for which an observer can be constructed with linear error dynamics. If a system can be transformed into the nonlinear observer form via a coordinate change, then one can reconstruct the state of the system through the inverse coordinate transformation of the state of an observer. For the continuous-time models Krener and Isidori[1] obtained a necessary and sufficient condition for a single-output system to be state equivalent to the nonlinear observer form, and the result was extended to the multi-output systems by Krener and Respondek[2] and Xia and Gao[6]. Marino[4] obtained a necessary and sufficient condition for the existence of an input independent diffeomorphism which transforms a single-output system into the nonlinear observer form.

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The problem of designing nonlinear observers for autonomous discrete-time systems was solved in [3]. In this paper we characterize the class of systems which are state equivalent to the nonlinear observer form and derive necessary and sufficient conditions for a discrete-time system to be state equivalent to the nonlinear observer form in the multi-input, multi-output case, as well as in the multi-input, single-output case, using the techniques in [5].

2. Definition and Preliminaries

Let M be a smooth n-dimensional manifold. By smooth, we mean infinite differentiability. We denote by $\mathbb{R}$ the real line. Since the problems addressed in this paper are local in nature, we identify manifold M with an open neighborhood of the origin in $\mathbb{R}^n$. Let T_xM denote a tangent space at $x \in M$. Given $X(t,x) \in T_xM$ for each $t \in \mathbb{R}$, we denote by $\Phi_t^X(p_0)$ the solution of $d\Phi/dt = X(t,\Phi)$, $\Phi(t_0) = p_0$.

Definition. *Let U_1, U_2 be open subsets of a smooth manifold M and $\sigma : U_1 \to U_2$ be a local homeomorphism of class at least C^1. For vector field X over an open set V of M, we define $\mathrm{Ad}_\sigma X$ to be a vector field on $\sigma(U_1 \cap V)$ such that*

$$\mathrm{Ad}_\sigma X(p) = D\sigma \mid_{\sigma^{-1}(p)} X(\sigma^{-1}(p)),$$

where $D\sigma$ implies the Jacobian of σ.

Given $F : M \to M$, $h : M \to \mathbb{R}^m$, $h \circ F$ denotes a composite function $h(F(x))$. We also denote by F^n the n-times composite function $\overbrace{F \circ \cdots \circ F}^{n}$. We denote by ϵ_j a unit vector of $\mathbb{R}^n$ whose j-th component is the unity. The j-the component of a vector z is defined by z_j.

We call the following canonical structure a discrete-time nonlinear observer form

$$z(k+1) = Az(k) + \gamma(y(k)), \quad (1)$$

$$y(k) = cz(k), \quad (2)$$

where

$$A = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}, \quad c = [\, 0 \cdots 0 \; 1 \,]$$

and $\gamma : \mathbb{R} \to \mathbb{R}^n$ is a smooth function. If a single-output autonomous discrete-time nonlinear system

$$x(k+1) = F(x(k)), \quad (3)$$

$$y(k) = h(x(k)) \quad (4)$$

is transformed into a nonlinear observer form (1),(2) by a (local) diffeomorphism $z = T(x)$, then we say that the system (3),(4) is (locally) ***state equivalent*** to the system (1),(2) and vice versa. We can construct an asymptotic observer for the system (1),(2) in such a way that

$$\hat{z}(k+1) = A\hat{z}(k) + l(c\hat{z}(k) - y(k)) + \gamma(y(k)). \quad (5)$$

Then the state observation error $e = \hat{z} - z$ satisfies the error equation $e(k+1) = (A+lc)e(k)$. Thus, if all the eigenvalues of $A + lc$ lie inside the unit circle of the complex plane $\mathbb{C}$, the state observation error e_k converges asymptotically to zero as $k \longrightarrow \infty$. Then, $\hat{x}(k) \equiv T^{-1}(\hat{z}(k))$ converges to $x(k)$.

We define the observability matrix $\mathcal{O}$ and a vector field g as follows:

$$\mathcal{O}(x) = \begin{bmatrix} \frac{\partial h}{\partial x} \\ \frac{\partial (h \circ F)}{\partial x} \\ \vdots \\ \frac{\partial (h \circ F^{n-1})}{\partial x} \end{bmatrix} (x), \quad g(x) = \mathcal{O}^{-1}(x) \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}.$$

Then, a necessary and sufficient condition for the system (1),(2) to be state equivalent to the nonlinear observer form (3),(4) is given in the following theorem.

Theorem 1 *[3]. There exists a local diffeomorphism which transforms the system (1),(2) into the nonlinear observer form (3),(4) if and only if for all x in an open neighborhood of a fixed point x_e of F,*

(i) $\mathcal{O}(x)$ is full-rank,

(ii) $\left[\mathrm{Ad}^i_F g,\ \mathrm{Ad}^j_F g\right] = 0, \qquad 0 \le i, j \le n-1.$

Nonlinear observer form for a multi-input, single-output discrete-time system is given by

$$z(k+1) = Az(k) + \alpha(y(k)) + \sum_{i=1}^{p} \beta_i(y(k))u_i(k), \quad (6)$$

$$y(k) = cz(k), \quad (7)$$

where $\alpha : \mathbb{R} \to \mathbb{R}^n$ and $\beta_i : \mathbb{R} \to \mathbb{R}^n$ are smooth functions. Similarly to the autonomous case, we can construct an observer such that

$$\hat{z}(k+1) = A\hat{z}(k) + l(c\hat{z}(k) - y(k)) + \alpha(y(k)) + \sum_{i=1}^{p} \beta_i(y(k))u_i(k).$$

Repeating the same argument, if all the eigenvalues of $A + lc$ lie inside the unit circle of $\mathbb{C}$, the state observation error $e(k) = \hat{z}(k) - z(k)$ converges asymptotically to zero as $k \longrightarrow \infty$.

3. A Discrete-time System Model

Consider a multi-input, single-output system

$$x(k+1) = H\big(x(k), u(k)\big), \quad (8)$$

$$y(k) = h(x(k)), \quad (9)$$

where $H : M \times \mathbb{R}^p \to M$, $h : M \to \mathbb{R}$ are smooth functions and $u = [u_1, \cdots, u_p]^T$ is the input vector. We assume that $(x_e, 0)$ is a fixed point of the map H, i.e., $H(x_e, 0) = x_e$ and that $h(x_e) = 0$.

To investigate the structure of the system (6), we define its autonomous part by the map $\psi : \mathbb{R}^n \to \mathbb{R}^n$

$$\psi(z) = Az + \alpha(z_n) = \begin{bmatrix} \alpha_1(z_n) \\ z_1 + \alpha_2(z_n) \\ \vdots \\ z_{n-1} + \alpha_n(z_n) \end{bmatrix}.$$

Since discrete dynamics inherits its structure from the continuous-time system by sampling, it is more reasonable to assume that the autonomous part of the system (6), i.e., ψ is a local diffeomorphism. To meet such a requirement, we need to assume that

$$\left.\frac{\partial \alpha_1}{\partial z_n}\right|_{z_n=0} \neq 0. \quad (10)$$

For each u we also define a diffeomorphism $\varphi_u : \mathbf{R}^n \to \mathbf{R}^n$ by

$$\varphi_u(\xi) = \xi + \sum_{i=1}^{p} \tilde{\beta}_i(\xi_1) u_i,$$

where $\tilde{\beta}_i : \mathbf{R} \to \mathbf{R}^n$, $\tilde{\beta}_i(\xi_1) \equiv \beta_i \circ \alpha_1^{-1}(\xi_1)$. Then, we can express the system (6) equivalently as a composition of the two diffeomorphisms:

$$z(k+1) = \varphi_{u(k)} \circ \psi(z(k)). \tag{11}$$

Suppose that the system (8),(9) is state equivalent to the nonlinear observer form (6),(7) in a neighborhood U_{x_e} of x_e and that $z = T(x)$ is the coordinate transformation map which transforms (8),(9) into (6),(7). Transforming back the equivalent expression (11) of the system (6) via a coordinate change $x = T^{-1}(z)$, we obtain

$$x(k+1) = T^{-1}\big(\varphi_{u(k)} \circ \psi(z(k))\big) = \tilde{\varphi}_{u(k)} \circ \tilde{\psi}(x(k)), \tag{12}$$

where $\tilde{\varphi}_{u(k)} = T^{-1} \circ \varphi_{u(k)} \circ T$ and $\tilde{\psi} = T^{-1} \circ \psi \circ T$. Therefore, comparing (12) with (8) we obtain

$$H\big(x(k), u(k)\big) = \tilde{\varphi}_{u(k)} \circ \tilde{\psi}(x(k)).$$

Note that $\tilde{\varphi}_0(x(k)) = x(k)$ since $\varphi_0(\xi) = \xi$. Hence, $\tilde{\psi}(x(k)) = H(x(k), 0)$ and $\tilde{\varphi}_{u(k)}(x(k)) = H(\tilde{\psi}^{-1}(x(k)), u(k))$. Summarizing this, we obtain the following lemma.

Lemma 1. *If the multi-input, single-output system (8),(9) is state equivalent to the nonlinear observer form (6),(7) satisfying (10), then (8) can be represented by the following canonical structure:*

$$x(k+1) = \phi_{u(k)} \circ F(x(k)),$$

where $F : M \to M$, $F(x) = H(x, 0)$ is a diffeomorphism on M, $\phi_u : M \to M$, $\phi_u(x) = H(F^{-1}(x), u)$ is a diffeomorphism on M for each $u \in \mathbb{R}^p$, and $\phi_0(x) = x$.

Remark: If a system is locally state equivalent to the nonlinear observer form, all statement in Lemma 1 becomes local argument.

4. Observers for Single-output Systems

Lemma 1 states that if a multi-input, single-output system is state equivalent to the nonlinear observer form, then it can be represented as a composition of the two diffeomorphisms, ϕ_u and F. Thus, we consider the following canonical form for the characterization of state equivalence to the nonlinear observer form:

$$x(k+1) = \phi_{u(k)} \circ F(x(k)), \tag{13}$$

$$y(k) = h(x(k)). \tag{14}$$

Here, we assume $F(x_e) = x_e$ and let U_{x_e} be an open neighborhood of x_e. We further assume that $F : U_{x_e} \to F(U_{x_e})$ is a local diffeomorphism, and that $h : U_{x_e} \to \mathbf{R}$ is smooth and $h(x_e) = 0$. We also assume that $\phi_u : U_{x_e} \to \phi_u(U_{x_e})$ is a local diffeomorphism for each u in an open neighborhood of $0 \in \mathbf{R}^p$.

Suppose that a local diffeomorphism T from U_{x_e} onto its image $V_o (= T(U_{x_e}))$ transforms the system (13),(14) into the nonlinear observer form (6),(7). Let $G(z(k), u(k)) = Az(k) + \alpha(y(k)) + \sum_{i=1}^{p} \beta_i(y(k)) u_i(k)$ and $\mathcal{X} = T^{-1}$. Then, since $x(k+1) = \mathcal{X}(z(k+1))$, we obtain

$$\phi_{u(k)} \circ F(x(k)) = \mathcal{X}(z(k+1)). \tag{15}$$

It follows from the structure of G that $\frac{\partial G}{\partial z_j} = \epsilon_{j+1}$, $1 \le j \le n-1$. Differentiating the both sides of (15) with respect to z_j, we obtain

$$\frac{\partial \phi_{u(k)}}{\partial x}\Big|_{F(x(k))} \frac{\partial F}{\partial x}\Big|_{x(k)} \frac{\partial \mathcal{X}}{\partial z_j}(z(k)) = \frac{\partial \mathcal{X}}{\partial z}\Big|_{z(k+1)} \frac{\partial G}{\partial z_j}(z(k), u(k))$$
$$= \frac{\partial \mathcal{X}}{\partial z_{j+1}}(z(k+1)), \quad 1 \le j \le n-1, \tag{16}$$

$$\frac{\partial \phi_{u(k)}}{\partial x}\Big|_{F(x(k))} \frac{\partial F}{\partial x}\Big|_{x(k)} \frac{\partial \mathcal{X}}{\partial z_n}(z(k)) = \frac{\partial \mathcal{X}}{\partial z}\Big|_{z(k+1)} \frac{\partial G}{\partial z_n}(z(k), u(k)). \tag{17}$$

We can rewrite the left hand side of (16) as

$$\frac{\partial \phi_{u(k)}}{\partial x}\Big|_{F(x(k))} \frac{\partial F}{\partial x}\Big|_{F^{-1} \circ F(x(k))} \frac{\partial \mathcal{X}}{\partial z_j}(\mathcal{X}^{-1} \circ F^{-1} \circ F(x(k)))$$
$$= \frac{\partial \phi_{u(k)}}{\partial x}\Big|_{F(x(k))} \frac{\partial F}{\partial x}\Big|_{F^{-1} \circ F(x(k))} (D\mathcal{X} \frac{\partial}{\partial z_j})(\mathcal{X}^{-1} \circ F^{-1} \circ F(x(k)))$$
$$= \frac{\partial \phi_{u(k)}}{\partial x}\Big|_{F(x(k))} \mathrm{Ad}_F \mathrm{Ad}_{\mathcal{X}} \frac{\partial}{\partial z_j}(F(x(k)))$$
$$= \mathrm{Ad}_{\phi_{u(k)}} \mathrm{Ad}_F \mathrm{Ad}_{\mathcal{X}} \frac{\partial}{\partial z_j}(x(k+1)).$$

Therefore, $\frac{\partial \mathcal{X}}{\partial z_{j+1}}$ being independent of u implies that $\mathrm{Ad}_{\phi_u(k)} \mathrm{Ad}_F \mathrm{Ad}_{\mathcal{X}} \frac{\partial}{\partial z_j}$ is independent of u. Also note that the necessary and sufficient condition for $\mathrm{Ad}_{\phi_{u(k)}} \mathrm{Ad}_F \mathrm{Ad}_{\mathcal{X}} \frac{\partial}{\partial z_j}$ to be independent of u is

$$\mathrm{Ad}_{\phi_{u(k)}}\mathrm{Ad}_F\mathrm{Ad}_{\mathcal{X}}\frac{\partial}{\partial z_j}\big(x(k+1)\big) = \mathrm{Ad}_F\mathrm{Ad}_{\mathcal{X}}\frac{\partial}{\partial z_j}\big(x(k+1)\big). \quad (18)$$

Letting $u(k) = 0$, we can check the identity of (18). Then, (18) is equal to for $1 \le j \le n-1$

$$\mathrm{Ad}_{\phi_{u(k)}}\mathrm{Ad}_F\frac{\partial \mathcal{X}}{\partial z_j}(z(k+1)) = \mathrm{Ad}_F\frac{\partial \mathcal{X}}{\partial z_j}(z(k+1)). \quad (19)$$

Therefore, from (16),(17), we obtain

$$\mathrm{Ad}_F\frac{\partial \mathcal{X}}{\partial z_j}(z(k+1)) = \frac{\partial \mathcal{X}}{\partial z_{j+1}}(z(k+1)), \quad 1 \le j \le n-1, \quad (20)$$

$$\mathrm{Ad}_F\frac{\partial \mathcal{X}}{\partial z_n}(z(k+1)) = \frac{\partial \mathcal{X}}{\partial z}\Big|_{z(k+1)}\frac{\partial G}{\partial z_n}(z(k),u(k)). \quad (21)$$

Hence, we obtain

$$\frac{\partial \mathcal{X}}{\partial z_{j+1}}(z(k+1)) = \mathrm{Ad}_F^j\frac{\partial \mathcal{X}}{\partial z_1}(z(k+1)), \quad 1 \le j \le n-1, \quad (22)$$

$$\frac{\partial \mathcal{X}}{\partial z}\Big|_{z(k+1)}\frac{\partial G}{\partial z_n}(z(k),u(k)) = \mathrm{Ad}_F^n\frac{\partial \mathcal{X}}{\partial z_1}(z(k+1)). \quad (23)$$

On the basis of the above result we obtain the following proposition.

Proposition 1. *There exists a local diffeomorphism which transforms the system (13),(14) into the nonlinear observer form (6),(7) satisfying (10) if and only if for all (u,x) in an open neighborhood of $(0,x_e) \in \mathbb{R}^p \times M$*

(i) $\mathcal{O}(x)$ *is full-rank,*

(ii) $[\,\mathrm{Ad}_F^i g,\ \mathrm{Ad}_F^j g\,] = 0, \quad 0 \le i,j \le n-1,$

(iii) $\mathrm{Ad}_{\phi_u}\mathrm{Ad}_F^j g(x) = \mathrm{Ad}_F^j g(x), \quad 1 \le j \le n-1,$
where $g(x) = \mathcal{O}^{-1}(x)[0,\cdots,0,1]^T$.

PROOF: *(Necessity)* Since $h(x(k)) = h\circ\mathcal{X}(z(k)) = cz(k)$,

$$< \frac{\partial h}{\partial x}(\mathcal{X}(z(k))),\ \frac{\partial \mathcal{X}}{\partial z_j}(z(k)) > \;=< \frac{\partial h}{\partial x}(\mathcal{X}(z(k))),\ \mathrm{Ad}_F^{j-1}\frac{\partial \mathcal{X}}{\partial z_1}(z(k)) >= \delta_{jn},$$

where δ_{jn} is the Kronecker delta. Thus, we obtain

$$\mathcal{O}(x(k))\frac{\partial \mathcal{X}}{\partial z}(z(k)) = \begin{bmatrix} 0 & \cdots & 0 & 1 \\ \vdots & . & . & \times \\ 0 & . & \times & \vdots \\ 1 & \times & \cdots & \times \end{bmatrix}, \quad (24)$$

where $\times$ denotes an arbitrary function. The nonsingularity of $\frac{\partial \mathcal{X}}{\partial z}$ and the right hand side of (24) implies that $\mathcal{O}(x)$ is full-rank. Therefore, we obtain

$$\frac{\partial \mathcal{X}}{\partial z_1}(z(k)) = \mathcal{O}^{-1}(x(k))[0,\cdots,0,1]^T$$

which can be identified with $g(x(k))$. Since $\{\frac{\partial \mathcal{X}}{\partial z_1},\ \mathrm{Ad}_F\frac{\partial \mathcal{X}}{\partial z_1},\ \cdots,\ \mathrm{Ad}_F^{n-1}\frac{\partial \mathcal{X}}{\partial z_1}\}$, i.e., $\{g,\ \mathrm{Ad}_F g,\cdots,\ \mathrm{Ad}_F^{n-1}g\}$ are push-forwarded vectors of $\{\frac{\partial}{\partial z_1},\ \frac{\partial}{\partial z_2},\cdots,\frac{\partial}{\partial z_n}\}$, we obtain condition *(ii)*. It follows directly from (19),(22) that condition *(iii)* holds.

(Sufficiency) We define a map $\mathcal{X}$ from an open neighborhood V_o' of $0 \in \mathbb{R}^n$ onto its image in M by

$$\mathcal{X}(z) = \Phi_{z_1}^{g}\circ\Phi_{z_2}^{\mathrm{Ad}_F g}\circ\cdots\circ\Phi_{z_n}^{\mathrm{Ad}_F^{n-1}g}(x_e). \quad (25)$$

Since the vector fields $g, \mathrm{Ad}_F g,\cdots,\mathrm{Ad}_F^{n-1}g$ commute, we obtain

$$\frac{\partial \mathcal{X}}{\partial z}(z(k)) = [\,g,\mathrm{Ad}_F g,\cdots,\mathrm{Ad}_F^{n-1}g\,](\mathcal{X}(z(k))). \quad (26)$$

From the definition of g and (26) we obtain (24). Since $\mathcal{O}(x)$ is full-rank, $\frac{\partial \mathcal{X}}{\partial z}$ is nonsingular in V_o'. Therefore, $\mathcal{X}$ is a local diffeomorphism. Choosing $T = \mathcal{X}^{-1}$ as a coordinate transformation map, we obtain

$$z(k+1) = \mathcal{X}^{-1}\circ\phi_{u(k)}\circ\mathcal{X}\circ\mathcal{X}^{-1}\circ F\circ\mathcal{X}(z(k)). \quad (27)$$

Utilizing (26) we obtain that

$$\begin{aligned}\frac{\partial(\mathcal{X}^{-1}\circ F\circ\mathcal{X})}{\partial z}(z) &= [\,g,\mathrm{Ad}_F g,\cdots,\mathrm{Ad}_F^{n-1}g\,]^{-1}_{(F(x))} \frac{\partial F}{\partial x}(x)[\,g,\mathrm{Ad}_F g,\cdots,\mathrm{Ad}_F^{n-1}g\,](x)\\ &= [\,g,\mathrm{Ad}_F g,\cdots,\mathrm{Ad}_F^{n-1}g\,]^{-1}_{(F(x))} [\,\mathrm{Ad}_F g,\mathrm{Ad}_F^2 g,\cdots,\mathrm{Ad}_F^{n}g\,](F(x)).\end{aligned}$$

This implies that $\frac{\partial}{\partial z_j}(\mathcal{X}^{-1}\circ F\circ\mathcal{X})(z) = \epsilon_{j+1}$ for $1 \le j \le n-1$. Thus, we deduce that $\mathcal{X}^{-1}\circ F\circ\mathcal{X}$ is a linear function of $z_1,\cdots,z_{n-1}$. Hence, we obtain for some $\alpha : \mathbb{R} \to \mathbb{R}^n$ that

$$\mathcal{X}^{-1}\circ F\circ\mathcal{X}(z) = Az + \alpha(z_n). \quad (28)$$

On the other hand, notice that

$$\frac{\partial(\mathcal{X}^{-1}\circ\phi_u\circ\mathcal{X})}{\partial z}(z) = [\,g,\mathrm{Ad}_F g,\cdots,\mathrm{Ad}_F^{n-1}g\,]^{-1}_{(\phi_u(x))} \frac{\partial \phi_u}{\partial x}(x)[\,g,\mathrm{Ad}_F g,\cdots,\mathrm{Ad}_F^{n-1}g\,](x).$$

From condition *(iii)* we obtain $\frac{\partial}{\partial z_j}(\mathcal{X}^{-1}\circ\phi_u\circ\mathcal{X})(z) = \epsilon_j$ for $2 \le j \le n$. This implies that

$$\mathcal{X}^{-1}\circ\phi_u\circ\mathcal{X}(z) = z + \sum_{i=1}^{p} \tilde{\beta}_i(z_1)u_i. \qquad (29)$$

Therefore, we can conclude from (27)-(29) that

$$z(k+1) = Az(k) + \alpha(z_n(k)) + \sum_{i=1}^{p} \beta_i(z_n(k))u_i(k),$$

where $\beta_i(z_n(k)) = \tilde{\beta}_i\circ\alpha_1(z_n(k))$. Further, we obtain $y(k) = cz(k)$, since

$$\frac{\partial(h\circ\mathcal{X})}{\partial z_j}(z(k)) =< \frac{\partial h}{\partial x}(x(k)),\ \mathrm{Ad}_F^{j-1}g(x(k)) >= \delta_{jn}.$$

■

Remark: Condition *(iii)* implies that the vector field $Ad_F^j g$ is invariant with respect to the Ad_{ϕ_u} operation for $1 \le j \le n-1$, whose parallel condition in the continuous-time case is the condition *ii)* of Theorem 3.1 of [4]. If manifold M is isomorphic to $\mathbf{R}^n$ and the maps ϕ_u, F and $\mathcal{X}$ are global diffeomorphisms, then the system (13),(14) is globally state equivalent to the system (6),(7).

Corollary 1. *There exists a local diffeomorphism which transforms the system (8),(9) into the nonlinear observer form (6),(7) satisfying (10) if and only if for all (u,x) in an open neighborhood of $(0,x_e) \in \mathbb{R}^p \times M$*

(i) $\mathcal{O}(x)$ *is full-rank,*

(ii) $[\,\mathrm{Ad}_F^i g,\ \mathrm{Ad}_F^j g\,] = 0, \quad 0 \le i,j \le n-1,$

(iii) $\mathrm{Ad}_{\phi_u}\mathrm{Ad}_F^j g(x) = \mathrm{Ad}_F^j g(x), \quad 1 \le j \le n-1,$

where $g(x) = \mathcal{O}^{-1}(x)[0,\cdots,0,1]^T$, $F(x) = H(x,0)$, and $\phi_u(x) = H(F^{-1}(x),u)$.

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