각도 측정치만을 이용한 로봇을 위한 강인한 제어기 설계

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Robust Linear Tracking Controller Design for Manipulators Using Only Position Measurements

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Abstract

In this note, we propose a method for designing a robot controller which can suppress the effects of both the model uncertainty and noisy velocity measurements. The controller is an output feedback compensator of which the constant gains are given in terms of a Riccati equation and a Lyapunov equation. The controller guarantees not only uniform boundedness but uniform ultimate boundedness. The stability result is local but the region can be arbitrarily enlarged at the expense of large control gain. The control law needs neither the exact knowledge of the physical robot parameters nor clean velocity measurements.

I. Introduction

As manipulators are used more in many industries, considerable methods for designing robot controllers have been studied by many researchers. Most of them are based on the assumptions not only that the dynamics of the robot are exactly known but that the full state, the angular position and velocity of the joints, is available. In practice, because the dynamics are difficult to model accurately due to uncertainties on such as the payload, the assumption of good model matching is not realistic. And, while the position measurement are obtained very accurately through encoders the velocity measurements are typically obtained tachometers and contaminated by noise.

Poor model matching and noisy state will degrade the dynamic performance of the robot. Those problems may be solved through either adaptive schemes or robust schemes which use some kind of time-derivative approximation computed from clean position measurements as the velocity of the joints. But the approximated velocity is not meaningful at high sampling rates and it is difficult to quantify the effects of the approximated velocity in the closed-loop stability as well as on the tracking precision. Thus, methods for designing a robust controller which can suppress the effects of both the model uncertainty and noisy velocity measurements are needed.

Recently, several authors studied methods for robot control using only position measurements [1-4]. Because the methods in [1-3] need the exact knowledge of the physical robot parameters, they are sensitive to errors in the model parameters and rather complicate. The problems are solved by C. C. Wit and N. Fixot in [4]. They propose a robot control scheme via estimated velocity feedback which is guaranteed to be locally exponentially stable under model parameter inaccuracies and input torque bounded disturbances. But the method uses a discontinuous control which cause the robot to vibrate in a high frequency structural mode.

In this note, we propose a robust output feedback compensator design method for manipulators. The controller guarantees not only uniform boundedness but uniform ultimate boundedness. Though the control law does not lead to perfect tracking, it guarantees to track to within an arbitrarily precision. The control is a simple linear time-invariant type one and the constant control parameters are obtained in terms of the solutions of a Riccati equation and a Lyapunov equation. The stability result is local but the region can be arbitrarily enlarged at the expense of large control gains. The control law needs neither the exact knowledge of the physical robot parameters nor clean velocity measurements. Also, the control law allows quick response in an on-line implementation because the method uses a simple linear feedback control.

We will use the following notations. $\|\cdot\|$ denotes the Euclidian vector norm, and $\lambda_{min(max)}(\cdot)$ represents the operation of taking the minimum (maximum) eigenvalue.

II. Model Properties and Assumptions

The dynamics for robot manipulators with n degrees of freedom are generally described by the following nonlinear differential equations:

$$M(q(t))\ddot{q}(t)+C(q(t),\dot{q}(t))\dot{q}(t)+G(q(t))=u(t) \quad (1)$$

where $q(t)\in R^n$ is the joint angle vector; $M(q(t))\in R^{n\times n}$ is the inertia matrix, which is symmetric and positive definite for any $q(t)$; $C(q(t),\dot{q}(t))\dot{q}(t)\in R^n$ is the centrifugal and Coriolis torque; $G(q(t))\in R^n$ is the gravitational torque; $u(t)\in R^n$ is the applied joint torque. In the following, $M(q(t))$, $C(q(t),\dot{q}(t))$, and $G(q(t))$ will be denoted as M, C, and G respectively when necessary. For convenience, the argument t is sometimes omitted when no confusion is likely to arise.

In the above dynamic model, one can find the following important properties.

P1 :[1][5] Given a proper definition of the matrix C, the matrix $\dot{M}-2C$ is skew symmetric. One possible definition for the elements of C which leads to the skew-symmetry of $\dot{M}-2C$ is

$$C_{ij}(q,\dot{q})=\frac{1}{2}\left[\dot{M}_{ij}+\sum_{k=1}^{n}\left(\frac{\partial M_{ik}}{\partial q_j}-\frac{\partial M_{jk}}{\partial q_i}\right)\dot{q}_k\right], i,j=1,\ldots,n. \quad (2)$$

P2 :[1][6] Given two $n\times 1$ vectors x and y, (2) implies

$$C(q,x)y=C(q,y)x. \quad (3)$$

P3 :[1][6] The matrices M, G, and C are bounded in q, and C is linear with respect to $\dot{q}$.

Now we make the following assumptions in our analysis.

A1 : There is no limit on the torque or force that can be delivered by the actuator.

A2 : Output variables of (1) is the joint displacements.

A3 : The given trajectory for tracking q_d is twice differentiable and q_d, $\dot{q}_d$, and $\ddot{q}_d$ are bounded in all $t\geq 0$.

The dynamic properties of manipulators and above assumptions will be utilized to ensure the robust stability of robot systems under consideration.

III. Problem Formulation

Let $x^T=[x_1^T,x_2^T]=[q^T-q_d^T,\dot{q}^T-\dot{q}_d^T]$. Then (1) can be described in state space as follows:

$$\dot{x}=Ax+Bu+\Delta Ax+\Delta Bu+\tilde{B}M^{-1}(q)d \quad (4)$$

where

$$A=\begin{bmatrix}0 & I\\ 0 & 0\end{bmatrix}\quad B=\tilde{B}\hat{B}\quad \Delta A=-\tilde{B}[0\ \ M^{-1}D]\quad \Delta B=\tilde{B}M^{-1}-B \quad (5a)$$

$$\tilde{B}=\begin{bmatrix}0\\ I\end{bmatrix}\quad D=C(q,x_2)+2C(q,\dot{q}_d) \quad (5b)$$

$$d=-(G(q)+C(q,\dot{q}_d)\dot{q}_d+M(q)\ddot{q}_d) \quad (5c)$$

and $\hat{B}\in R^{n\times n}$ is a positive definite matrix which will later be specified.

Consider an output feedback compensator given by

$$\dot{z}=K_{11}x_1+K_{12}z \qquad z(0)=z_0 \quad (6)$$

$$u=K_{21}x_1+K_{22}z \quad (7)$$

where $z\in R^n$, and K_{11}, K_{12}, K_{21}, and K_{22} are $n\times n$ gain matrices. At this point, we remark that the controller can be an observer-based one but we restrict our analysis to the output feedback compensator because we can analyze similarly in the observer-based control case.

Now, based on the dynamic properties of robots and the previous assumptions we introduce additional assumptions for further use in the stability analysis of the proposed robot control system. The need for the following assumptions will be clarified in the subsequent sections.

A4 : We know the positive constants, α and β, satisfying

$$\beta I\leq M^{-1}(q)\leq \alpha I, \qquad \forall q\in R^n \quad (8)$$

A5 : We know the matrix $\bar{D}$ which is any matrix satisfying

$$D^TD\leq \bar{D} \qquad \forall x\in\{x\mid \|x\|\leq r_0\} \quad (9)$$

for some positive constant r_0.

A6 : We know the positive constant δ which is satisfying for given $\dot{q}_d$ and $\ddot{q}_d$

$$\|d\|\leq\delta \qquad \forall q\in R^n \quad (10)$$

Thus, our design problem is formulated as choosing the parameters K_{11}, K_{12}, K_{21}, and K_{22} of (6) and (7) such that all the closed-loop system responses of (4), (6), and (7) starting from within some prescribed region satisfies uniform ultimate boundedness within an arbitrarily small ball centered at the zero state.

IV. Main Result

Consider the following matrix Riccati equation:

$$A^TP_x+P_xA-P_x\tilde{B}\tilde{B}^TP_x+2\alpha^2\tilde{B}\bar{D}\tilde{B}^T+Q_x=0 \quad (11)$$

where $Q_x\in R^{2n\times 2n}$ is a positive definite matrix. We can see from standard results that a unique solution matrix exists for (11). And, consider the following Lyapunov equation:

$$(1+\zeta+\gamma\alpha(\alpha+\beta))P_eP_e-(P_eL+L^TP_e)+Q_e=0 \quad (12)$$

where $Q_e\in R^{n\times n}$ is a positive definite matrix,

$$\gamma=\frac{2+\zeta}{\beta(\alpha+\beta)},\qquad P_*=\tilde{B}^TP_x\tilde{B}, \quad (13)$$

and ζ is a positive constant to be specified in the following. Also the solution of the Lyapunov equation (12) always exists.

Let the control parameters be

$$K_{11} = -L\,L \qquad K_{12} = -L \tag{14a}$$

$$K_{21} = -\gamma B^T P_x [I \quad L^T]^T \quad K_{22} = -\gamma B^T P_x \tilde{B} \tag{14b}$$

Then, by introducing

$$e \triangleq [-L \quad I]x - z \tag{15}$$

we can get

$$u = -\gamma B^T P_x x + \gamma B^T P_x \tilde{B} e \tag{16}$$

$$\dot{e} = -Le + \gamma M^{-1} B^T P_x \tilde{B} e + \tilde{B}^T \Delta A x - \gamma M^{-1} B^T P_x x + M^{-1} d \tag{17}$$

$$\dot{x} = (A - \gamma B B^T P_x) x + \Delta A x - \gamma \Delta B B^T P_x x + \gamma \tilde{B} M^{-1} B^T P_x \tilde{B} e + \tilde{B} M^{-1} d \tag{18}$$

Now choose $\hat{B}$ of (3) as follows:

$$\hat{B} = \frac{(\alpha+\beta)}{2} I \tag{19}$$

Let $\bar{x}^T = [x^T, e^T]$ and the Lyapunov function $V(\bar{x})$ be

$$V(\bar{x}) \triangleq V_x(x) + V_e(e) = x^T P_x x + e^T P_e e \tag{20}$$

Using (8), (9), (11), (13) and the following matrix identity

$$XY^T + YX^T \le XX^T + YY^T \tag{21}$$

for any appropriate dimensioned matrices X and Y, we can show that in the region of $B_{r_0} \triangleq \|\bar{x}\| \le r_0$

$$\dot{V}_x \le -x^T Q_x x - \zeta x^T P_x \tilde{B} \tilde{B}^T P_x x - \alpha^2 x^T \tilde{B} \tilde{D} \tilde{B}^T x + 2\gamma x^T P_x \tilde{B} M^{-1} B^T P_x \tilde{B} e + 2x^T P_x \tilde{B} M^{-1} d \tag{22}$$

After similar manipulations, the following inequality is obtained for $\dot{V}_e$

$$\dot{V}_e \le -e^T Q_e e - \zeta e^T P_e P_e e + \alpha^2 x^T \tilde{B} \tilde{D} \tilde{B}^T x - 2\gamma x^T P_x \tilde{B} M^{-1} B^T P_x \tilde{B} e + 2e^T P_e M^{-1} d \tag{23}$$

Thus the following inequality is satisfied in B_{r_0}.

$$\dot{V}(\bar{x}) \le -\lambda_1 \|\bar{x}\|^2 - \zeta \|[\tilde{B}^T P_x \quad P_e]\bar{x}\|^2 + 2\alpha\delta \|[\tilde{B}^T P_x \quad P_e]\bar{x}\| \tag{24}$$

where $\lambda_1 = \min\{\lambda_{\min}(Q_x), \lambda_{\min}(Q_e)\}$.

Now we choose

$$\zeta = \frac{4\alpha^2\delta^2}{\lambda_1 \varepsilon^2} \qquad 0 < \varepsilon < \frac{1}{\lambda} r_0 = \left[\frac{\lambda_2}{\lambda_3}\right]^{1/2} r_0 \tag{25}$$

where $\lambda_2 = \min\{\lambda_{\min}(P_x), \lambda_{\min}(P_e)\}$, $\lambda_3 = \max\{\lambda_{\max}(P_x), \lambda_{\max}(P_e)\}$, and $\lambda\varepsilon$ is the radius of the small ball centered at $\bar{x}=0$ within which we want to guarantee uniform ultimate boundedness of $\bar{x}(t)$ i.e. $\lambda\varepsilon$ is the constant which determines the tracking precision. Then the following are true:

1) If $\|[\tilde{B}^T P_x \quad P_e]\bar{x}\| \le \frac{2\alpha\delta}{\zeta}$ in B_{r_0}

$$\dot{V}(\bar{x}) \le -\lambda_1 \|\bar{x}\|^2 + \lambda_1 \varepsilon^2 \tag{26}$$

2) If $\|[\tilde{B}^T P_x \quad P_e]\bar{x}\| > \frac{2\alpha\delta}{\zeta}$ in B_{r_0}

$$\dot{V}(\bar{x}) \le -\lambda_1 \|\bar{x}\|^2 \tag{27}$$

After all, in B_{r_0}

$$\dot{V}(\bar{x}) \le -\lambda_1 \|\bar{x}\|^2 + \lambda_1 \varepsilon^2 \tag{28}$$

Now, we are ready to establish our main result.

Theorem 1 : Suppose that *A1* ~ *A6* are satisfied and consider the closed-loop system of (4), (6), and (7) with (14). Then the following properties hold:

1) Uniform Boundedness: Given any $r \in [0, \frac{r_0}{\lambda})$,

$$\|\bar{x}(t_0)\| \le r \;\rightarrow\; \|\bar{x}(t)\| \le \Omega(r), \; \forall t \ge t_0 \tag{29}$$

where

$$\Omega(r) \triangleq \begin{cases} \lambda r & \text{if } r > \varepsilon \\ \lambda\varepsilon & \text{otherwise} \end{cases} \tag{30}$$

2) Uniform Ultimate Boundedness: Given any $\bar{\varepsilon} > \lambda\varepsilon$ and any $r \in [0, \frac{r_0}{\lambda})$,

$$\|\bar{x}(t_0)\| \le r \;\rightarrow\; \|\bar{x}(t)\| \le \bar{\varepsilon}, \; \forall t \ge t_0 + T(\bar{\varepsilon}, r) \tag{31}$$

where

$$T(\bar{\varepsilon}, r) \triangleq \begin{cases} \dfrac{\lambda_3 r^2 - \lambda_2 \bar{R}^2}{\lambda_1 \bar{R}^2 - \lambda_1 \varepsilon^2} & \text{if } r > \bar{R} \\ 0 & \text{otherwise} \end{cases} \quad \text{and } \bar{R} = \frac{\bar{\varepsilon}}{\lambda} \tag{32}$$

Proof :Immediate from (28), and the result of [7].

At this point, we remark that through the method in [4] if $\alpha/\beta > (\sqrt{2}+1)/(\sqrt{2}-1)$ i.e. if the eigenvalues of the inertia matrix $M(q)$ vary much or if the load variation is large, then simple choices of the nominal inertia matrix $\hat{B}$ like (19) is impossible, which leads to using a more precise model, but our method allows simple choices. Also, it should be noted that by selecting a constant positive definite matrix as $\hat{B}$ differently from (19) we can draw similar result and our approach can be straightforwardly applied to mechanical systems the dynamic equations of which can be described like (1).

V. Conclusion

We propose a robust output feedback compensator design method for manipulators. The control law needs neither the exact knowledge of the physical robot parameters nor clean velocity measurements. The controller guarantees not only uniform boundedness but uniform ultimate boundedness. Because the proposed method uses a simple linear feedback control we can get quick response in an on-line implementation.

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