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A STABILIZATION OF AN INVERTED PENDULUM BY A NONLINEAR CONTROL LAW

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ABSTRACT

This paper presents a stabilization technique for unstable systems. An inverted pendulum, which is a typical unstable mechanical system, is considered and stabilized by a nonlinear control. The stabilization problem in this system is related to that in postural control of human being. In this paper, the variable structure control (VSC) is applied to the stabilization problem. Robustness by the VSC and that by a conventional linear feedback controller are compared.

1 INTRODUCTION

The rapid progress of science has been made recently, and many complex systems are constructed. These systems involve problems of stabilization. Since such systems may be unstable. It is interesting to know which kind of a control law is effective in stabilizing systems. In this study, an inverted pendulum is stabilized by a linear control law and a nonlinear control law, and the performances by the controllers are compared. State feedback control (a pole assignment or an optimal regulator) is used as the linear control law, and variable structure control law is used as the nonlinear control law. If the state feedback control is used as a conventional control method, precise mathematical models of the systems are required. In practice, it is, however, impossible to do so, since the mathematical models can not represent system properties completely due to uncertainties, which is caused by some errors such as ones in constructing models and determining parameters, etc. Hence, it seems that variable structure control law, which yields some robustness properties against the uncertainties, is effective in controlling systems. In section 4, mathematical models of inverted pendulums are given. The outline of VSC design is simply described in section 5. In section 6, VSC design of the inverted pendulum are described. Simulations and experimental results are shown respectively in section 5 and section 6.

2 CONTROLLED OBJECT

Controlled object is a cart-pendulum as shown in Fig. 1. The horizontal displacement of pivot on the carriage and the rotational angle of the pendulum are measured by each potentiometer. The carriage is moved horizontally by the motor.

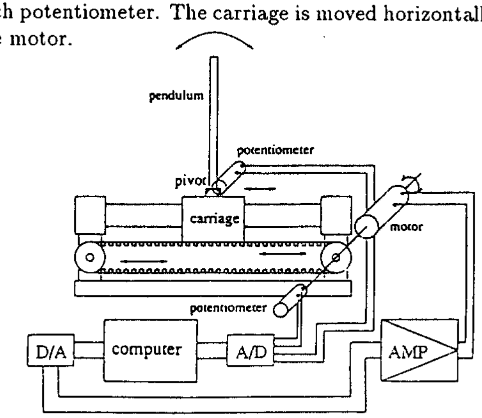

Fig.1. Controlled object

3 MATHEMATICAL MODELS

In this section, mathematical models of a single inverted pendulum and a double inverted pendulum are presented.
(1) Inverted pendulum

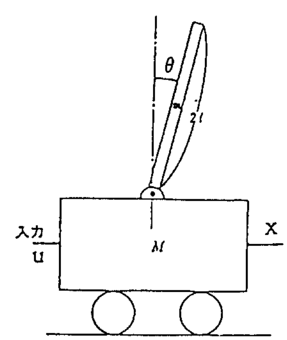

Fig.2. Inverted pendulum

x: the horizontal displacement of pivot on the carriage
θ: the rotational angle of the pendulum
M: the mass of the carriage
m: the mass of the pendulum
l: the half length of the pendulum

F: the friction coefficient for the linear motion of the carriage
C: the friction coefficient for the rotary motion of the pendulum
u: the voltage of the input
G: the gain coefficient
I_G: the moment of inertia with respect to the center of gravity ($= \frac{ml^2}{3}$)

Selecting (x, θ) as generalized coordinates and considering $u_x = Gu, u_\theta = 0$, then Lagrange's equation yields

$$(M+m)\ddot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta + F\dot{x} = Gu \quad (1)$$

$$\frac{4}{3}ml^2\ddot{\theta} + ml\ddot{x}\cos\theta + C\dot{\theta} - mgl\sin\theta = 0 \quad (2)$$

Linearizing the above equation (1) and (2) and choosing the state variables $x = (x_1\ x_2\ x_3\ x_4) = (x\ \theta\ \dot{x}\ \dot{\theta})$, we obtain the linear state space model of the inverted pendulum

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (3)$$

$$A = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{m^2gl^2}{N} & \frac{(I_G+ml^2)F}{N} & \frac{-Cml}{N} \\ 0 & \frac{-(M+m)mgl}{N} & \frac{-mlF}{N} & \frac{C(M+m)}{N} \end{pmatrix}$$

$$B = \begin{pmatrix} 0 & 0 & \frac{-G(I_G+ml^2)}{N} & \frac{Gml}{N} \end{pmatrix}^T$$

where $N = m^2l^2 - (M+m)(I_G+ml^2)$

Selecting (x, θ) as outputs, then the output equation is described by

$$y(t) = Cx(t)$$

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

(2) Double inverted pendulum

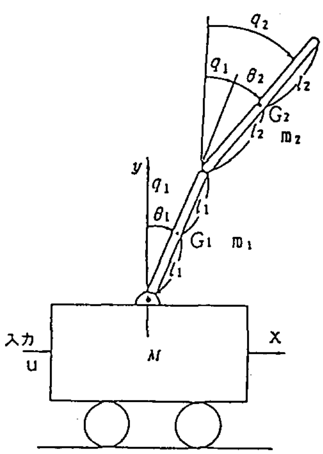

Fig.3. Double inverted pendulum

q_1: the rotational angle of the link 1
q_2: the rotational angle of the link 2
m_1: the mass of the link 1
m_2: the mass of the link 2
l_1: the half length of the link 1
l_2: the half length of the link 2
D_x: the friction coefficient for the linear motion of the carriage
D_1: the friction coefficient for the rotary motion of the link 1
D_2: the friction coefficient for the rotary motion of the link 2
Q: the gain coefficient
I_1: the moment of inertia with respect to the center of gravity of link 1 ($= \frac{m_1l_1^2}{3}$)
I_2: the moment of inertia with respect to the center of gravity of link 2 ($= \frac{m_2l_2^2}{3}$)

The other notations (x, M, u) are the same meanings as ones of the inverted pendulum.

Selecting (x, q_1, q_2) as generalized coordinates and considering $u_x = Qu, u_{q_1} = u_{q_2} = 0$, then Lagrange's equation yields

$$\begin{aligned} (M+m_1+m_2)\ddot{x} + m_2l_2(\ddot{q}_2\cos q_2 - \dot{q}_2^2\sin q_2) + D_x\dot{x} & \\ +(m_1+2m_2)l_1\ddot{q}_1\cos q_1 - (m_1+2m_2)l_1\dot{q}_1^2\sin q_1 &= Qu \\ (m_1+2m_2)l_1\ddot{x}\cos q_1 + \{(m_1+4m_2)l_1^2 + I_1\}\ddot{q}_1 + D_1\dot{q}_1 & \\ +2m_2l_1l_2\{\ddot{q}_2\cos(q_2-q_1) - \dot{q}_2^2\sin(q_2-q_1)\} & \\ -D_2(\dot{q}_2-\dot{q}_1) - (m_1+2m_2)l_1g\sin q_1 &= 0 \\ m_2l_2\ddot{x}\cos q_2 + 2m_2l_1l_2\{\ddot{q}_1\cos(q_2-q_1) + \dot{q}_1^2\sin(q_2-q_1)\} & \\ +(m_2l_2^2+I_2)\ddot{q}_2 + D_2(\dot{q}_2-\dot{q}_1) - m_2l_2g\sin q_2 &= 0 \end{aligned}$$

Linearizing the above equation and choosing the state variables $x = (x_1\ x_2\ x_3\ x_4\ x_5\ x_6) = (x\ q_1\ q_2\ \dot{x}\ \dot{q}_1\ \dot{q}_2)$, we obtain the linear state space model of the double inverted pendulum

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (4)$$

$$A = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} \\ 0 & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} \\ 0 & a_{62} & a_{63} & a_{64} & a_{65} & a_{66} \end{pmatrix}$$

$$B = \begin{pmatrix} 0 & 0 & 0 & \frac{Q(G^2-FL)}{R} & \frac{Q(BL-GC)}{R} & \frac{Q(CF-GB)}{R} \end{pmatrix}^T$$

where

$A = M + m_1 + m_2$
$B = (m_1 + 2m_2)l_1$
$C = m_2l_2$
$D = D_x$
$F = (m_1 + 4m_2)l_1^2 + I_1$
$G = 2m_2l_1l_2$
$H = D_1 + D_2$
$I = -D_2$
$J = -(m_1 + 2m_2)l_1g$
$L = m_2l_2^2 + I_2$
$P = -m_2l_2g$
$R = B^2L - FAL + G^2A - 2GBC + FC^2$

and

$$
\begin{aligned}
a_{42} &= \frac{J(GC-BL)}{R} & a_{43} &= \frac{P(BG-FC)}{R} \\
a_{44} &= \frac{D(FL-G^2)}{R} & a_{45} &= \frac{HGC+IBG-HBL-IFC}{R} \\
a_{46} &= \frac{I(GC-GB-BL+FC)}{R} & & \\
a_{52} &= \frac{J(LA-C^2)}{R} & a_{53} &= \frac{P(BC-GA)}{R} \\
a_{54} &= \frac{D(GC-BL)}{R} & a_{55} &= \frac{BCI-GAI-HC^2+HLA}{R} \\
a_{56} &= \frac{I(GA+LA-BC-C^2)}{R} & & \\
a_{62} &= \frac{J(BC-GA)}{R} & a_{63} &= \frac{P(FA-B^2)}{R} \\
a_{64} &= \frac{D(BG-CF)}{R} & a_{65} &= \frac{BCH-GAH-IB^2+IFA}{R} \\
a_{66} &= \frac{I(B^2-FA+BC-GA)}{R} & &
\end{aligned}
\tag{5}
$$

Selecting (x, q_1, q_2) as outputs, then the output equation is described by

$$
\begin{aligned}
y(t) &= Cx(t) \\
C &= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}
\end{aligned}
$$

4 NONLINEAR CONTROL LAW

4.1 Variable Structure Control

Variable Structure Control(VSC) is a switching feedback control. This variable structure control law gives an effective and robust method in controlling nonlinear systems. VSC use a switching feedback control to drive the state trajectory of the nonlinear system onto a surface which is specified by a designer, and to maintain the system trajectory on this surface for all subsequent time. This surface is called the switching surface because if state trajectory of the system is above the surface, a control path has one gain and if state trajectory is below the surface, a control path has another gain. The dynamics restricted to this surface represent the controlled system behavior. By selecting the switching surface properly, VSC can stabilize the system more effectively. VSC design breaks down into two phases. Phase1 needs to construct switching surfaces so that the system restricted to the switching surface represents a desired behavior. Phase2 needs to construct switching feedback gain which drive the state trajectory to the switching surface and maintain it forever.

4.2 System Model

This paper considers the linear dynamical system described by

$$
\begin{aligned}
\dot{x}(t) &= Ax(t) + Bu(t) &&: A(n \times n), B(n \times m) \\
y(t) &= Cx(t) &&: C(l \times n)
\end{aligned}
\tag{6}
$$

4.3 The Switching Surface

The switching surface $\sigma(x) = 0$ is a (n-m)-dimensional manifold determined by the intersection of m(n-1)-dimensional switching surfaces $\sigma_i(x) = 0$. The switching surfaces are designed such that the system response restricted to $\sigma(x) = 0$ has a desired behavior such as stability.

In general nonlinear switching surfaces are possible. But for the simplicity of design, linear switching surfaces are more popular. So this paper uses linear switching surfaces of the form

$$\sigma(t) = Sx(t) = 0 \tag{7}$$

where S is an $m \times n$ matrix.

4.4 Sliding Modes

After switching surfaces are designed, the next important aspect of VSC is to guarantee the existence of a sliding mode. If in the vicinity of the switching surface $\sigma(x) = 0$, the velocity vectors of the state trajectory always points toward the switching surface, then a sliding mode exists. Therefore if the state trajectory intersects the sliding surface, the value of the state trajectory or "representative point" remains within a neighborhood of $\{x|\sigma(x) = 0\}$. If a sliding mode exists on $\sigma(x) = 0$, then $\sigma(x)$ is called a sliding surface.

4.5 Conditions for the Existence of a Sliding Mode

Existence of a sliding mode requires stability of the state trajectory to the sliding surface $\sigma(x) = 0$ at least in a neighborhood of $\{x|\sigma(x) = 0\}$. In other words, the representative point must approach the surface at least asymptotically. The largest such neighborhood is called the region of attraction.

The existence problem resembles a generalized stability problem. Specifically, stability to the switching surface requires selecting a generalized Lyapunov function $V(t, x)$ which is positive definite and has a negative time derivative in the region of attraction.

The structure of the function $V(t, x, \sigma)$ determines the ease with which one computes the actual feedback gains implementing a VSC design. For poorly chosen Lyapunov functions, the computations of the feedback gain can be untenable.

For all single input systems, a suitable Lyapunov function is $V(t, x) = \frac{1}{2}\sigma^2(x)$ which clearly is globally positive definite. In VSC, $\dot{\sigma}$ will depend on the control and hence if switched feedback gains can be chosen so that

$$\frac{dV}{dt} = \frac{1}{2}\frac{d\sigma^2}{dt} = \sigma\frac{d\sigma}{dt} < 0 \tag{8}$$

in the domain of attraction, then the state trajectory converges to the surface and is restricted to the surface for all subsequent time. The feedback gains which would implement an associated VSC design are straightforward to compute.

4.6 The Method of Equivalent Control

The method of equivalent control is a means to determine the system motion restricted to the switching surface $\sigma(x) = 0$. Suppose at t_0, the state trajectory intercepts the switching surface and a sliding mode exists for $t \geq t_0$. The conditions for the existence of a sliding mode are that

$\dot{\sigma}(x(t)) = 0$ and $\sigma(x(t)) = 0$ for all $t \geq t_0$. From the chain rule $\dot{\sigma}(x(t)) = \left[\frac{\partial\sigma}{\partial x}\right]\dot{x} = 0$. Substituting for $\dot{x}$ yields

$$\left[\frac{\partial\sigma}{\partial x}\right]\dot{x} = \left[\frac{\partial\sigma}{\partial x}\right](Ax(t) + Bu_{eq}) = 0 \qquad (9)$$

where u_{eq} is the so-called equivalent control which solves this equation. Now we select a linear switching surface as a switching surface. Hence $\sigma(x) = Sx = 0$, $\frac{\partial\sigma}{\partial x} = S$. Then

$$S\dot{x} = S(Ax(t) + Bu_{eq}) = 0 \qquad (10)$$

Substituting this u_{eq} into (6), then the motion of (6) describes the behavior of the system restricted to the switching surface providing the initial condition $x(t_0)$ satisfies $\sigma(x(t_0)) = 0$.

To compute u_{eq}, suppose that SB is nonsingular. Then

$$u_{eq} = -(SB)^{-1}SAx \qquad (11)$$

Therefore, given $\sigma(x(t_0)) = 0$, the dynamics of the system on the switching surface for $t \geq t_0$ is given by

$$\dot{x} = \left[I - B(SB)^{-1}S\right]Ax \qquad (12)$$

This equation can be advantageously used in constructing a switching surface.

Notice that (12) and the constraint $\sigma(x) = 0$ determine the system motion on the switching surface. In other words, in the sliding mode, the equivalent system must satisfy not only the n-dimensional state dynamics (12), but also the "m" algebraic equations, $\sigma(t) = 0$. The motion on the switching surface will be governed by a reduced order set of equations. This reduction of order happens due to the set of state variable constraints, $\sigma(x) = 0$.

4.7 The diagonalization Method

The diagonalization method needs to constructing a new control u^* by a nonsingular transformation , $Q^{-1}\frac{\partial\sigma}{\partial x}B = Q^{-1}SB$, of the original control u defined as

$$u^* = Q^{-1}(t,x)SBu \qquad (13)$$

where $Q(t,x)$ is an arbitrary $m \times m$ diagonal matrix with elements $q_i(t,x)(i = 1,\ldots,m)$ such that $\inf|q_i(t,x)| > 0$ for all $t \geq 0$ and all x. Often $Q(t,x)$ is chosen as the identity. The state equation using u^* is described by

$$\dot{x} = Ax + B[SB]^{-1}Q(t,x)u^* \qquad (14)$$

Although this new control structure looks more complicated, the structure of $\dot{\sigma}(t) = 0$ permits us to independently select the m-entries of u^* to satisfy the sufficient conditions for the existence and reachability of a sliding mode. Once u^* is known, the required u is obtained from $u = (SB)^{-1}Qu^*$. For existence and reachability of a sliding mode it is enough to satisfy the condition $\sigma^T(t)\dot{\sigma} < 0$. In terms of u^*

$$\dot{\sigma}(t) = SAx + Qu^*(t) \qquad (15)$$

Thus if the entries u^{*+} and u^{*-} are selected to satisfy

$$\begin{array}{ll} q_i u_i^{*+} < -(S_{i1}\ S_{i2} \ldots S_{in})Ax \quad if \quad \sigma_i > 0 \\ q_i u_i^{*-} > -(S_{i1}\ S_{i2} \ldots S_{in})Ax \quad if \quad \sigma_i < 0 \end{array} \quad (i = 1 \sim m) \qquad (16)$$

then sufficient conditions for the existence and reachability are satisfied. As mentioned above, the control actually implemented is

$$u = (SB)^{-1}Qu^* \qquad (17)$$

5 CALCULATION

Now this VSC design method is applied to the system of the inverted pendulum. The experimental results of a double inverted pendulum were not obtained because the precision of instruments is not enough to control. Hence this paper mainly deals with a single inverted pendulum.

5.1 The Method of Equivalent Control

From (3), the state equation of the inverted pendulum is described by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \qquad (18) \\ A &= \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{m^2gl^2}{N} & \frac{(I_G+ml^2)F}{N} & \frac{-Cml}{N} \\ 0 & \frac{-(M+m)mgl}{N} & \frac{-mlF}{N} & \frac{C(M+m)}{N} \end{pmatrix} \\ &= \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & a_{32} & a_{33} & a_{34} \\ 0 & a_{42} & a_{43} & a_{44} \end{pmatrix} \\ B &= \begin{pmatrix} 0 & 0 & \frac{-G(I_G+ml^2)}{N} & \frac{Gml}{N} \end{pmatrix}^T \\ &= \begin{pmatrix} 0 & 0 & b_3 & b_4 \end{pmatrix}^T \end{aligned}$$

Let $S = \begin{pmatrix} s_1 & s_2 & s_3 & s_4 \end{pmatrix}$,then

$$\begin{aligned} \dot{x}(t) &= \left[I - B(SB)^{-1}S\right]Ax(t) \qquad (19) \\ &= \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & t_1 & t_2 & t_3 \\ 0 & t_4 & t_5 & t_6 \end{pmatrix} x \end{aligned}$$

where

$$\begin{aligned} t_1 &= \frac{(a_{32}b_4 - a_{42}b_3)s_4}{b_3s_3 + b_4s_4} \\ t_2 &= \frac{-b_3s_1 + (a_{33}b_4 - a_{43}b_3)s_4}{b_3s_3 + b_4s_4} \\ t_3 &= \frac{-b_3s_2 + (a_{34}b_4 - a_{44}b_3)s_4}{b_3s_3 + b_4s_4} \\ t_4 &= \frac{(a_{42}b_3 - a_{32}b_4)s_3}{b_3s_3 + b_4s_4} \\ t_5 &= \frac{-b_4s_1 + (a_{43}b_3 - a_{33}b_4)s_3}{b_3s_3 + b_4s_4} \\ t_6 &= \frac{-b_4s_2 + (a_{44}b_3 - a_{34}b_4)s_3}{b_3s_3 + b_4s_4} \end{aligned}$$

i.e.

$$\begin{aligned} \dot{x}_1 &= x_3 \qquad (20) \\ \dot{x}_2 &= x_4 \qquad (21) \\ \dot{x}_3 &= t_1x_2 + t_2x_3 + t_3x_4 \qquad (22) \\ \dot{x}_4 &= t_4x_2 + t_5x_3 + t_6x_4 \qquad (23) \end{aligned}$$

From the condition of the existence of a sliding mode $\sigma(t) = Sx = 0$

$$s_1x_1 + s_2x_2 + s_3x_3 + s_4x_4 = 0 \qquad (24)$$

Let $s_4 = 1$,then

$$x_4 = -(s_1x_1 + s_2x_2 + s_3x_3) \qquad (25)$$

Substituting (25) into (20) $\sim$ (23),

$$\begin{aligned} \dot{x}_1 &= x_3 && (26)\\ \dot{x}_2 &= -(s_1x_1 + s_2x_2 + s_3x_3) && (27)\\ \dot{x}_3 &= -s_1t_3x_1 + (t_1 - s_2t_3)x_2 + (t_2 - s_3t_3)x_3 && (28) \end{aligned}$$

Consequently, the reduced order equivalent linear system is

$$\dot{x} = \begin{pmatrix} 0 & 0 & 1 \\ -s_1 & -s_2 & -s_3 \\ -s_1t_3 & (t_1 - s_2t_3) & (t_2 - s_3t_3) \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \qquad (29)$$

Then the characteristic equation of (29) is

$$\lambda^3 + (s_2 + s_3t_3 - t_2)\lambda^2 + (s_1t_3 - s_2t_2 + s_3t_1)\lambda + s_1t_1 = 0 \qquad (30)$$

By choosing $S = \begin{pmatrix} s_1 & s_2 & s_3 \end{pmatrix}$ properly, we can arrange poles arbitrarily.

5.2 Diagonalization Method

Let $Q = 1, Q^{-1} = 1$ and recall $S = \begin{pmatrix} s_1 & s_2 & s_3 & s_4 \end{pmatrix}$, then sufficient conditions for the existence of a sliding mode are

$$\begin{aligned} qu^{*+} &< -(s_1\ s_2\ s_3\ s_4)Ax \quad if \quad \sigma > 0 \\ qu^{*-} &> -(s_1\ s_2\ s_3\ s_4)Ax \quad if \quad \sigma < 0 \end{aligned} \qquad (31)$$

Let $-SAx = h_1x_1 + h_2x_2 + h_3x_3 + h_4x_4$ and $u^* = K^*x$ where $K^* = \begin{pmatrix} k_1^* & k_2^* & k_3^* & k_4^* \end{pmatrix}$, to satisfy the above condition of a sliding mode, $K^* = \begin{pmatrix} k_1^* & k_2^* & k_3^* & k_4^* \end{pmatrix}$ must satisfy the following.

$$k_i^* = \begin{cases} < h_i & \sigma x_i > 0 \\ > h_i & \sigma x_i < 0 \end{cases} \quad (i = 1 \sim 4) \qquad (32)$$

Let the size of each switching gain $(k_1e\ k_2e\ k_3e\ k_4e)$. Since $u = (SB)^{-1}Qu^*$ and $u = Kx$, $K = \begin{pmatrix} k_1 & k_2 & k_3 & k_4 \end{pmatrix}$ must satisfy the following.

$$k_i = \begin{cases} < (h_i - k_ie)(SB)^{-1} & \sigma x_i > 0 \\ > (h_i + k_ie)(SB)^{-1} & \sigma x_i < 0 \end{cases} \quad (i = 1 \sim 4) \qquad (33)$$

6 SIMULATION

The value of each parameter is the following.

$$\begin{aligned} &M = 1.425kg \quad m = 0.241kg \\ &l = 0.4m \quad F = 12.6kg/s \\ &G = 16.6N/V \quad C = 0.000515kg \cdot m^2/s \\ &g = 9.81m/s^2 \end{aligned} \qquad (34)$$

The standard Runge-Kutta method was used for simulation.

6.1 State Feedback Control

The initial conditions are $x = 0m$, $\dot{x} = 0m/s$, $\theta = 0.14rad$ and $\dot{\theta} = 0rad/s$.

Choosing $K = \begin{pmatrix} -1.50 & -5.50 & -2.00 & -1.30 \end{pmatrix}$ yields poles $\lambda_1 = -2.27$, $\lambda_2 = -6.17$ $\lambda_3 = -2.47 + 7.97i$, $\lambda_4 = -2.47 - 7.97i$. The corresponding figures are shown in Fig.4 and Fig.5.

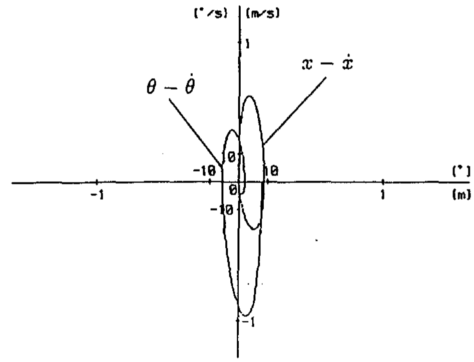

Fig.4. State space

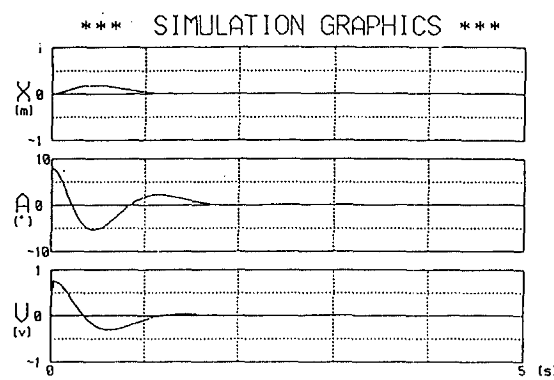

Fig.5. Simulation

6.2 Variable Structure Control

The initial conditions are $x = 0m$, $\dot{x} = 0m/s$, $\theta = 0.2rad$ and $\dot{\theta} = 0rad/s$.

Choosing $S = \begin{pmatrix} 1.016362 & 4.149764 & 1.355703 & 1 \end{pmatrix}$ yields poles $\lambda_1 = -1.00$, $\lambda_2 = -6.00 + 0.03i$ $\lambda_3 = -6.00 - 0.03i$. And let $k_1e = 1$, $k_2e = 10$, $k_3e = 2$, $k_4e = 1$. The corresponding figures are shown in Fig.6 and Fig.7.

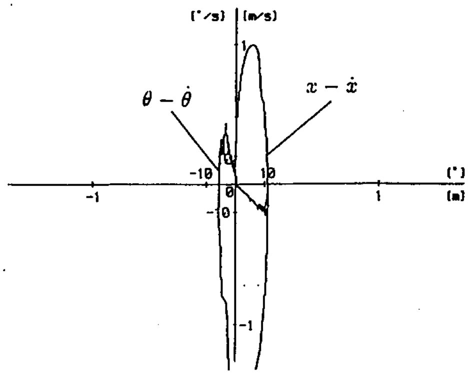

Fig.6. State space

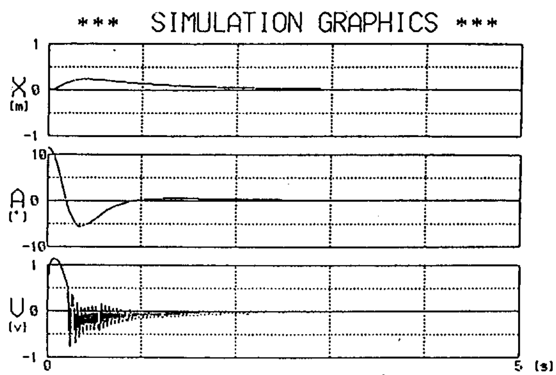

Fig.7. Simulation

7 EXPERIMENT

7.1 State Feedback Control

Choosing $K = \begin{pmatrix} -2.24 & -7.31 & -2.41 & -1.66 \end{pmatrix}$ yields poles $\lambda_1 = -3.03+2.56i$, $\lambda_2 = -3.03-2.56i$, $\lambda_3 = -5.15+3.37i$, $\lambda_4 = -5.15 + 3.37i$. The corresponding figure is shown in Fig.8.

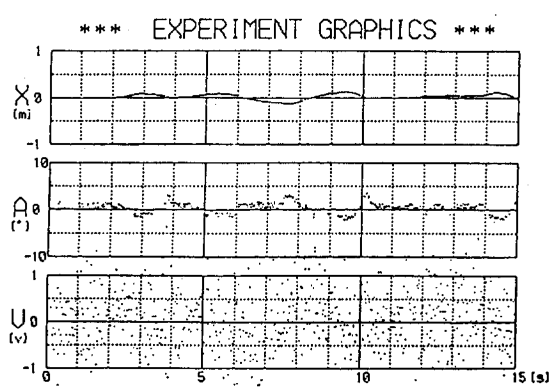

Fig.8. Experiment

7.2 Variable Structure Control

Choosing $S = \begin{pmatrix} 3.839674 & 4.584338 & 1.547620 & 1 \end{pmatrix}$ yields poles $\lambda_1 = -6.00$, $\lambda_2 = -4.24+8.48i$ $\lambda_3 = -4.24-8.48i$. And let $k_1e = 0.3$, $k_2e = 0.1$, $k_3e = 0.3$, $k_4e = 0.1$. The corresponding figure is shown in Fig.9.

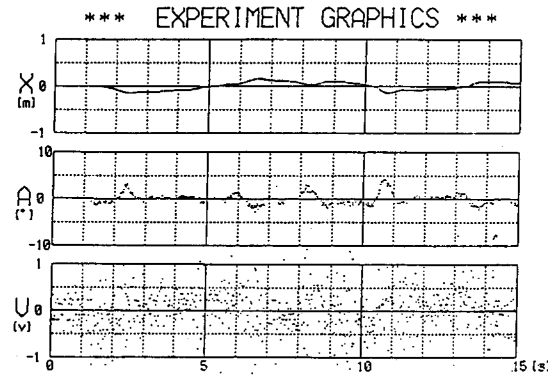

Fig.9. Experiment

8 CONCLUSION

An inverted pendulum, which is a unstable mechanical system, has been stabilized by using state feedback control and variable structure control. From the experimental results, a big difference between the two controls was not observed. The reason would be that nonlinearity of the system is small in the neighborhood of system equilibrium. It seems that VSC will be effective for systems with large nonlinearities. –e.g., mechanical systems with coulomb frictions and robotics.

The problems which should be considered next are how to choose the size of each switching gain and how to design an optimal control about VSC.

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