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A Method of Optimal Regulator Using a Pseudo-linearized Transformation of Nonlinear Term

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Abstract

For a single-input nonlinear system, the transformation which transposes the nonlinear system to a controllable-like canonical system has been proposed [1]. However this method islimited by a single-input system and it is difficult to apply the method actually. In this paper we propose a method which transposes the nonlinear system with multi-input into an equivalent pseudo-linear system. And we apply the pseudo-linear system to a linear optimal regulator. To confirm the effectiveness of the proposed method, a transient stability control of the generator with an excitor and a governor is considered.

1. Introduction

A number of papers concerned with a control of nonlinear systems have been proposed [1]~[13]. Among them, the approach which transposes the nonlinear system into a linear system is thought to be one of the most useful methods.

Hunt and Su et al. showed a method of transforming a nonlinear system into a linear one in the whole state space. Reboulet and Champetier generalized the method of Hunt etal.. They proposed a technique for the transformation of a nonlinear system into a linear one [2]. But the method is limited to the case with a single input and also it is difficult to compute a transformation matrix for the complex system. To be useful in actual application, the method has to be a simple one. With these points in mind, we propose a simple method which transforms the nonlinear system with multi-input into an equivalent pseudo-linear system.

We consider a transient stability control of the generator with an excitor and a governor in a power system to confirm the effectiveness of the method.

2. Pseudo-linearized Transformation of Nonlinear Terms

Consider the nonlinear systems in which the state vector only has the nonlinearity.

$$\dot{x}=f(x)+Bu \tag{1}$$

We rewrite the nonlinear differential equation as the following.

$$\dot{x}=Ax+Bu+\{f(x)-Ax\} \tag{2}$$

where x is an n-th order state vector, u is an r-th order control vector, $f(x)$ is an n-th order vector function. A is a linearized matrix of nonlinear function $f(x)$ and B is an $n\times r$ matrix.

In general, the non-zero elements of vector function $f(x)$ and matrix B are few. Moreover, in many cases as in Eq(3), the row in which the nonlinear term exists is not coincident with the one in which a control input exists.

$$\begin{aligned}
&\cdots\cdots\cdots\cdots\\
\dot{x}_i&=a_{i1}x_1+a_{i2}x_2+\cdots+a_{ii}x_i+\cdots+a_{ik}x_k+\cdots+a_{in}x_n+f_i(x)\\
&\cdots\cdots\cdots\cdots\\
\dot{x}_k&=a_{k1}x_1+a_{k2}x_2+\cdots+a_{ki}x_i+\cdots+a_{kk}x_k+\cdots+a_{kn}x_n+u\\
&\cdots\cdots\cdots\cdots
\end{aligned} \tag{3}$$

So, we introduce a linearized transformation and transpose the nonlinear term $f_i(x)$ into the row in which the control input u exists. We define a pseudo-linear variable x_k^* instead of a nonlinear term in Eq. 3.

$$k a_{ik} x_k^* = a_{ik} x_k + f_i(x) \tag{4}$$

where k is a transformation coefficient.

We replace the nonlinear term with a pseudo-linear variable and transpose the nonlinear term from the original row to the row in which a control input exists.

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\dot{x}_i = a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{ii}x_i + \cdots + a_{ik}x_k^* + \cdots + a_{in}x_n$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\dot{x}_k^* = \frac{a_{k1}}{k}x_1 + \frac{a_{k2}}{k}x_2 + \cdots + \frac{a_{ki}}{k}x_i + \cdots + a_{kk}x_k^* + \cdots + \frac{a_{kn}}{k}x_n + \frac{1}{k}u$$

$$- \frac{a_{kk} f_i(x)}{k a_{ik}} + \Sigma \frac{\partial f_i(x)}{k \partial x_j}(a_{j1}x_1 + \cdots + a_{jn}x_n)$$

$$\cdots\cdots\cdots\cdots\cdots\cdots \tag{5}$$

Then, the nonlinear term is transposed to the k-th row and the i-th row is pseudo-linearized. By iterations of the procedure the whole system can be pseudo-linearized. Then, the nonlinear terms are absorbed by the control inputs respectively.

3. Optimal regulator for pseudo-linear system

In this section we derive an optimal regulator for the pseudo-linear system which is transposed from the original nonlinear system.

Setting $z = f(x) - Ax$ and $\beta = (BB^T)^{-1}B^T z$ in Eq(2) we obtain Eq(6).

$$\dot{x} = Ax + B(u + \beta) \tag{6}$$

Setting $u^* = u + \beta$, Eq(6) becomes an apparent linear system. We call this linear system a pseudo-linear system.

$$\dot{x} = Ax + Bu^* \tag{7}$$

As the cost function we define the following

$$J = \int \{ x^T Q x + (u^* - \beta)^T R (u^* - \beta) \} dt \tag{8}$$

Supposing the following functional Eq. 9 which describes the minimum value of Eq. 8, we have the optimal control input Eq. (10).

$$V(x, t) = x^T K_1 x + 2x^T K_2 \beta + \beta^T K_3 \beta \tag{9}$$

$$u = -R^{-1}B^T(K_1 x + K_2 \beta) \tag{10}$$

Where matrices K_1, K_2 are the solutions which satisfy the following equations.

$$\frac{\partial K_1}{\partial t} + Q - K_1 B R^{-1} B^T K_1 + K_1 A + A^T K_1 = 0 \tag{11}$$

$$\frac{\partial K_2}{\partial t} - K_1 B R^{-1} B^T K_2 + K_1 B + A^T K_2 = 0 \tag{12}$$

Matrix K_2 can be also computed by the Matrix K_1.

$$K_2 = (K_1 B R^{-1} B^T - A^T)^{-1} K_1 B \tag{13}$$

4. System equations of generator in power system

As a numerical example, We consider a transient stability control to damp the electro-mechanical oscillations associated with the transient conditions in power system.

We consider the next system equations of a one-machine infinite bus model system in power system. A one-machine infinite bus model system is given in Appendix[3].

$$\left.\begin{aligned}
\dot{x}_1 &= -a_1 x_1 + a_1 x_2 + a_2 x_3 \cos(a_3 - x_4) \\
\dot{x}_2 &= -a_4 x_2 + a_4 x_3 \\
\dot{x}_3 &= -a_5 x_3 + a_5 u_1 \\
\dot{x}_4 &= x_5 \\
\dot{x}_5 &= -x_5 + a_6 x_6 - a_7 (x_1 + a_8)^2 + a_4 \\
&\quad - a_9 (x_1 + a_8) \cos(a_3 - x_4) \\
\dot{x}_6 &= -a_{10} x_6 + a_{10} x_7 \\
\dot{x}_7 &= -a_{11} x_7 + a_{11} u_2
\end{aligned}\right\} \tag{14}$$

Replacing the nonlinear terms by the new pseudo-linear variables we get the next equations.

$$\dot{x}_1 = -a_1 x_1 + k_1 a_1 x_2^* + b_1 x_5$$

$$\dot{x}_2^* = a_{21} x_1 + a_{22} x_2^* + k_3 a_{23} x_3^* + a_{24} x_4$$

$$+ a_{25} x_5 + a_{26} x_6^*$$

$$\dot{x}_3^* = -a_5 x_3^* + (a_5 / k_3)(u_1 + \beta_1)$$

$$\left.\begin{aligned}\dot{x}_4 &= x_5\\ \dot{x}_5 &= -b_2x_1 - b_3x_4 - x_5 + k_2a_6x_6^*\\ \dot{x}_6^* &= a_{61}x_1 + a_{62}x_2^* + a_{64}x_4 + a_{65}x_5\\ &\quad + a_{66}x_6^* + k_4a_{67}x_7^*\\ \dot{x}_7^* &= -a_{11}x_7^* + (a_{11}/k_4)(u_2+\beta_2)\end{aligned}\right\} \quad (15)$$

where

$$\beta_1 = [c_{21}CO\cdot FF1 + a_5(CX\cdot CO + c_{27}x_5^2SI) + (c_{24}CO + CX\cdot SI - c_{27}x_5^2CO)x_5 + 2c_{27}x_5FF5\cdot SI + c_{25}CO\cdot FF5 + c_{26}FF6\cdot CO] / a_5a_{23}$$

$$\beta_2 = [(c_{61}CO + c_{67}x_5SI + 2d_{61}x_1 + d_{62}x_2^* + d_{65}x_5)FF1 + (c_{62}CO + d_{62}x_1)FF2 + CY\cdot SI\,x_5 + c_{65}FF5\cdot CO + a_{11}\{CY\cdot CO + (c_{67}x_1 + c_{68})x_5SI + DX\} + (c_{67}x_1 + c_{68})SI\cdot FF5 - (c_{67}x_1 + c_{68})x_5^2CO + d_{65}x_1FF5] / (a_{11}a_{67})$$

where

$$k_1a_1x_2^* = a_1x_2 + a_2x_5CO - b_1x_5$$

$$k_3a_{23}x_3^* = a_{23}x_3 + CX\cdot CO + c_{27}x_5^2SI$$

$$k_2a_6x_6^* = a_6x_6 - a_7(x_1+a_8)^2 + a_4 - a_9(x_1+a_8)CO + b_2x_1 + b_3x_4$$

$$k_4a_{67}x_7^* = a_{67}x_7 + CY\cdot CO + (c_{67}x_1 + c_{68})x_5SI + DX$$

$$FF1 = -a_1x_1 + k_1a_1x_2^* + b_1x_5$$

$$FF2 = a_{21}x_1 + a_{22}x_2^* + k_3a_{23}x_3^* + a_{24}x_4 + a_{25}x_5 + a_{26}x_6^* - a_5x_3^*$$

$$FF5 = -b_2x_1 - b_3x_4 - x_5 + k_2a_6x_6^*$$

$$FF6 = a_{61}x_1 + a_{62}x_2^* + a_{64}x_4 + a_{65}x_5 + a_{66}x_6^* + k_4a_{67}x_7^*$$

$$CO = \cos(a_3 - x_4)$$

$$SI = \sin(a_3 - x_4)$$

$$CX = c_{21}x_1 + c_{24}x_4 + c_{25}x_5 + c_{26}x_6^*$$

$$CY = c_{60} + c_{61}x_1 + c_{62}x_2^* + c_{65}x_5$$

$$DX = d_{60} + d_{61}x_1^2 + d_{62}x_1x_2^* + d_{65}x_1x_5$$

5. Numerical simulation and Discussion

Setting the constants of the machine in Fig. A1 to the following,

$M=0.06$	$D=0.06$
$X_d=0.320$(p.u.)	$X_d'=0.084$(p.u.)
$Y_{11}=0.266-j1.53$(p.u.)	$Y_{12}=0.18+j1.08$(p.u.)
$T_{d0}=5.0$(sec)	$T_t=0.3$(sec)
$T_A=0.02$(sec)	$T_E=0.04$(sec)
$T_{gv}=0.1$(sec)	$K_A=1.0$
$K_{EX}=1.0$ $V=1.0$(p.u.)	
$E_0=x_{10}=u_{10}=1.482$(p.u.)	$\delta_0=x_{40}=0.4363$(p.u.)
$P_0=x_{60}=u_{20}=1.5$(p.u.)	

we obtain the following parameter values.

$a_1=0.313$	$a_2=0.404$	$a_3=0.969$
$a_4=25.0$	$a_5=50.0$	$a_6=16.667$
$a_7=4.433$	$a_8=1.482$	$a_9=18.248$
$a_{10}=3.333$	$a_{11}=10.0$	

It is assumed that the disturbance is produced by a three phase short-circuit fault at the generator bus in the simulations. The phase plane trajectories with no control are shown in Fig. 1. The critical reclosing time t_c with no control is 0.28 seconds.

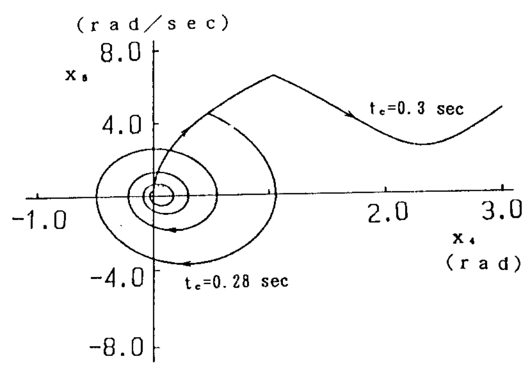

Fig. 1 Phase plane trajectories

For a small disturbance a linear optimal regulator is useful and no scheme for nonlinearity is neccessary. For the disturbance t_c= 0.3 sec a mere linear optimal control doesn't stabilize the generator and power system falls into an unstable state. However, the trajectories using the proposed control scheme stabilizes the machine successfully.

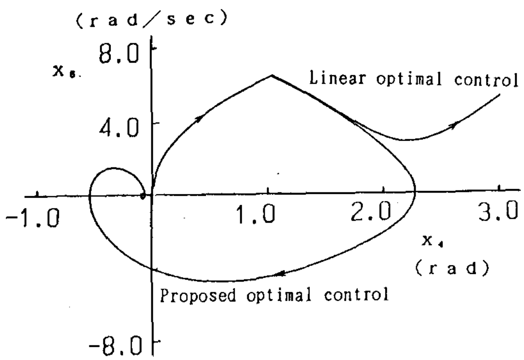

Fig. 2 Phase plane trajectories
(t_c=0.3 sec)

Trajectories for the large disturbance t_c=0.6 are shown in Fig. 3. The time responses of control inputs u_1, u_2 and state variables x_3, x_7 are shown in Fig. 4. In this case the system seems to be stable, but these situations are only numerically successful, and are not able to be realized actually, because the control inputs necessary are over their threshold values.

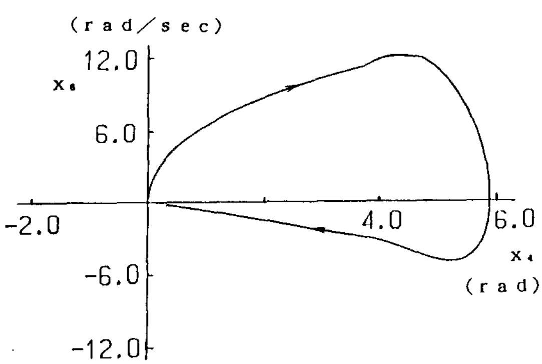

Fig. 3 Phase plane trajectory
(t_c=0.6 sec)

If we set the threshold values of the control inputs, the trajectory of x_4 for the disturbance tc=0.4 is a limit at which the machine can be stabilized. It is shown in Fig. 5.

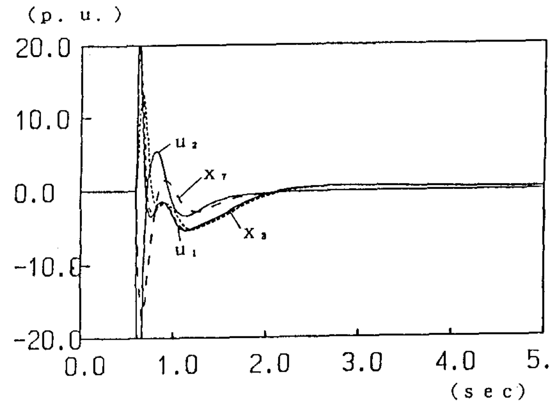

Fig. 4 Time responses of state variables
and control variables

The time responses of control inputs u_1, u_2 and state variables x_3, x_7 are shown in Fig. 6.

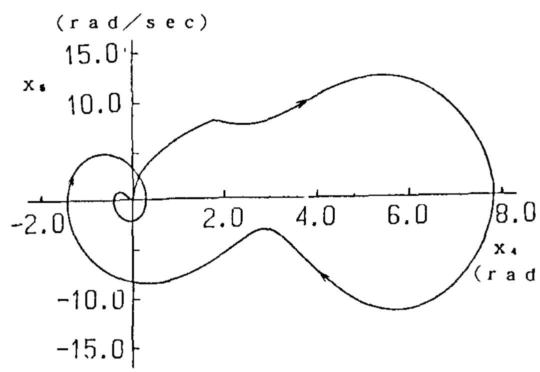

Fig. 5 Phase plane trajectory
(t_c=0.4 sec)

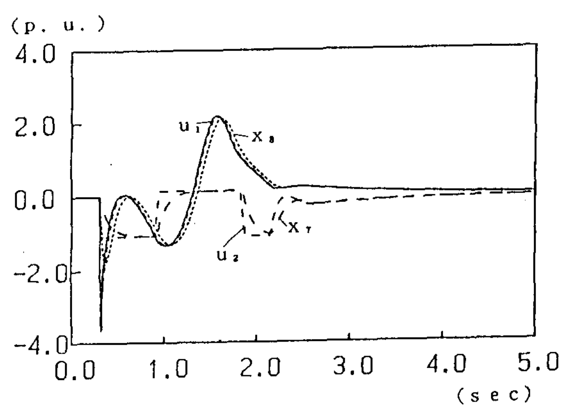

Fig. 6 Time responses of state variables and
control variables
($-7.5 \leq u_1 \leq 7.5 \quad 0.45 \leq u_2 \leq 1.65$)

In the simulations we set the parameter of the pseudo-linearized transformation to $[k_1, k_2, k_3, k_4] = [1,1,1,1]$. If we set the parameter to $[k_1, k_2, k_3, k_4] = [5,5,5,5]$, the trajectories coresponding to Fig. 2 becomes Fig. 7. It is seen that the control characteristics in Fig. 7 are improved.

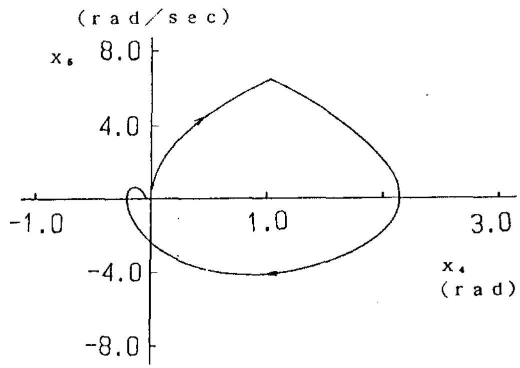

Fig. 7 Phase plane trajectory (t_c=0.3 sec)
$[k_1, k_2, k_3, k_4] = [5,5,5,5]$

In the next paper we will consider the influ-ences of the parameter k, and the problem of weight coefficients of the matrices Q or R.

5. Conclusion

In this paper we showed that a useful regulator for a nonlinear system is obtained by using the pseudo-linearization. It may not always be possible to transform the nonlinear term into pseudo-linear one. A phisical image of the weight coefficients for the pseudo-variables is also not clear. There may be some theoretical problems in this method. Nevertheless, It is thought that the transformation proposed here is feasible in many actual systems and that the method is useful as the regulator to stabilize the system.

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Appendix

A one-machine infinite bus model system used for simulation study is shown in Fig. A1. State variables consist of the internal induced voltage E_I, phase angle δ, rotor speed $d\delta/dt$, turbine output power P, excitation voltage E_{fd}, and governor valve position X_p.

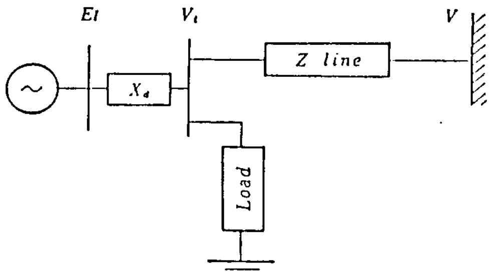

Fig. A1 One-machine infinite bus system

In Fig. A1, the differential equation of a generator in transient state is given as follows.

$$M\frac{d^2\delta}{dt^2}+D\frac{d\delta}{dt}+P_e=P$$

$$E_I+T_{do}\frac{dE_q'}{dt}=E_{fd}$$

$$P+T_t\frac{dP}{dt}=X_{gv}$$

The generator output P_e is given by

$$P_e=E_I^2Y_{11}COS\theta_{11}+E_IVY_{12}COS(\theta_{12}-\delta)$$

where $Y_{11}\angle\theta_{11}$、$Y_{12}\angle\theta_{12}$ are the admittances respectively. Internal induced voltage E_I and voltage behind transient reactance E_q' have the following relation.

$$E_I=E_q'+(X_d-X_d')I_d$$

$$I_d=-E_IY_{11}\sin\theta_{11}-VY_{12}\sin(\theta_{12}-\delta)$$

where I_d is a d-axis armature current and X_d, X_d' are d-axis synchronous reactance and d-axis transient reactance respectively.

$$\frac{dE_{fd}}{dt}=-\frac{E_{fd}}{T_E}+\frac{K_{EX}}{T_E}E_{fdo}$$

$$\frac{dE_{fdo}}{dt}=-\frac{E_{fdo}}{T_A}+\frac{K_A}{T_A}u_1$$

Valve opening of speed governor X_{gv} is related by the control input u_2 as the following.

$$\frac{dX_{gv}}{dt}=-\frac{1}{T_{gv}}X_{gv}+\frac{1}{T_{gv}}u_2$$

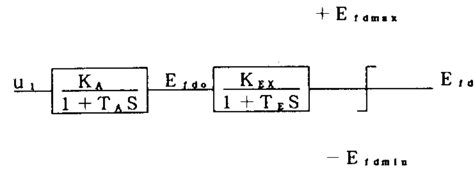

Fig. A2 Excitor control system

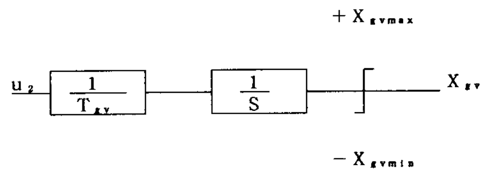

Fig. 3A Governor control system

Letting

$x_1=E_I$ $x_2=E_{fd}$ $x_3=E_{fdo}$

$x_4=\delta$ $x_5=d\delta/dt$

$x_6=P$ $x_7=X_{gv}$

we obtain

$$\dot{x}_1=-\frac{1}{kT_{do}}x_1+\frac{1}{kT_{do}}x_2+\frac{(Xd-Xd')VY_{12}}{k}x_5\cos(\theta_{12}-x_4)$$

$$\dot{x}_2=-\frac{1}{T_E}x_2+\frac{K_{EX}}{T_E}x_3$$

$$\dot{x}_3=-\frac{1}{T_A}x_3+\frac{K_A}{T_A}u_1$$

$$\dot{x}_4=x_5$$

$$\dot{x}_5=-\frac{VY_{12}\cos(\theta_{12}-x_4)}{M}x_1-\frac{Y_{11}\cos\theta_{11}}{M}x_1^2-\frac{D}{M}x_5+\frac{1}{M}x_6$$

$$\dot{x}_6=-\frac{1}{T_t}x_6+\frac{1}{T_t}x_7$$

$$\dot{x}_7=-\frac{1}{T_{gv}}x_7+\frac{1}{T_{gv}}u_2$$

where $k=1+(Xd-Xd')VY_{11}SIN\theta_{11}$.