'91 KACC 1991. 10. 22~24

IDENTIFICATION OF SINGLE VARIABLE CONTINUITY LINEAR SYSTEMS WITH STABILITY CONSTRAINTS FROM SAMPLES OF INPUT-OUTPUT DATA

Zhao-Qing Huang, Jian-Feng Ao

Institute of Chemical Engineering
South China University of Technology
Guangzhou, 510641, P. R. CHINA

Abstract

Identification theory for linear discrete system has been presented by a great many reference, but research works for identification of continuous-time system are less than preceding identification. In fact, a great many systems for engineering are continuous-time systems, hence, research for identification of continuous-time system has important meaning. This paper offers the following results:

1.Corresponding relations for the parameters of continuous-time model and discrete model may be shown,when single input-output system has general characteristic roots.

2.To do identification of single variable continuity linear system with stability constraints from samples of input-output data, it is necessary to use optimization with stability constraints.

3. Main results of this paper may be explained by a simple example.

INTRODUCTION

There are many publications on linear discrete systems identification theory, but few on continuous-time systems identification. Considering the fact that most of engineering systems are continuous and there is no direct and efficient algorithm to identify continuous-time systems by means of transforming continuous systems into discrete ones, it is important and meaningful to carry out some research on this area. In [1] it has been pointed out that there exist simple corresponding relationships between parameters of continuous and discrete time systems model when all characteristic roots of single variable system are single ones. In this paper, the relationships have been generalized in the case of general systems with single variable.

The model stability is another important problem in systems identification. Considering influence of disturbance and of many factors, it is possible that identification models are instable even if practical systems are stable ones. For the purpose to solve the problems of model stability, authors connect stability criteria as stability constraint conditions in identification process, thereby, the identification problem can be transformed into an optimum one with inequality constraints. In the end of this paper, there is an example of computer simulation.

1. SYSTEMS CONTINUITY TIME OBSERVABLE AND JORDAN CANONICAL FORM

Consider an observable canonical state equation of a single input-output linear, constant-coefficient, dynamical system as follows:

$$\dot{X} = AX + BU \quad (1)$$
$$Y = CX \quad (2)$$

in which,

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & & 1 \\ -a_n, & -a_{n-1} & \cdots & & -a_1 \end{bmatrix} \qquad B = \begin{bmatrix} b_1 \\ b_2 \\ \cdot \\ \cdot \\ \cdot \\ b_n \end{bmatrix} \quad (3)$$

$$C = [1, 0, \ldots\ldots 0] \quad (4)$$

It is convenient to identify the system since there are only unknown parameters a_i and b_i $(i=1,2,\ldots,n_1)$ existed.Since the general form of systems state equation may be transformed into observable canonical one by linear transformation, then the discussions are carried out on the basis of original forms of (1) and (2).

Suppose to know characteristic roots of the system, it is possible to construct transform matrix through characteristic roots. Usually, we suppose the system has a pair of conjugate complex roots, single q's multiple roots and p's single real roots as follows:

$\lambda_1 = \sigma + j\omega$, $\lambda_2 = \sigma - j\omega$ conjugate complex root,
λ q's multiple root,
$\lambda_{31}, \lambda_{32}, \ldots\ldots, \lambda_{3p}$ p's single root.

Then, to construct following real transform matrix

$$M = [\xi \,\vdots\, \eta \,\vdots\, M_1 \,\vdots\, M_2] \quad (5)$$

in which,

$$\xi = [1, \sigma, (\sigma^2+\omega^2)^{1/2}\cos(2tg^{-1}\frac{\omega}{\sigma}), \ldots, (\sigma^2+\omega^2)^{(n-1)/2}\cos[(n-1)tg^{-1}\frac{\omega}{\sigma}]]^T$$

$$\eta = [1, \omega, (\sigma^2+\omega^2)^{1/2}\sin(2tg^{-1}\frac{\omega}{\sigma}), \ldots, (\sigma^2+\omega^2)^{(n-1)/2}\sin[(n-1)tg^{-1}\frac{\omega}{\sigma}]]^T \quad (6)$$

and the form of M_1 and M_2 is shown in [3].

Take a linear transformation:

$$\hat{X} = M^{-1}X \quad (7)$$

and we have Jordan canonical form of the system:

$$\dot{\hat{X}} = \hat{A}\hat{X} + \hat{B}u \quad (8)$$

$$Y = \hat{C}\hat{X} \quad (9)$$

$$\hat{A} = M^{-1}AM = \begin{bmatrix} \sigma, \omega \\ -\omega, \sigma \end{bmatrix} \dot{+} \begin{bmatrix} \lambda_2 & 1 & & \\ & \lambda_2 & 1 & \\ & & \ddots & 1 \\ & & & \lambda_2 \end{bmatrix} \dot{+} \begin{bmatrix} \lambda_{31} & & & \\ & \lambda_{32} & & \\ & & \ddots & \\ & & & \lambda_{3p} \end{bmatrix} \quad (10)$$

$$\hat{B} = M^{-1}B, \qquad \hat{C} = CM$$

in which, sign $\dot{+}$ represents diagonal arrangement.

2. STATE SPACE DISCRETIZATION AND DISCRETE JORDAN CANONICAL FORM

Utilizing modified method of state space discretization [5] for (8) and (9), we get discrete model of the system as follows:

$$\hat{X}_{k+1} = \hat{F}\hat{X}_k + \hat{G}u_k \quad (11)$$

$$Y_k = \hat{C}\hat{X}_k \quad (12)$$

in which,

$$\hat{F} = EXP(\hat{A}T) = \begin{bmatrix} e^{\sigma T}\cos\omega T, & e^{\sigma T}\sin\omega T \\ -e^{\sigma T}\sin\omega T, & e^{\sigma T}\cos\omega T \end{bmatrix} \dot{+} \begin{bmatrix} e^{\lambda_2 T}, Te^{\lambda_2 T}, T^2 e^{\lambda_2 T}/2, \ldots, T^{q-1} e^{\lambda_2 T}/(q-1)! \\ \ddots \\ Te^{\lambda_2 T} \\ e^{\lambda_2 T} \end{bmatrix} \dot{+} \begin{bmatrix} e^{\lambda_{31} T} & & & \\ & e^{\lambda_{32} T} & & \\ & & \ddots & \\ & & & e^{\lambda_{3p} T} \end{bmatrix} \quad (13)$$

$$\hat{G} = \int_0^T EXP(\hat{A}t)\hat{B}dt = (I+(1/2!)\hat{A}T+(1/3!)A^2T^2+\ldots)^T\hat{B} \quad (14)$$

From (13), it is clear that the system has a pair of conjugate complex roots $\sigma \pm j\omega$, q's multiple root $e^{\lambda_2 T}$ and p's single real roots $e^{\lambda_{31} T}, \ldots, e^{\lambda_{3p} T}$, which are in correspondence with characteristic roots of continuous systems.

Since the second diagonal matrix $\hat{F}_2$ in $\hat{F}$ is not Jordan canonical form, it is necessary to search a transform matrix to transform $\hat{F}_2$ into Jordan canonical form with multiple roots. By means of induction, q's characteristic vectors Q_1, $Q_2, \ldots, Q_q$ for $\hat{F}_2$ may be shown as follows:

$$Q_1 = [1, 0, \ldots, 0]^T \quad (15)$$

and

$$Q_{K+1} = [v_{1,K+1}, v_{2,K+1}, \ldots, v_{q,K+1}]^T \qquad k = 1, 2, \ldots, q-1 \quad (16)$$

in which

$$v_{K+1,K+1} = v_{K,K} \,/\, Te^{\lambda_2 T} \quad (17)$$

$$v_{i,k+1} = \frac{1}{Te^{\lambda_2 T}}\left(v_{i-1,k} - \sum_{j=2}^{k+2-i} \frac{T^j}{j!} e^{\lambda_2 T} v_{j+i-1,k+1}\right) \qquad i = k, k-1, \ldots, 2 \quad (18)$$

$$v_{1,k+1} = 0, \quad v_{i,k+1} = 0 \qquad i > k+1 \quad (19)$$

If $Q = [Q_1, Q_2, \ldots, Q_q]$ (20)

then

$$Q^{-1}\begin{bmatrix} e^{\lambda_2 T}, Te^{\lambda_2 T}, \ldots, T^{q} e^{\lambda_2 T}/(q-1)! \\ \ddots \quad \ddots \quad \vdots \\ \ddots \quad \ddots \; Te^{\lambda_2 T} \\ \ddots \; e^{\lambda_2 T} \end{bmatrix} Q$$

$$= \begin{bmatrix} e^{\lambda_2 T} & 1 & & \\ & \ddots & \ddots & \\ & & \ddots & 1 \\ O & & & e^{\lambda_2 T} \end{bmatrix}$$

And substitute transform matrix

$$N = I_2 \dotplus Q \dotplus I_p \quad (21)$$

and transformation

$$\hat{X}_k = N\tilde{X}_k \quad (22)$$

into (11) and (12), then the Jordan canonical form of the system is

$$\tilde{X}_{k+1} = \tilde{F}\tilde{X}_k + \tilde{G}u_k \quad (23)$$

$$Y_k = \tilde{C}\tilde{X}_k \quad (24)$$

in which,

$$\tilde{F} = N^{-1}\hat{F}N = \begin{bmatrix} e^{\sigma T}\cos\omega T, & e^{\sigma T}\sin\omega T \\ -e^{\sigma T}\sin\omega T, & e^{\sigma T}\cos\omega T \end{bmatrix} \dotplus \begin{bmatrix} e^{\lambda_{21} T} & 1 & & \\ & \ddots & \ddots & \\ & & \ddots & 1 \\ & & & e^{\lambda_2 T} \end{bmatrix} \dotplus \begin{bmatrix} e^{\lambda_{31} T} & & \\ & \ddots & \\ & & e^{\lambda_3 T} \end{bmatrix} \quad (25)$$

$$\tilde{G} = N^{-1}\hat{G}, \qquad \tilde{C} = \hat{C}N \quad (26)$$

Synthesize (1)-(26), we have the relationships between observable canonical form of systems continuity models and Jordan canonical form of systems discreteness models as follows:

$$\tilde{F} = N^{-1}M^{-1}e^{AT}MN \quad (27)$$

$$\tilde{G} = N^{-1}M^{-1}\int_0^T e^{At}B\,dt \quad (28)$$

$$\tilde{C} = CMN \quad (29)$$

3. DISCRETE TRANSFER FUNCTION

Discrete transfer function of the system is

$$H(Z) = \tilde{C}(ZI-\tilde{F})^{-1}\tilde{G} \quad (30)$$

Through correspondent operation, the concrete form of H(Z) is.

$$H(Z) = \frac{(\tilde{c}_1\tilde{g}_1 + \tilde{c}_2\tilde{g}_2)z + (\tilde{c}_1\tilde{g}_2 - \tilde{c}_2\tilde{g}_1)e^{\sigma T}\sin\omega T + (\tilde{c}_1\tilde{g}_1 - \tilde{c}_2\tilde{g}_2)e^{\sigma T}\cos\omega T}{(z-e^{\sigma T}\cos\omega T)^2 + (e^{\sigma T}\sin\omega T)^2}$$

$$+ \sum_{k=1}^{q} \frac{\sum_{i=0}^{q-k} \tilde{c}_{3+i}\,\tilde{g}_{k+2+i}}{(z-e^{\lambda_2 T})^k} + \sum_{i=1}^{p} \frac{\tilde{c}_{q+2+i}\,\tilde{g}_{q+2+i}}{z-e^{\lambda_{3i} T}} \quad (31)$$

in which, $\tilde{c}_i$, $\tilde{g}_i$ represents parameters in matrix $\tilde{G}$ and $\tilde{C}$, respectively.

From (31), it is clear that the relationships between systems parameters of discrete state models and coefficients of discrete transfer function. Therefore,it is easy to know that if coefficients of H(Z) have been identified, identified values of model parameters of systems discrete state can be achieved by means of partial fraction expansion. And then, based on the relationships (27)-(29), we can get systems continuity time state model.

For the system with multiple input and output variables, if its observable cononical form is of regular diagonal block, above considerations are also suitable to be used.

4. SYSTEM IDENTIFICATION WITH STABLE CONSTRAINTS

As mentioned above, if characteristic value of continuous-time model of the system is λ_i, then characteristic root of discrete model is $e^{\lambda_i T}$.Thereby, the stability between continuous and discrete model is correspondent each other. So then for the purpose to guarantee the stability of identified continuous-time model, it is enough to guarantee the stability of identified discrete model.

Discrete transfer function of the system is

$$H(Z) = G(Z)/D(Z) \quad (32)$$

in which, $G(Z)$ and $D(Z)$ are polynomials of z, and the order of $G(Z)$ is not larger than of $D(Z)$. Since we suppose the system is observable and controllable, that is, $G(Z)$ and $D(Z)$ have no common factors. Therefore, if the discrete model is stable, the all zeros of $D(Z)$ should fall inside unit circle of complex plane.

Similar to Routh-Hurwitz criterion [6] it is possible to utilize modified Schur-Cohn criterion to identify real coefficient polynomial:

$$D(Z) = Z^{n} + d_1 Z^{n-1} + \ldots\ldots d_{n-1} Z + d_n \qquad (33)$$

its all zeros fall inside unit circle or not.

Here, the division algorithm of Schur-Cohn criterion is used, and its procedure is, the first step to construct a reciprocal polynomial of $D(Z)$, that is, $D^{(\alpha)}(Z)=Z^{n}D(Z^{-1})$. The roots of $D^{(\alpha)}(Z)$ are the reciprocals of the roots of $D(Z)$ and $|D^{(\alpha)}(Z)|=|D(Z)|$ on the unit circle. The next step is to divide $D^{(\alpha)}(Z)$ by $D(Z)$starting at the high-power end to obtain a quotient $\alpha_0=d_0/d_n$ and remainder $D_1^{(\alpha)}(Z)$ of degree n-1 or less so that.

$$D^{(\alpha)}(Z)/D(Z) = \alpha_0 + D_1^{(\alpha)}(Z)/D(Z)$$

The division process is repeated with $D_1^{(\alpha)}(Z)$ and its reciprocal polynomial $D_1(Z)$ and the sequence $\alpha_0, \ldots, \alpha_{n-2}$ is generated according to the rule.

$$D_k^{(\alpha)}(Z)/D_k(Z) = \alpha_k + D_{k+1}^{(\alpha)}(Z)/D_k(Z) \quad \text{for } k=0,1,\ldots,n-2, \qquad (34)$$

where $D_o(Z)=D(Z)$ and at each step $D_k(Z)$ is considered to be a polynomial of degree n-k regardless of its actual degree. Therefore, some α's may be zero.

Sufficient and necessary conditions that zeros of $D(Z)$ all are inside the unit circle are:

$$\begin{array}{lll} (1)\ D(1) > 0 & & \\ (2)\ D(-1) < 0 & \text{for } n \text{ odd number} & \\ \quad\quad\quad\ \ > 0 & \text{for } n \text{ even number} & (35) \\ (3)\ |\alpha_k| < 1 & \text{for } k = 0,1,\ldots\ldots,n-2 & \end{array}$$

For the purpose to apply above criterion as stability constraint conditions in the procedure of practical systems identification, first, from (32) we have input-output difference equation of the system:

$$y(k) = -d_1 y(k-1) - \ldots -d_n y(k-n) + g_1 u(k-1) + \ldots + g_n u(k-n) \qquad (36)$$

According to the method of least square, to define residual error of (36) is.

$$e(k) = y(k) + d_1 y(k-1) + \ldots + d_n y(k-n) - g_1 u(k-1) - \ldots - g_n u(k-n) \qquad (37)$$

Through N times of measuring input and output of the system, we have the following matrix equation:

$$E(\theta) = y - H\theta, \qquad (38)$$

in which,

$$e = [e(n),e(n+1),\ldots\ldots,e(N)]^{T} \qquad (39)$$

$$y = [y(n),y(n+1),\ldots\ldots,y(N)]^{T} \qquad (40)$$

$$\theta = [+d_1,+d_2,\ldots\ldots,+d_n,-g_1,-g_2,\ldots\ldots,-g_n]^{T} \qquad (41)$$

$$H=-\begin{bmatrix} y(n-1), & y(n-2),\ldots,y(0), & u(n-1),\ldots,u(0) \\ y(n), & y(n-1),\ldots,y(1), & u(n),\ \ldots,u(1) \\ \vdots & \vdots & \vdots \\ y(N-1), & y(N-2),\ldots,y(N-n), & u(N-1),\ldots,u(N-n) \end{bmatrix} \qquad (42)$$

Since former n parameters in θ are coefficients of characteristic polynomial $D(Z)$, then the problem to identify the system with stability constraints by the method of least square become to solve the following optimization problem with constraints:

$$\min J = E^{T}(\theta)E(\theta) \qquad (43)$$

s.t. (35)

Generally, optimization problem (43) is a nonlinear one, and its solving may use suitable nonlinear programming algorithms, then systems model is stable.

5. SIMULATION RESULTS

For the purpose to test proof of above discussions, there are some simulation procedures of low-order systems. And in the case of big noise disturbance. there are stability problems occurred for identified models.As an example, there is a system:

$$\dot{X}(t) = \begin{bmatrix} 0 & 1 \\ -0.01 & -0.2 \end{bmatrix} X(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \qquad (44)$$

$$y(t) = [1,\ 1]x(t) + \beta_w W(t) \qquad (45)$$

in which, $W(t)$ is a random disturbance variable, and β_w is a disturbance strength coefficient.

Characteristic roots of the system are multiple root $\lambda = -0.1$, and it is stable.

Similarly, we transform this model into discrete model by taking the period T, then we have Jordan canonical form of discrete system:

$$\tilde{F} = \begin{bmatrix} e^{-.1T} & 1 \\ 0 & e^{-.1T} \end{bmatrix} \qquad (46)$$

$$\tilde{G} = \begin{bmatrix} -10(T+10)e^{-0.1T} + 100 \\ 10T(1-e^{-0.1T})e^{-0.1T} \end{bmatrix} \quad (47)$$

$$\tilde{C} = [0.9,\ 1/Te^{-0.1T}] \quad (48)$$

From above matrixes, it is possible to calculate precise value of H(Z) and compare it with identified parameter values.

Based on the method of least square, to identify discrete transfer function:

$$H(Z) = (g_1 Z+g_2)/(d_2+d_1 Z+Z^2) \quad (49)$$

Here, Schur-Cohn criterion (35) becomes:

$$1 + d_1 + d_2 > 0 \quad (50)$$

$$1 - d_1 + d_2 > 0 \quad (51)$$

$$|d_2| < 0 \quad (52)$$

The subset in R^2 surrounded by above inequalities is shown in Fig. 1.

For convenience sake, to take computer random function RND(K) as random noise 'V(K) which is uniform distribution in the interval [0,1]. If sampling period is 0.8 seconds, input function $u(t)=\cos t+e^{-0.8t}\cos 2t$, $\beta_w=0$ and β_w = 30%,sample size is 200, identified values of parameters are shown in Table 1. As a kind of comparisons, there are precise parameter values listed in it.

From the results of Table 1, it is clear that if there is no disturbance existed, identified parameter values are close to precise values, and the model is stable. When there exists strong disturbance, it is necessary to consider the stability problem of identified model, because identified parameter values can not satisfy Schur-Cohn criterion. But if we give stable constraints for the system, then it is possible to guarantee the stability of identified discrete models,so further induced continuous-models are also stable.

REFERENCES

[1] Sinha, N. K., "Estimation of Transfer Function of Continuous-Time System from Sampled Data", Proc. IEE, 119, pp. 612-614, 1972.

[2] Hsia, T. C., "On Sampled Data Approach to Parameter Identification of Continuous Linear System", IEEE Trans. AC-17, pp. 247-249, 1972.

[3] Sinha, N. K. and Rozsa, P., Some Canonical Forms for Linear Multivariable Systems", Int. J. Control, 23, pp. 865-883, 1976.

[4] Sinha, N. K. and Lastman, G. J., "A transformation Algorithm for the Identification of Continuous-Time Multivariable System from Discrete Data", Electron Lett., 17, pp. 779-780, 1981.

[5] Sinha, N. K. and Zhou, Q. J., "Discrete-Time Approximation of Multivariable Continuous-Time System", IEE Proc., Vol. 130, Pt. D, No. 3, pp. 103-110, May, 1983.

[6] Tretter, S. A., "Introduction to Discrete-Time Signal Processing", John Wiley & Sons Inc., 1976.

APPENDIX

Table.1

Parameter	d_1	d_2	g_1	g_2	β_w
Precise Value*	-1.846	0.852	1.042	-.451	/
Identified Value	-1.839	0.857	1.081	-.470	0
Identified Value	-2.018	1.005	1.204	-.621	0.03
Identified Value**	-1.913	0.981	1.138	-.552	0.03

in which, * - Calculated values;
** - with stable constraints.

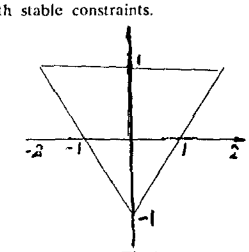

Fig.1