'91 KACC 1991. 10. 22~24

Sliding Mode Control with Adaptive VSS Observer

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ABSTRACT

The conventional sliding mode control and variable structure control (VSC) of nonlinear uncertain system are well known for their robust property and simplity of control law. However, the use of them is only pardonable on the assumption that the upper-bound of parameter variation or nonlinearity is known and that the complete information about state is available. though the former has been solved with adaptive robust control theory recently, the latter seems not to be solved. In this paper, we try to solve this problem using the technique of VSS adaptive robust control theory. That is, we propose a VSS adaptive observer and a sliding mode control incorporated with this observer. We can prove the robust stability of the closed system applying the Lyapunov's second methed.

I. INTRODUCTION

Sliding mode control is researched quite a lot recently for its robust property and simplity of control law, namely it can be realized relatively simply only know the upper-bound of nonlinearity and/or uncertain varing parameters. However, the use of them is only pardonable on the assumption that the complete information about state is available and the upper-bound of parameter variation and/or nonlinearity is known. Although the latter can be solved with the idea of adaptive robust control theory which identifies the upper-bound during the control process, the former seems not to be solved yet.

In this paper, we try to solve this problem using the technique of VSS adaptive robust control theory and observer theory. Firstly, we propose a VSS adaptive observer algorithm to the system where the bound of uncertain nonlinearity and/or parameter variation is not known, by means of the idea of VSS robust adaptive control. Secondly, we realize the the sliding mode control incorporating the state and upperbound estimated by the observer. The stability of the closed loop system can be proved by the Lyapunov's second method. Lastly, in order to verify the validity of the control method, we make a simulation of a 3-state 2-input system.

FOOTNOTES: In this paper, we means the VSS control the cntrol method researched by Gutman[3] Corless[4] et al. This method is similar to sliding mode control, lays a switching surface where the gain is switched. However the control law doesnot restain the state to the surface like sliding mode control.

II. DEFINITION & ASSUMPTION

Consider the multi-input multi-output continuous time control system

$$\dot{x}(t) = Ax(t) + Bh(t, x) + Bu(t) \qquad (1a)$$

$$y(t) = Cx(t) \qquad (1b)$$

where (A, B) is controllable and (C, A) is observable. $x \in R^n$, $u \in R^m$, $y \in R^p$ represent state variable, input variable, output variable respectively, $A \in R^{n\times n}$, $B \in R^{n\times m}$, $C \in R^{p\times n}$ are known constant matrices. $h(t, x) \in R^{m\times 1}$ represent the nonlinearity and uncertain parameter variation.

The purpose of this paper is that construct a adaptive VSS observer which observes the state by means of output, and realizes the sliding mode control with the observed states and guarantees the stability of the closed loop system.

Because (A, B) is controllable, for some arbitrary $Q_1>0$, there exists uniquely a positive definite and symmetric matrix P_1 which satisfies the Riccati equation.

$$P_1A + A^TP_1 - P_1BB^TP_1 + 2Q_1 = 0 \qquad (2)$$

Here we define Λ which will be used later as switching variable coefficient as

$$\Lambda \triangleq B^TP_1 \qquad (3)$$

And because (C, A) is observable, there exists a constant matrix $K \in R^{n\times p}$ such that $\sigma[A_o] \in C^-$ where

$$A_o = A - KC \qquad (4)$$

and C^- is the open left-half plane. consequently, for some arbitrary $Q_2>0$, there exists uniquely a $P_2>0$ such that

$$P_2A_o + A_o^TP_2 = -2Q_2 \qquad (5)$$

is satisfied.

Consider the following three assumptions pertaining to the system (1).

A1) There exists a matrix $F \in R^{m\times p}$ such that

$$(FC, A_o, B)$$

is strictly positive real. This is the same as that there exists P_2 which is the solution of (5), such that

$$FC = B^T P_2 \quad (6)$$

is satisfied[10].

A2) For the uncertainty $h(t, x)$ there exists a scalar upper-bound function $\rho(t, y, \beta) \geqq 0$ which is known in form but may be unknown for value $\beta \in R^k$, such that

$$\| h(t, x) \| \leqq \rho(t, y, \beta) \quad (7)$$

where $\|\cdot\|$ represents ℓ_2 norm, and β is introduced to measure the size of the upper-bound ρ.

A3) The function $\rho(t, y, \beta)$ defined in assumption A2) is C^1 and concave, that is

$$\rho(t, y, \beta^1) - \rho(t, y, \beta^2) \geqq (\beta^1 - \beta^2)^T \frac{\partial \rho}{\partial \beta}\Big|_{\beta^1} \quad (8a)$$

where,

$$\frac{\partial \rho}{\partial \beta} \triangleq \begin{bmatrix} \partial\rho/\partial\beta_1 \\ \partial\rho/\partial\beta_2 \\ \vdots \\ \vdots \\ \partial\rho/\partial\beta_k \end{bmatrix} \quad (8b)$$

On the assumption above, we propose a adaptive robust observer and realize the general sliding mode control with states estimated in this observer.

III. ADAPTIVE VSS OBSERVER

Let the error difference between the observer dynamics estimate and the true state be denote by

$$e(t) = \hat{x}(t) - x(t) \quad (9a)$$

and the output difference multiplied with matrix F be denoted by

$$\alpha(t) = F(\hat{y}(t) - y(t))$$
$$: \hat{y}(t) \triangleq C\hat{x}(t) \quad (9b)$$

Consider the following nonlinear observer dynamical equation.

$$\dot{\hat{x}}(t) = A_o\hat{x} + Ky + Bu + B\delta(t, y, \tilde{\beta}) \quad (10)$$

$$: \delta = \begin{cases} -\dfrac{\alpha}{\|\alpha\|}\rho(t, y, \tilde{\beta}) & \text{for } \alpha \neq 0 \\ 0 & \text{for } \alpha = 0 \end{cases}$$

where $\tilde{\beta}$ represents the estimated value of β and follows the following adaptive algorithm.

$$\dot{\tilde{\beta}} = L\frac{\partial \rho}{\partial \beta}\Big|_{\tilde{\beta}} \cdot \|\alpha\| \quad (11)$$

where L>0 is introduced to regulate the adaption velocity. For the observer dynamics proposed in (10)、(11) the theorem 1 is satisfied.

Theorem 1. Given system (1), and the observer dynamics governed by (10),(11). if assumptions A1)~A3) are valid, then

$$\lim_{t\to\infty}[\hat{x}(t) - x(t)] = \lim_{t\to\infty} e(t) = 0 \quad (12)$$

Proof: The error difference between the output of observer and the true state can be obtained by differentiating (9a) and inserting (1), (4), (10).

$$\begin{aligned} \dot{e} &= \dot{\hat{x}} - \dot{x} \\ &= A_o\hat{x} + Ky + Bu + B\delta - Ax - Bh - Bu \\ &= A_o e + B\delta - Bh \\ &= \begin{cases} A_o e - \dfrac{B\alpha}{\|\alpha\|}\rho(t, y, \tilde{\beta}) - Bh & \text{for } \alpha \neq 0 \\ A_o e - Bh & \text{for } \alpha = 0 \end{cases} \end{aligned} \quad (13)$$

In order to prove the convergency of $e(t)$ to 0, choose the Lyapunov function candidate about $e(t)$ as

$$V(t) = V_1(t) + V_2(t) \quad (14a)$$
$$V_1(t) \triangleq (1/2)\, e^T P_2 e \quad (14b)$$
$$V_2(t) \triangleq (1/2)(\tilde{\beta} - \beta)^T L^{-1}(\tilde{\beta} - \beta) \quad (14c)$$

where P_2 is the solution of (5), and L is the same as in (11).

Differentiating V_1 yields

$$\dot{V}_1(t) = (1/2)\,\dot{e}^T P_2 e + (1/2)\, e^T P_2 \dot{e} \quad (15)$$

substituting(13) to (15) and paying attention to $FC = B^T P_2$, $e^T C^T F^T = \alpha^T$, $|\alpha^T h| \leqq \|\alpha\|\rho$ we have,

$$\begin{aligned} \dot{V}_1(t) &= (1/2)\, e^T(P_2 A_o + A_o^T P_2)e \\ &\quad - e^T\frac{P_2 B\alpha}{\|\alpha\|}\rho(t, y, \tilde{\beta}) - e^T P_2 B h \\ &= -e^T Q_2 e \\ &\quad - e^T\frac{C^T F^T \alpha}{\|\alpha\|}\rho(t, y, \tilde{\beta}) - e^T C^T F^T h \\ &= -e^T Q_2 e - \|\alpha\|\rho(t, y, \tilde{\beta}) - \alpha^T h \\ &\leqq -e^T Q_2 e - \|\alpha\|\rho(t, y, \tilde{\beta}) \\ &\quad + \|\alpha\|\rho(t, y, \beta) \end{aligned} \quad (16a)$$

for $\alpha \neq 0$, and

$$\begin{aligned} \dot{V}_1(t) &= (1/2)\, e^T(P_2 A_o + A_o^T P_2)e - e^T P_2 B h \\ &= -e^T Q_2 e - \alpha^T h = -e^T Q_2 e \end{aligned} \quad (16b)$$

for $\alpha = 0$.

For the derivative of V_2, differentiating V_2 and inserting (11) and assumption 8a), we have

$$\begin{aligned} \dot{V}_2(t) &= (\tilde{\beta} - \beta)^T L^{-1}\dot{\tilde{\beta}} \\ &= (\tilde{\beta} - \beta)^T\frac{\partial \rho}{\partial \beta}\Big|_{\tilde{\beta}}\|\alpha\| \\ &\leqq \rho(t, y, \tilde{\beta})\|\alpha\| - \rho(t, y, \beta)\|\alpha\| \end{aligned} \quad (17)$$

Combining (16), (17), obtains

$$\dot{V}(t)=\dot{V}_1(t)+\dot{V}_2(t)\leqq -e^TQ_2e \tag{18}$$

for all α. It is obviously a negative definite function of $e(t)$. Therefore, $e(t)\to 0$ ($t\to\infty$).

Q. E. D.

IV. Sliding mode control with estimated states

Here we construct the sliding mode control system with state and upper-bound estimated in the observer (10), (11).

Let the switching surfaces be defined as

$$s(t)\triangleq \Lambda\hat{x}(t) \tag{19}$$

where matrix Λ is defined in (3).

The sliding mode control law is constructed as follows.

$$u(t)=-[k\|\hat{x}\|+\rho(t,y,\tilde{\beta})]\frac{s}{\|s\|} \tag{20}$$

$$: k>\|(\Lambda B)^{-1}\Lambda A\|$$

It should be noted that it becomes the same as usual sliding mode control law when $\tilde{\beta}$ is replaced by true value β and $\hat{x}$ by true value $x(t)$.

When control law (20) is used, next theorem is satisfied.

Theorem 2. Given system (1) and the control law (20) with $\hat{x}$, $\tilde{\beta}$ estimated in adaptive VSS observer (10), (11). if assumptions A1)~A3) are valid, the closed loop system is asymptotic stable. that is

$$\lim_{t\to\infty}[\hat{x}(t)-x(t)]=\lim_{t\to\infty}e(t)=0 \tag{21}$$

$$\lim_{t\to\infty}x(t)=0 \tag{22}$$

Proof: It is obvious that if x, $e\to 0$ are guaranteed, then (21), (22) are satisfied. In order to see the stability of the closed loop system, choose the Lyapunov function candidate about e, $\hat{x}$, $\tilde{\beta}$ as

$$V(t)=V_0(t)+\gamma_1V_1(t)+\gamma_1V_2(t)+\gamma_2V_3(t) \quad \cdots\cdots\cdots \ (23)$$

where γ_1, $\gamma_2>0$ are some positive constants and how to choose them will be shown later. $V_0(t)$, $V_1(t)$, $V_2(t)$, $V_3(t)$ are part Lyapunov function candidate respectively shown as follows.

$$V_0(t)=(1/2)\hat{x}^TP_1\hat{x} \tag{24a}$$

$$V_1(t)=(1/2)e^TP_2e \tag{24b}$$

$$V_2(t)=(1/2)(\tilde{\beta}-\beta)^TL^{-1}(\tilde{\beta}-\beta) \tag{24c}$$

$$V_3(t)=(1/2)s^T(\Lambda B)^{-1}s \tag{24d}$$

Let us differentiate V_i respectively.

For V_0, from (24a), (2), (4), (10), we have

$$\begin{aligned}
\dot{V}_0(t)&=(1/2)\dot{\hat{x}}^TP_1\hat{x}+(1/2)\hat{x}^TP_1\dot{\hat{x}}\\
&=(1/2)[A_o\hat{x}+Ky+Bu\\
&\qquad -B\frac{\alpha}{\|\alpha\|}\rho(t,y,\tilde{\beta})]^TP_1\hat{x}\\
&\quad+(1/2)\hat{x}^TP_1[A_o\hat{x}+Ky+Bu\\
&\qquad -B\frac{\alpha}{\|\alpha\|}\rho(t,y,\tilde{\beta})]\\
&=(1/2)[A\hat{x}-KCe+Bu\\
&\qquad -B\frac{\alpha}{\|\alpha\|}\rho(t,y,\tilde{\beta})]^TP_1\hat{x}\\
&\quad+(1/2)\hat{x}^TP_1[A\hat{x}-KCe+Bu\\
&\qquad -B\frac{\alpha}{\|\alpha\|}\rho(t,y,\tilde{\beta})]\\
&=(1/2)\hat{x}^T(P_1A+A^TP_1-P_1BB^TP_1)\hat{x}\\
&\quad+(1/2)\hat{x}^TP_1BB^TP_1\hat{x}\\
&\quad-\hat{x}^TP_1KCe+\hat{x}^TP_1Bu\\
&\quad-\rho(t,y,\tilde{\beta})\hat{x}^TP_1B\frac{\alpha}{\|\alpha\|}
\end{aligned} \tag{25a}$$

from (3), (19), we have $\hat{x}^TP_1BB^TP_1\hat{x}=s^Ts=s^T\Lambda\hat{x}$, thus

$$\begin{aligned}
\dot{V}_0(t)&=-\hat{x}^TQ_1\hat{x}+(1/2)s^T\Lambda\hat{x}-\hat{x}^TP_1KCe\\
&\qquad-[k\|\hat{x}\|+\rho(t,y,\tilde{\beta})]\|s\|\\
&\qquad-s^T\frac{\alpha}{\|\alpha\|}\rho(t,y,\tilde{\beta})\\
&\leqq-\lambda_{min}(Q_1)\|\hat{x}\|^2\\
&\quad+\|\Lambda/2\|\cdot\|s\|\cdot\|\hat{x}\|\\
&\quad+\|P_1KC\|\|\hat{x}\|\|e\|-k\|s\|\|\hat{x}\|
\end{aligned} \quad \cdots\cdots\cdots \ (25b)$$

where $\lambda_{min}(Q_1)$ represents the minimum eigenvalue of Q_1.

About V_1, V_2, we can use the results of (16), (17), the following inequalities are satisfied

$$\begin{aligned}
\dot{V}_1(t)&\leqq-e^TQ_2e-\|\alpha\|\rho(t,y,\tilde{\beta})\\
&\qquad+\|\alpha\|\rho(t,y,\beta)\\
&\leqq-\lambda_{min}(Q_2)\|e\|^2\\
&\qquad-\|\alpha\|[\rho(t,y,\tilde{\beta})-\rho(t,y,\beta)]
\end{aligned} \tag{26}$$

$$\dot{V}_2(t)\leqq\|\alpha\|[\rho(t,y,\tilde{\beta})-\rho(t,y,\beta)] \tag{27}$$

lastly for V_3, translating same as (25), we have

$$\begin{aligned}
\dot{V}_3(t)&=s^T(\Lambda B)^{-1}\dot{s}\\
&=s^T(\Lambda B)^{-1}\Lambda[A\hat{x}-KCe+Bu\\
&\qquad-B\frac{\alpha}{\|\alpha\|}\rho(t,y,\tilde{\beta})]\\
&=s^T(\Lambda B)^{-1}\Lambda A\hat{x}-s^T(\Lambda B)^{-1}\Lambda KCe\\
&\qquad+s^Tu-s^T\frac{\alpha}{\|\alpha\|}\rho(t,y,\tilde{\beta})\\
&=s^T(\Lambda B)^{-1}\Lambda A\hat{x}-s^T(\Lambda B)^{-1}\Lambda KCe\\
&\qquad-\|s\|[k\|\hat{x}\|+\rho(t,y,\tilde{\beta})]\\
&\qquad-s^T\frac{\alpha}{\|\alpha\|}\rho(t,y,\tilde{\beta})\\
&\leqq-[k-\|(\Lambda B)^{-1}\Lambda A\|]\|s\|\|\hat{x}\|\\
&\qquad+\|\Lambda^T(\Lambda B)^{-1}\Lambda KC\|\|\hat{x}\|\|e\|
\end{aligned} \tag{28}$$

Substituting the above results into $\dot{V}$, yields

$$\dot{V}=\dot{V}_0+\gamma_1\dot{V}_1+\gamma_1\dot{V}_2+\gamma_2\dot{V}_3$$
$$\leqq-\lambda_{min}(Q_1)\|\hat{x}\|^2+(\|P_1KC\|+\gamma_2\|\Lambda^T(\Lambda B)^{-1}\Lambda KC\|)\|\hat{x}\|\|e\|$$
$$-\gamma_1\lambda_{min}(Q_2)\|e\|^2$$
$$-[\gamma_2(k-\|(\Lambda B)^{-1}\Lambda A\|)+k-\|\Lambda/2\|]\|\hat{x}\|\|s\| \quad (29)$$

Here if γ_2 is choosed as

$$\gamma_2>\frac{max(0,\|\Lambda/2\|-k)}{(k-\|(\Lambda B)^{-1}\Lambda A\|)} \quad (30a)$$

the last term of (29) becomes nonpositive. and if γ_1 is choosed

$$\gamma_1>\frac{(\|P_1KC\|+\gamma_2\|\Lambda^T(\Lambda B)^{-1}\Lambda KC\|)^2}{4\lambda_{min}(Q_1)\lambda_{min}(Q_2)} \quad \cdots\cdots\cdots (30b)$$

the first three terms of (29) become negative about $\hat{x}$, e (refer to supplement). Therefore the total of (29) becomes negative definite to e, $\hat{x}$. so that $e(t)\rightarrow 0$, $\hat{x}\rightarrow 0$, $x(t)=\hat{x}(t)-e(t)\rightarrow 0$. namely the combined system is asymptotic stable.

Q. E. D.

Note that although we can not guarantee that $\tilde{\beta}$ converge to the true value of β, because the convergency of e, x is guaranteed, our purpose is attained.

Moreover, because that the control law is discontinuous in the vicinity of $s=0$, it may become the cause of chattering and may excite high-frequency neglected in the course of modelling. The basic way to avoid this is to alter the control law (20) to follows such that the inputs become continuous.

$$u(t)=\begin{cases}-[k\|\hat{x}\|+\rho(t,y,\tilde{\beta})]\dfrac{s}{\|s\|} & \text{for } \|s\|\geqq\varepsilon \quad (31a)\\ -[k\|\hat{x}\|+\rho(t,y,\tilde{\beta})]\dfrac{s}{\varepsilon} & \text{for } \|s\|<\varepsilon \quad (31b)\end{cases}$$

V. EXAMPLE AND SIMULATION

A simulation has been made to assure the validity of the observer and the contol law. System (1) is considered with

$$A=\begin{bmatrix}2&4&0\\3&-5&0\\2&0&-4\end{bmatrix}\quad B=\begin{bmatrix}1&1\\1&-1\\2&0\end{bmatrix}$$
$$C=\begin{bmatrix}1&1&0\\1&-1&0\end{bmatrix}$$
$$h(t,x)=\begin{bmatrix}\beta_1\sin x_3+\beta_2(x_1^2-x_2^2)\\ \beta_2(x_1^2+x_2^2)\end{bmatrix} \quad (32)$$

$\beta_1>0$, $\beta_2>0$ represent the uncertain parameters. Because A is unstable, feedback gain K is chosen as

$$K=\begin{bmatrix}4&0\\0&3\\1&1\end{bmatrix}\quad A_o=\begin{bmatrix}-2&0&0\\0&-2&0\\0&0&-4\end{bmatrix} \quad (33)$$

such that A_o is stable. When Q_2 in Lyapunov equation (5) is selected as follows , P_2 can be obtained

$$Q_2=\begin{bmatrix}4&2&-3\\2&4&-3\\-3&-3&4\end{bmatrix}\quad P_2=\begin{bmatrix}2&1&-1\\1&2&-1\\-1&-1&1\end{bmatrix} \quad (34)$$

It is obvious that assumption A1) is satisfied with $F=I$.

Because

$$\|h\|\leqq\beta_1+\beta_2(|y_1|+|y_2|)^2/2 \quad (35)$$

the upper-bound function ρ can be choosed as

$$\rho=\beta_1+\beta_2(|y_1|+|y_2|)^2/2 \quad (36)$$

where the uncertain parameters β_1, β_2 can be used as the upper-bound parameter as they are. Do like this the parameter adaptive algorithm (10) becomes

$$\dot{\tilde{\beta}}_1=L_1\|\alpha\| \quad (37a)$$
$$\dot{\tilde{\beta}}_2=L_2(|y_1|+|y_2|)^2\|\alpha\|/2 \quad (37b)$$

For the Riccati equation (2), when Q_1 is selected as follows, P_1 can be obtained as

$$Q_1=\begin{bmatrix}.5&0&0\\0&1&0\\0&0&2\end{bmatrix}\quad P_1=\begin{bmatrix}3.65&1.77&-0.42\\1.77&1.00&-0.25\\-0.42&-0.25&0.49\end{bmatrix} \quad (38)$$

so that from (3), switching variable coefficient becomes

$$\Lambda=B^TP_1=\begin{bmatrix}4.58&2.30&0.30\\1.89&0.74&-0.17\end{bmatrix} \quad (39)$$

In the simulation, the values $\beta_1=1$, $\beta_2=2$ are put. and the values of coefficients of control side are put as

$$L_1=1,\ L_2=1,\ k=7.4 \quad (40)$$

So the control input u becomes

$$u=-[7.4\|\hat{x}\|+\tilde{\beta}_1+\tilde{\beta}_2(|y_1|+|y_2|)^2/2]s/\|s\| \quad (41)$$

The simulation results are shown in Fig. 1～Fig. 8. Fig. 1～Fig. 4 show the results when standard control law is used. Fig. 5～Fig. 8 show the results when approximate one is used, where $\varepsilon=0.5$.

From Fig. 2, Fig. 6 we can see that although $\tilde{\beta}$ failed to reach the true value β, because the control object is achieved, it is of course the expecting thing. Where $\tilde{\beta}$ will converge would be left as the theme hereafter.

VI. CONCLUSION

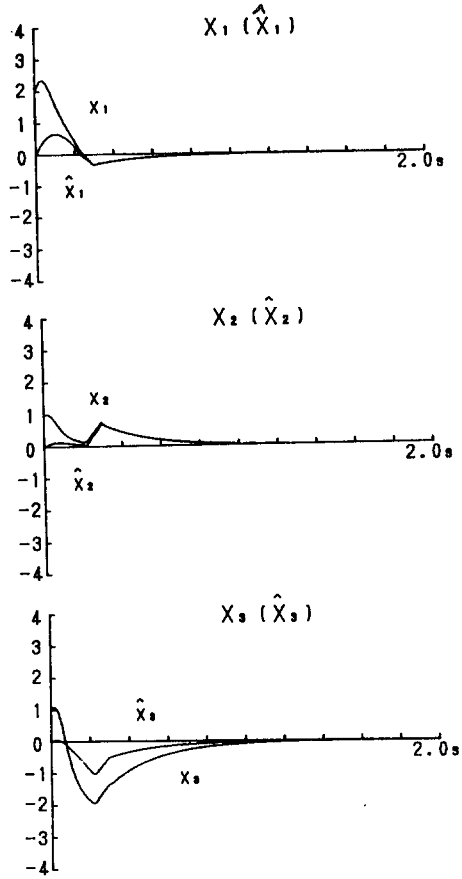

Fig. 1 states of plant and observer (standard)

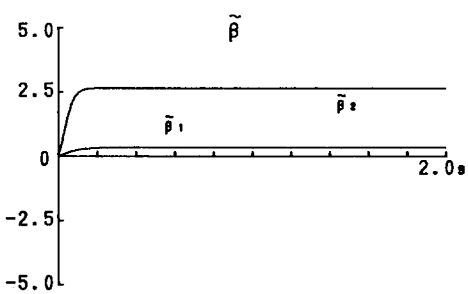

Fig. 2 gain parameters (standard law)

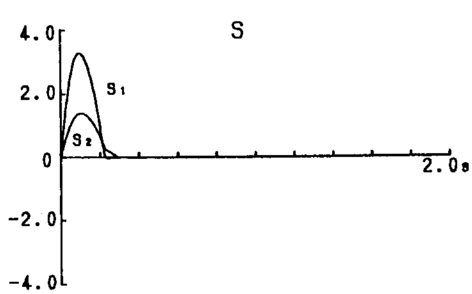

Fig. 3 switching variables (standard law)

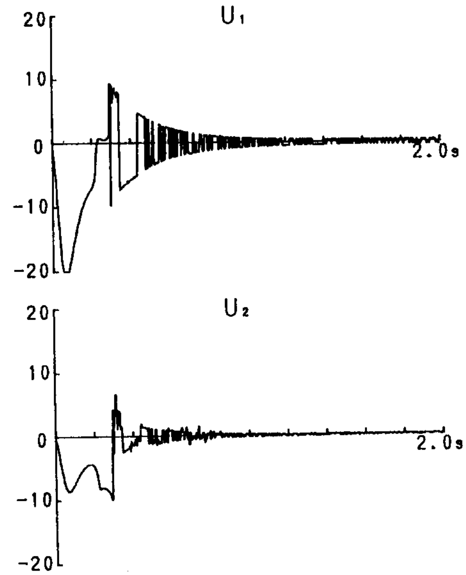

Fig. 4 inputs (standard law)

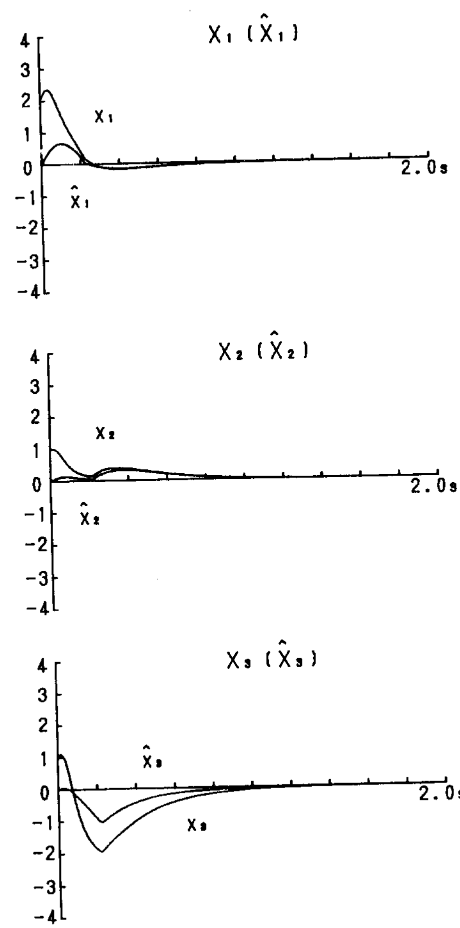

Fig. 5 states of plant and observer (approximate)

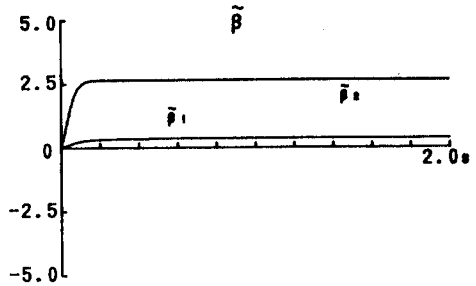

Fig. 6 gain parameters (approximate law)

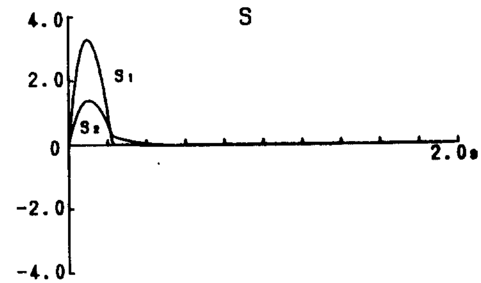

Fig. 7 switching variables (approximate law)

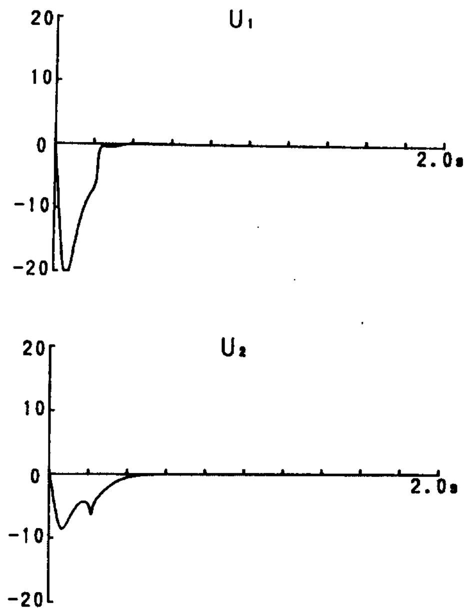

Fig. 8 inputs (approximate law)

The conventional sliding mode control is based on the assumption that state feedback is possible and that the upper-bound of uncertainty is known. For this, in this paper we present a adaptive VSS observer which estimate the states while the upper-bound parameter of uncertainty is identified. And realized the usual sliding mode control with the observed states. The stability of the closed system is assured by Lyapunov's second method and simulation.

SUPPLEMENT

The first three terms of (29) can be expressed in the form of

$$J = -a\|\hat{x}\|^2 - b\|e\|^2 + 2c\|\hat{x}\|\|e\| \quad (A1)$$

where

$$a = \lambda_{min}(Q_1) \qquad b = \gamma_1 \lambda_{min}(Q_2)$$

$$c = [\|P_1 K C\| + \gamma_2 \|\Lambda^T(\Lambda B)^{-1}\Lambda K C\|]/2$$

if $a > 0,\ b > 0,\ c^2 < ab$ (A2)

is satisfied, $J < 0$ when $\hat{x} \neq 0,\ e \neq 0$, and $J = 0$ when $\hat{x} = e = 0$. namely J is negative definite.

From condition (A2), (30b) can be obtained.

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