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Parameter Learning of Algebraic Systems

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Abstract

We present a parameter estimator which operates in the domain of iteration sequence. The scheme can be applied to identify unknown algebraic systems whose uncertainty is parametric.

1 Introduction

We present in this note a parameter estimator for linear or nonlinear systems whose dynamics are linear with respect to a set of selected parameters. The estimator operates along the iteration sequence for each fixed time, which makes it equally applicable to time-varying system parameters as well as time-constant parameters. This marks a difference from the usual estimator which operates in the time domain[1, 3] and mostly adequate for time-constant parameter estimation.

2 Algebraic Systems and Parameter Estimation

Consider the following algebraic system in which system parameters $\phi(t) \in R^l$ are linearly related to the system dynamics

$$y(t) = W^T(t)\phi(t), \tag{1}$$

where $y(t) \in R^n$ and $W(t) \in R^{l\times n}$. Each element of the regression matrix $W(t)$ consists of linear or nonlinear functions of the system inputs, states and/or outputs. For the given class of algebraic systems (1) it is required to find the system parameter vector $\phi(t) \in R^l$ for $t \in [0,t_f](t_f < \infty)$ which are time-constant or time-varying. Of numerous parameter identification methods, our approach is to estimate the unknown system parameters along the iteration sequence for each $t \in [0,t_f]$, so that time-variation in system parameters can be handled simultaneously and convergence of the estimated parameters is independent of the time duration t_f.

1. Derivation of the Estimator

In deriving the estimator along the iteration, consider the following index:

$$J(t) = \frac{1}{2}\sum_{r=1}^{\infty}(y^r(t) - \hat{y}^r(t))^T(y^r(t) - \hat{y}^r(t)),$$

where an estimated system $\hat{y}^i(t)$ at the ith iteration is given by

$$\hat{y}^i(t) = W^{i^T}(t)\hat{\phi}^i(t).$$

Applying the gradient descent method, we obtain the following learning rule for estimation of unknown parameters at the jth iteration:

$$\hat{\phi}^{i+1}(t) = \hat{\phi}^i(t) + \beta^i S^{-1}W^i(t)e^i(t), \tag{2}$$

where $e^i(t) = y^i(t) - \hat{y}^i(t)$. The training factor β^i is set to be $0 < \beta^i < 2$ and the scaling matrix $S = S^T$ is positive definite.

A convergent result on the estimated system (1) is stated as follows:

Proposition 1: The error of the estimated system converges as follows:

$$i) \quad J^{i+1}(t) \leq J^i(t)$$

$$ii) \quad \lim_{i\to\infty} e^i(t) = 0,$$

where

$$J^i(t) = \int_0^t(\phi(\eta) - \hat{\phi}^i(\eta))^T S(\phi(\eta) - \hat{\phi}^i(\eta))d\eta$$

for all $t \in [0,t_f]$.

Proof: Let $\tilde{\phi}^i(t) = \phi(t) - \hat{\phi}^i(t)$ and let $\bar{\phi}(t) = \tilde{\phi}^{i+1}(t) - \tilde{\phi}^i(t)$. Then, the estimator (2) can be rewritten as

$$\bar{\phi} = -\beta^i S^{-1}W^i(t)e^i(t). \tag{3}$$

Further, let $\Delta J^i(t) = J^{i+1}(t) - J^i(t)$. Then,

$$\begin{aligned}\Delta J^i(t) &= \int_0^t(\bar{\phi}^T(\eta)S\bar{\phi}(\eta) + 2\bar{\phi}^T(\eta)S\tilde{\phi}^i(\eta)) \\ &= \int_0^t(\beta^{i^2}e^{i^T}(\eta)W^{i^T}(\eta)S^{-1}W^i(\eta)e^i(\eta)\end{aligned}$$

$$- \quad 2\beta^i e^{i^T}(\eta) e^i(\eta))d\eta$$

$$= \quad -\beta^i \int_0^t (e^{i^T}(\eta)(2I - \beta^i W^{i^T}(\eta) S^{-1} W^i(\eta)) e^i(\eta)) d\eta.$$

Now, choose the scaling matrix S^{-1} so that $W^{i^T}(t)S^{-1}W^i(t) < I$ for all $t \in [0, t_f]$. Then, it becomes

$$\Delta J^i(t) = -\beta^i \int_0^t (e^{i^T}(\eta)(2I - \beta^i W^{i^T}(\eta) S^{-1} W^i(\eta)) e^i(\eta)) d\eta \leq 0.$$

Hence, $i)$ follows. Assuming boundedness of the initial parameter estimation, the nonnegative sequence $\{J^i(t)\}$ converges to a fixed value. Hence, $\Delta J^i(t) \to 0$ as $i \to \infty$ resulting in $e^i(t) \to 0$. Q.E.D.

The following definition of PE(Persistent Excitation) in the domain of iteration sequence is related to convergence of parameter estimation.

(Definition)[2]: A matrix function $W^i(t) : R_+ \to R^{l \times n}$ is *persistently exciting along the iteration* if there exist positive constants σ_1, σ_2 and a positive integer N such that

$$\sigma_1 I \leq \sum_{r=i}^{i+N} W^r(t) W^{r^T}(t) \leq \sigma_2 I$$

for $t \in [0, t_f]$.

Proposition 2: The estimated parameters converges to unknown system parameters, provided that the regression matrix is persistently exciting along the iteration sequence.

$$\lim_{i \to \infty} \hat{\phi}^i(t) = \phi(t)$$

for $t \in [0, t_f]$.

Proof: It is trivial that the estimator (2) can be written as

$$\begin{aligned} \tilde{\phi}^i &= \phi - \hat{\phi}^i \\ &= \phi - \hat{\phi}^{i+1} + \beta S^{-1} W^i e^i \\ &= \tilde{\phi}^{i+1} + \beta S^{-1} W^i e^i, \end{aligned} \quad (4)$$

and

$$\tilde{\phi}^{i+n} = \tilde{\phi}^{i+N+1} + \beta \sum_{r=i+n}^{i+N} S^{-1} W^r e^r, \quad (5)$$

for $1 \leq n \leq N$. Multiplying W^{i+n-1^T} on both sides of the parameter estimator (4) at the $(i+n-1)th$ iteration, we obtain:

$$\begin{aligned} W^{(i+n-1)^T} \tilde{\phi}^{(i+n)} &= W^{(i+n-1)^T} \tilde{\phi}^{(i+n-1)} \\ &- \beta W^{(i+n-1)^T} S^{-1} W^{(i+n-1)} \\ &\quad e^{(i+n-1)}. \end{aligned} \quad (6)$$

Let

$$\begin{aligned} S_N^{n-1} &\equiv W^{(i+n-1)^T} \tilde{\phi}^{(i+n-1)} \\ &- \beta \sum_{r=i+n-1}^{i+N} W^{(i+n-1)^T} S^{-1} W^r e^r \end{aligned}$$

for $1 \leq n \leq N+1$. Then, from the convergence results in *Proposition 1*, it is trivial to prove that for all $t \in [0, t_f]$,

$$\lim_{i \to \infty} S_N^{n-1} = 0 \quad \text{for } 1 \leq n \leq N+1. \quad (7)$$

On the other hand, in view of (5) and (6), S_N^{n-1} becomes

$$S_N^{n-1} = W^{(i+n-1)^T} \tilde{\phi}^{(i+N+1)} \quad \text{for } 1 \leq n \leq N+1. \quad (8)$$

Let S_N be a finite series with $N+1$ terms such that

$$S_N \equiv \sum_{n=1}^{N+1} S_N^{(n-1)^T} S_N^{(n-1)}.$$

Then, we obtain from (8)

$$S_N = \tilde{\phi}^{(i+N+1)^T} \sum_{r=i}^{i+N} (W^r W^{r^T}) \tilde{\phi}^{(i+N+1)}. \quad (9)$$

Applying the PE condition of W^i to the equation (9) leads us to

$$\begin{aligned} 0 &\leq \alpha_1 \tilde{\phi}^{(i+N+1)^T} \tilde{\phi}^{(i+N+1)} \leq S_N \\ &\leq \alpha_2 \tilde{\phi}^{(i+N+1)^T} \tilde{\phi}^{(i+N+1)}. \end{aligned} \quad (10)$$

Combining (10) with (7), we have $\lim_{i \to \infty} \hat{\phi}^i(t) = \phi(t)$. Q.E.D.

3 Conclusion

In this note, a parameter estimator which operates in the domain of iteration sequence is proposed for algebraic systems. The estimator may be used in identifying linear or nonlinear systems whose linear parametrization reduces them to algebraic systems.

It can be also shown that by adjusting the gain factor in the estimator, variations of the estimator such as *Orthogonal Projection, Least-Square, Exponentially Weighted Least-Square* methods etc. are derived in the domain of iteration sequence.

References

[1] B.D.O. Anderson, *Stability of Adaptive Systems: Passivity & Averaging Analysis*, The MIT Press, 1986.

[2] T. Kuc and Jin S. Lee, "An Adaptive Learning Control of Robot Manipulators", IEEE Conf. on Dec. Contr., U.K. Dec. 1990.

[3] S. Sastry and M. Bodson, *Adaptive Control: Stability, Convergence, and Robustness*, Prentice-Hall, Inc. Englewood Cliffs, New Jersey, 1989.