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Orthogonalization Principle for Hybrid Control of Robot Arms under Geometric Constraint

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Abstract: A principle of "orthogonalization" is proposed as an extended notion of hybrid (force and position) control for robot manipulators under geometric endpoint constraints. The principle realizes the hybrid control in a strict sense by letting position and velocity feedback signals be orthogonal in joint space to the contact force vector whose components are exerted at corresponding joints. This orthogonalization is executed via a projection matrix computed in real-time from a gradient of the equation of the surface in joint coordinates and hence both projected position and velocity feedback signals become perpendicular to the force vector that is normal to the surface at the contact point in joint space.

To show the important role of the principle in control of robot manipulators, three basic problems are analyzed, the first is a hybrid trajectory tracking problem by means of a "modified hybrid computed torque method", the second is a model-based adaptive control problem for robot manipulators under geometric endpoint constraints, and the third is an iterative learning control problem. It is shown that the passivity of residual error dynamics of robots follows from the orthogonalization principle and it plays a crucial role in convergence properties of both positional and force error signals.

1. Introduction

It is still claimed that even the most advanced industrial robots at the present stage lack versatility in fulfilling various tasks imposed on them. This may be caused from lack of studies on analysis of dynamics and control actions for robot manipulators that are subject to actual execution of those tasks. One of typical situations during execution of tasks is that an endpoint of the manipulator is in touch with a surface of an object in the task environment. Polishing, grinding, and even writing with a pen are such examples. In those situations, both position of the endpoint and force at the contact point of the robot manipulator must be controlled simultaneously. To solve such a control problem, Raibert and Craig [1] proposed the so-called "hybrid position/force control" by introducing a mode selection that distinguishes positional control components from force feedback control components in cartesian coordinates. However, this concept can be in principle applied for only the case that the contact surface is a flat plane in cartesian coordinates. Very recently, MaClamroch and Wang [2] gave a proof of local feedback stabilization using linear feedback for set-point control when both constant target position and contact force are given to the manipulator. In [2], there was in principle no need of use of the mode selection matrix. They [3] also dealt with a general tracking problem for constrained robots by using nonlinear feedback on the basis of a modified computed torque method. Kankaanranta and Koivo [4] treated the problem by using nonlinear transformation of the model in joint coordinates into the constraint frame and reducing the dimensionality of the model as likely in [3]. However, a control problem of tracking both endpoint position and contact force trajectories still remains unsolved when some of basic physical parameters of robots are unknown or uncertain, though an efficient adaptive control method to track a prescribed positional trajectory was recently proposed by Slotine and Li [5] to the case when the endpoint is free to move.

This paper attempts to extend the concept of hybrid position/force control to a general class of tasks in the case that the endpoint touches with a general smooth surface. Instead of using the mode selection in cartesian coordinates or formulating the constrained dynamic equation of motion expressed on the constraint manifold, the proposed method uses a so-called "Orthogonalization Principle" that distinguishes positional feedback signals from force feedback signals by introducing a projection matrix in joint coordinates. The projection matrix is defined so as to project velocity and position error signals onto the plane tangent to the contact surface described in joint-space at each instantaneous contact point. Hence, the residual error positional and velocity signals become automatically perpendicular to the force vector normal to the surface in joint-space.

To show the crucial role of the orthogonalization principle, three basic trajectory-tracking problems are analyzed, the first is a hybrid trajectory tracking control problem by means of a so-called "modified hybrid computed torque method", the second is a model-based adaptive control problem for manipulators under geometric endpoint constraints, and the last is an iterative learning control problem under the same situation. It is shown that the passivity of residual error robot dynamics follows from the orthogonalization principle and it plays a key role in convergence properties of both position and force error signals.

In section 2, robot dynamics under geometric constraints is introduced. In section 3 a hybrid tracking problem is analyzed under the assumption that the exact dynamic model of the manipulator is available. In section 4 a model-based adaptive control problem is treated in the case that there are some unknown or uncertain parameters which appear linearly in the manipulator dynamics. In section 5 an iterative learning control problem is treated on the basis of the orthogonalization principles again. These last two problems are also treated somewhat in detail in our recent separate papers ([6],[7]). Original discussions in section 4 are found in [8].

2. Robot Dynamics under Geometric Endpoint Constraints

Suppose that the manipulator endpoint is in touch with a surface as shown in Fig.1. The surface is described by a scalar function, $\varphi(x^1, x^2, x^3) = 0$, where $x = (x^1, x^2, x^3)^T$ denotes the cartesian coordinates (task coordinates) fixed at the inertial reference frame. The contact force arises in the direction of normal vector to the surface at point x and the contact friction arises in the direction of $-\dot{x}$ with magnitude $\xi(\|\dot{x}\|)\cdot\|\dot{x}\|$ where $\xi(\alpha)$ is a positive function of α. Then the robot dynamics is described in terms of joint coordinates $q = (q^1, \cdots, q^n)^T$ in the following form:

$$H(q)\ddot{q} + \left(B_0 + \frac{1}{2}\dot{H}(q) + S(q,\dot{q})\right)\dot{q} + g(q) = J_\varphi^T(q)f - \xi(\|\dot{x}\|)J_x^T(q)\dot{x} + u \quad (1)$$

where the first term of the right hand side denotes the contact force exerted at joints, $J_x(q)$ and $J_\varphi(q)$ denote the $3 \times n$ Jacobian matrix of x in q and $1\times n$ unit normal vector of $\varphi(x(q)) = 0$ in joint space, respectively, that is,

$$J_x(q) = \left(\frac{\partial x^i(q)}{\partial q^j}\right), \qquad J_\varphi(q) = \frac{\partial\varphi}{\partial x}J_x(q)/\left\|\frac{\partial\varphi}{\partial x}J_x(q)\right\|. \quad (2)$$

The left hand side of eq.(1) expresses that of a Lagrange equation for a Lagrangian $L = K - U$, where $K(= \dot{q}^T H(q)\dot{q}/2)$ denotes the kinetic energy and $U(= U(q))$ the potential energy. In this form, B_0 denotes a positive definite matrix representing damping factors, $H(q)$ an inertia matrix including inertial terms of internal load distribution of actuators, $g(q) = (\partial U/\partial q^1, \cdots, \partial U/\partial q^n)^T$, and u a vector of input torques generated at servo actuators. We consider in this paper a class of anthropoid robot manipulators that have only revolute-type joints as shown in Fig.1. It is well known that the inertia matrix $H(q)$ for such a class of robot arms is symmetric and positive definite and, moreover, each entry of $H(q)$ is constant or a trigonometric function of components of joint vector q. It is also well known that matrix $S(q,\dot{q})$ is linear and homogeneous in $\dot{q}$ and skew symmetric, in other words,

$$r^T S(q,\dot{q})r = 0 \quad (3)$$

for any n-vector r. First we note that the dynamics of eq.(1) under geometric constraints also satisfies the passivity condition between torque input vector u and velocity vector $\dot{q}$. In fact, note that $J_\varphi(q)\dot{q} = 0$ and therefore

$$\int_0^t \dot{q}^T(\tau)u(\tau)d\tau = \int_0^t \left[\dot{q}^T(\tau)B_0\dot{q}(\tau) + \xi(\|\dot{x}\|)\,\|\dot{x}\|^2\right]d\tau + V(t) - V(0), \quad (4)$$

where $V(t)$ is defined as the total energy described by

$$V(t) = \frac{1}{2}\dot{q}^T(t)H(q(t))\dot{q}(t) + U(q(t)). \quad (5)$$

Since the constant term of potential is arbitrary, it is reasonable to assume that

$$\min_q U(q) = 0. \quad (6)$$

Then, $V(t) \geq 0$, and therefore it follows that

$$\int_0^t \dot{q}^T(\tau)u(\tau)d\tau \geq -V(0) = \gamma \quad (7)$$

which shows the passivity of eq.(1) with respect to input u and output $\dot{q}$ even when the endpoint is geometrically constrained on a surface.

3. Orthogonalization Principle and Hybrid Tracking Control

Suppose that a pair of desired position trajectory $q_d(t)$ and desired force trajectory $f_d(t)$ on the surface is given in joint space. The problem is to find an efficient tracking control that makes the manipulator track asymptotically the specified position and force trajectories as $t \to \infty$. In this section we assume that the exact dynamic model of the manipulator together with friction characteristics of the contact is available. However, we reasonably suppose that the acceleration signal $\ddot{q}(t)$ is not available in real-time because of the law of causality. Instead we reasonably assume that the velocity and position signals $\dot{q}(t)$ and $q(t)$ and the momentum signal defined by

$$F(t) = \int_0^t f(\tau)d\tau \quad (8)$$

can be measured in real-time and used in real-time computation of a control input. In reality , note that the equation of motion under geometric endpoint constraint is expressed by eq.(1) and hence the triplet of $\dot{q}$, q and F can be regarded as a state vector.

Let us now introduce a signal called "nominal reference" that is defined by

$$\dot{q}_r = Q(q)\{\dot{q}_d - \alpha\Delta q\} + \beta J_\varphi^T(q)\Delta F \quad (9)$$

where $\alpha > 0$ and $\beta \geq 0$ are constants and

$$\Delta q = q - q_d, \quad \Delta F = F - F_d, \quad F_d(t) = \int_0^t f_d(\tau)d\tau. \quad (10)$$

The $n \times n$ matrix $Q(q)$ in eq.(9) is defined by

$$Q(q) = I - J_\varphi^T(q)J_\varphi(q) \quad (11)$$

which can be regarded as a projection matrix that projects vectors in joint space onto the plane tangent to the surface $\varphi(q) = 0$ at point q (See Fig.2). Since $\varphi(q) = 0$ as long as the manipulator endpoint is in touch with the surface, it holds that

$$J_\varphi(q)\dot{q} = 0 \quad (12)$$

and hence

$$Q(q)\dot{q} = \dot{q}, \quad Q(q)J_\varphi^T(q) = 0. \quad (13)$$

Later in the sequel we will deal with the signal between current velocity $\dot{q}$ and nominal reference $\dot{q}_r$, i.e.,

$$s = \dot{q} - \dot{q}_r = Q(q)\{\Delta\dot{q} + \alpha\Delta q\} - \beta J_\varphi^T(q)\Delta F = s_0 + s_1 \quad (14)$$

where

$$\Delta\dot{q} = \dot{q} - \dot{q}_d, s_0 = Q(q)\{\Delta\dot{q} + \alpha\Delta q\}, s_1 = -\beta J_\varphi T(q)\Delta F. \quad (15)$$

It is important to note that the first term in the right hand side of eq.(14) is orthogonal to the second term by virtue of eq.(13). In other words, the nominal reference signal $\dot{q}_r$ is composed of two parts which are orthogonal to each other, and hence the residual signal $s(= \dot{q} - \dot{q}_r)$ is divided into tow parts s_0 and s_1 that are orthogonal to each other, too. The design concept of "nominal signal" by this idea is called in this paper "Orthogonalization Principle".

In order to show the effectiveness of introduction of the nominal reference signal on the basis of Orthogonalization Principle, we consider a control input that can be on-line computed in real-time by means of a modified computed torque method, which is defined by the form

$$u(t) = H(q)\ddot{q}_r + \left\{B_0 + \frac{1}{2}\dot{H}(q) + S(q,\dot{q}) + \xi(\|J_x(q)\dot{q}\|)J_x^T(q)J_x(q)\right\}\dot{q}_r + g(q) - J_\varphi^T(q)\{f_d - \gamma\Delta F\}, \tag{16}$$

where γ is a positive constant. Note that the real-time computation of the right hand side does not include the real-time computation of $\ddot{q}$ but includes the differentiation of ΔF in time t. This implicitly assumes the use of a force sensor for on-line measurement of the force signal $f(t)$. However, we do not use this for real-time compensation of the contact force $f(t)$ arising in the right hand side of eq.(1). The control law defined by eq.(16) is called in this paper a "Modified Hybrid Computed Torque Method". Substituting this into eq.(1) yields

$$H(q)\dot{s} + \left\{B_0 + \frac{1}{2}\dot{H}(q) + S(q,\dot{q}) + \xi(\|J_x(q)\dot{q}\|)J_x^T(q)J_x(q)\right\}s = J_\varphi^T(q)\{\Delta f + \gamma\Delta F\}. \tag{17}$$

where s is the difference vector between $\dot{q}$ and $\dot{q}_r$ defined by eq.(14). Taking an inner product between s and eq.(17) gives rise to

$$\frac{d}{dt}\left[\frac{1}{2}\left\{s^T H(q)s + \beta\Delta F^2\right\}\right] = -s^T\left\{B_0 + \xi(\|\dot{x}\|)J_x^T(q)J_x(q)\right\}s - \beta\gamma\Delta F^2. \tag{18}$$

This implies that the quadratic quantity

$$V(s,\Delta F) = \frac{1}{2}\left\{s^T H(q)s + \beta\Delta F^2\right\} \tag{19}$$

plays a role of Lyapunov's function. In further detail, eq.(18) implies the existence of a constant $c > 0$ such that

$$\frac{d}{dt}V(s(t),\Delta F(t)) \le -cV(s(t),\Delta F(t)), \tag{20}$$

for $H(q)$ is positive definite and periodic in components of q and B_0 is also positive definite. From eq.(20) it follows that

$$V(s(t),\Delta F(t)) \le V(s(0),0)\exp(-ct) \tag{21}$$

since $\Delta F(0) = 0$ by definition of $F(t)$ and $\Delta F(t)$. Thus, one concludes that

$$s(t) \to 0, \Delta F(t) \to 0, \quad \text{and} \quad s_0(t) \to 0 \quad \text{as} \quad t \to \infty. \tag{22}$$

The speed of convergence is also of the order of exponentially decreasing in time t.

Let us now show the convergence of $\Delta f(t) \to 0$ and $\Delta q(t) \to 0$ as $t \to \infty$ in the following if magnitudes of the initial differences $s(0)$ is not so large and both $\dot{q}_d(t)$ and$\ddot{q}_d(t)$ are bounded uniformly. Multiplying both sides of eq.(17) by $J_\varphi(q)H^{-1}(q)$, we obtain

$$J_\varphi(q)\dot{s} + J_\varphi(q)H^{-1}(q)G(q,\dot{q})s = J_\varphi(q)H^{-1}(q)J_\varphi^T(q)\{\Delta f + \gamma\Delta F\}, \tag{23}$$

where we put

$$G(q,\dot{q}) = B_0 + \frac{1}{2}\dot{H}(q) + S(q,\dot{q}) + \xi(\|J_x(q)\dot{q}\|)J_x^T(q)J_x(q). \tag{24}$$

From definition of s in eq.(14) and differentiation of the equality $J_\varphi s = -\beta\Delta F$, it follows that

$$J_\varphi\dot{s} = -\dot{J}_\varphi s - \beta\Delta f. \tag{25}$$

Thus, substituting this into eq.(23), we obtain

$$\left(\beta + J_\varphi H^{-1}J_\varphi^T\right)\Delta f = \left(J_\varphi H^{-1}G - \dot{J}_\varphi\right)s - \gamma J_\varphi H^{-1}J_\varphi^T\Delta F. \tag{26}$$

Since $s(t) \to 0$ as $t \to \infty$ and $\dot{q}_d(t)$ is bounded uniformly in t, $\dot{q}(t)$ is also uniformly bounded (in detail, see Theorem 1 and Appendix A). Hence $\dot{J}_\varphi(q)$ is also uniformly bounded. On the other hand, all entries of $H^{-1}(q)$ are trigonometric functions of q and $G(q,\dot{q})$ is linear in $\dot{q}$ and periodic in q. This implies that $\left(J_\varphi H^{-1}G - \dot{J}_\varphi\right)$ is bounded uniformly in t. Thus, from eq.(26) together with eq.(22), we conclude that

$$\Delta f(t) \to 0 \quad \text{as } t \to \infty. \tag{27}$$

More strictly, if $f_d(t)$ is bounded from below by a positive value $f_0 > 0$, i.e.,

$$f_d(t) \ge f_0 > 0 \tag{28}$$

and the magnitude of the initial residual vector $s(0)$ is as small as the inequality

$$f(t) = f_d(t) + \frac{1}{\beta + J_\varphi H^{-1}J_\varphi^T}\left\{\left(J_\varphi H^{-1}G - \dot{J}_\varphi\right)s - \gamma J_\varphi H^{-1}J_\varphi^T\Delta F\right\} > 0 \tag{29}$$

holds during an early stage of maneuvering, i.e., $t \in [0, t_1]$ for some positive value $t_1 > 0$, then $f(t)$ remains positive for all $t \in [0,\infty)$ because $V(s(t),\Delta F(t)) \le V(s(0),\Delta F(0))$ and $s(t) \to 0$, $\Delta F \to 0$ as $t \to \infty$.

Finally we show the following under an appropriate condition of smoothness of the surface $\varphi(q) = 0$:

Theorem 1. There is a positive constants $\eta > 0$ depending on the smoothness of the surface such that, for any initial discrepancy $\Delta q(0)$ and $\Delta\dot{q}(0)$ satisfying $\|\dot{q}(0)\|/\alpha + \|\Delta q(0)\| < \eta$, eq.(21) implies

$$\Delta\dot{q}(t) \to 0 \text{ and } \Delta q(t) \to 0 \text{ as } t \to \infty. \tag{30}$$

provided that $\alpha > 0$ is large enough satisfying $\alpha/2 \ge c_0 \cdot \sup_{t\ge 0}\|\dot{q}_d(t)\|$, where c_0 is a positive constant that depends on the smoothness of the surface and the minimum radius of curvatures of the surface (in further detail, refer to Appendix A).

The proof of this will be given in Appendix A.

Finally it should be remarked that the momentum error ΔF in the desired signal $\dot{q}_r$ and at the same time in the residual signal s can be excluded by letting $\beta = 0$ for the sake of showing only the convergence of Δq and $\Delta\dot{q}$. However, it should be also pointed out that if Δq and $\Delta\dot{q}$ converge to zero as t tends to infinity then it is also possible to show the convergence of ΔF and Δf in a similar way to the argument presented just before stating Theorem 1.

It is also important to see that, even in ordinary cases without geometric constraints, the conventional computed torque method defined by

$$u(t) = H(q_d)\ddot{q}_d + \left\{B_0 + \frac{1}{2}\dot{H}(q_d) + S(q_d,\dot{q}_d)\right\}\dot{q}_d + g(q_d) \tag{31}$$

dose not ensure the convergence of tracking errors as $t \to \infty$ if $\Delta q(0) \ne 0$ or $\Delta\dot{q}(0) \ne 0$. Instead of eq.(31), the modified method defined by

$$u(t) = H(q)\ddot{q}_r + \left\{B_0 + \frac{1}{2}\dot{H}(q) + S(q,\dot{q})\right\}\dot{q}_r + g(q) \tag{32}$$

ensures the convergence $s(t)(= \Delta\dot{q} + \alpha\Delta q) \to 0$ as $t \to \infty$ and hence the convergence $\Delta q(t) \to 0$ as $t \to \infty$, even if $\Delta q(0) \ne 0$ and $\Delta\dot{q} \ne 0$.

4. Orthogonalization Principle in Adaptive Hybrid Control

As discussed in the literature ([5],[9]~[11]), the design of model-based adaptive controllers is based on the property that the manipulator dynamics can be parameterized as a linear form of a parameter vector Θ whose components are functions of unknown or uncertain masses and moments of inertia of the links. In other words, the left hand side of the dynamic equation (1) together with a term of contact friction can be rewritten as follows:

$$\begin{aligned} Y(q,\dot{q},\dot{q},\ddot{q})\Theta &= H(q)\ddot{q} \\ &+\left\{B_0+\frac{1}{2}\dot{H}(q)+S(q,\dot{q})+\xi(\|J_x(q)\dot{q}\|)J_x^T(q)J_x(q)\right\}\dot{q} \\ &+g(q), \end{aligned} \tag{33}$$

where Y is a known matrix without depending on masses and inertia of the links. The first $\dot{q}$ in $Y(q,\dot{q},\dot{q},\ddot{q})$ denotes the $\dot{q}$ appearing linearly and homogeneously in $H(q)$ and $S(q,\dot{q})$ and the second represents the linear form outside the bracket $\{\}$. In design of an adaptive controller, we usually use a matrix $Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)$ instead of the last two terms $\dot{q}$ and $\ddot{q}$ in $Y(q,\dot{q},\dot{q},\ddot{q})$. In detail, let

$$\begin{aligned} Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)\Theta &= H(q)\ddot{q}_r \\ &+\left\{B_0+\frac{1}{2}\dot{H}(q)+S(q,\dot{q})+\xi(\|J_x(q)\dot{q}\|)J_x^T(q)J_x(q)\right\}\dot{q}_r \\ &+g(q), \end{aligned} \tag{34}$$

where vector $\dot{q}_r$ is the nominal reference signal defined by eq.(9). It is implicitly assumed that all q, $\dot{q}$, $\dot{q}_r$ and $\ddot{q}_r$ and hence $Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)$ are computable in real-time. If the frictional coefficient $\xi(\|\dot{x}\|)$ is unknown, it should be parameterized by unknown coefficients which can be put into the vector Θ of unknown parameters.

Let us design an adaptive control law by the form

$$u = Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)\hat{\Theta}+\tau_r, \quad \tau_r = J_\varphi^T(q)\{f_d-\gamma\Delta F\}, \tag{35}$$

where $\hat{\Theta}$ is an estimation at time t of unknown parameters Θ (See Fig.3). Substitution of eq.(35) into eq.(1) yields

$$\begin{aligned} &\{Y(q,\dot{q},\dot{q},\ddot{q})-Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)\}\Theta \\ &+Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)(\Theta-\hat{\Theta}) = J_\varphi^T(q)\{\Delta f+\gamma\Delta F\} \end{aligned} \tag{36}$$

which can be written in the form

$$Y(q,\dot{q},s,\dot{s})\Theta+Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)\Delta\Theta = J_\varphi^T(q)\{\Delta f+\gamma\Delta F\} \tag{37}$$

where

$$\begin{aligned} Y(q,\dot{q},s,\dot{s})\Theta &= H(q)\dot{s} \\ &+\left\{B_0+\frac{1}{2}\dot{H}(q)+S(q,\dot{q})+\xi(\|\dot{x}\|)J_x^T(q)J_x(q)\right\}s, \end{aligned} \tag{38}$$

and s is defined by eq.(14). Estimation $\hat{\Theta}$ of unknown parameters Θ is updated according to the adaptive law

$$\hat{\Theta}(t) = \hat{\Theta}(0)-\int_0^t \Gamma^{-1}Y^T(q,\dot{q},\dot{q}_r,\ddot{q}_r)s(\tau)d\tau \tag{39}$$

which implies

$$\frac{d}{dt}\Delta\Theta = \Gamma^{-1}Y^T(q,\dot{q},\dot{q}_r,\ddot{q}_r)s \tag{40}$$

where $\Delta\Theta = \Theta-\hat{\Theta}$, because the unknown parameter vector Θ is fixed and hence $d\Delta\Theta/dt = -d\hat{\Theta}/dt$.

Now, bearing in mind the forms of eq.(38) and (40), let us take an inner product of both sides of eq.(37) with s. This results in the following equality:

$$\begin{aligned} &\frac{1}{2}\frac{d}{dt}\left\{s^TH(q)s+\Delta\Theta^T\Gamma\Delta\Theta\right\} \\ &\quad +s^T\left\{B_0+\xi(\|\dot{x}\|)J_x^T(q)J_x(q)\right\}s \\ &= s^TJ_\varphi^T(q)\{\Delta f+\gamma\Delta F\} = -\beta\left\{\gamma\Delta F^2+\frac{1}{2}\frac{d}{dt}\Delta F^2\right\}, \end{aligned} \tag{41}$$

which is reduced to

$$\begin{aligned} \frac{d}{dt}V(t) = &-s^T(t)\left\{B_0+\xi(\|\dot{x}\|)J_x(q(t))J_x^T(q(t))\right\}s(t) \\ &-\beta\gamma\Delta F^2(t), \end{aligned} \tag{42}$$

where

$$V(t) = \frac{1}{2}\left\{s^T(t)H(q(t))s(t)+\Delta\Theta^T(t)\Gamma\Delta\Theta(t)+\beta\Delta F^2\right\}. \tag{43}$$

Since V is positive definite in s, $\Delta\Theta$, and ΔF and the right hand side of eq.(42) is negative definite in s and ΔF, eq.(42) implies

$$\lim_{t\to\infty} s(t) = 0 \text{ and } \lim_{t\to\infty}\Delta F(t) = 0 \text{ as } t\to\infty. \tag{44}$$

On the other hand, s is defined by eq.(14) and hence eq.(44) implies

$$s_0(t)\to 0 \text{ as } t\to\infty. \tag{45}$$

This leads to Theorem 1 again, though the proof of it should be devised more carefully than that given in Appendix.

Finally it should be noted that if $\Delta\Theta \to 0$ as $t\to 0$ then the convergence of Δf can be also proved as in section 3.

5. Learning Control under Geometric Endpoint Constraints

Consider now iterative learning for a class of robot tasks in which the manipulator endpoint is in touch with a surface $\varphi(q) = 0$ (see Fig.1). In [7], we propose the following learning update law:

$$u_{k+1}(t) = u_k(t)-\Phi Q(q_k(t))\Delta\dot{q}_k(t) \tag{46}$$

$$\sigma_{k+1}(t) = \sigma_k(t)+\psi\Delta f_k(t) \tag{47}$$

where $\Delta q = q-q_d$, $\Delta f_k = f_k-f_d$, and q_d and f_d are given desired position and force trajectories respectively. It can be assumed that the manipulator is governed by the equation of motion:

$$\begin{aligned} H(q)\ddot{q}+&\left\{B_0+\frac{1}{2}\dot{H}(q)+S(q,\dot{q})+\xi(\|\dot{x}\|)J_x^T(q)J_x(q)\right\}\dot{q} \\ &+g(q) = J_\varphi^T(q)f+v, \end{aligned} \tag{48}$$

$$v = u-A(q-q_d)-B_1\dot{q}+J_\varphi^T(q)\sigma. \tag{49}$$

That is, at the kth trial the manipulator's motion is subject to

$$\begin{aligned} H(q_k)\ddot{q}_k+&\left\{B+\frac{1}{2}\dot{H}(q_k)+S(q_k,\dot{q}_k)+\xi(\|\dot{x}_k\|)J_x^T(q_k)J_x(q_k)\right\}\dot{q}_k \\ &+A\Delta q_k+g(q_k) = J_\varphi^T(q_k)f_k+J_\varphi^T(q_k)\sigma_k+u_k \end{aligned} \tag{50}$$

where $B = B_0+B_1$. The problem is to prove the convergence of Δq_k and Δf_k with increasing k for the system of recursive equations of eqs.(46), (47), and (50). Note that in eq.(46) the velocity error signal is projected by a projection matrix $Q(q)$ onto the plane tangent to the surface $\varphi(q) = 0$.

Now, to derive a residual error equation for the manipulator, let

$$\begin{aligned} H(q_d)Q_d\ddot{q}_d+&\left(BQ_d+\frac{1}{2}\dot{H}(q_d)Q_d+H(q_d)\dot{Q}_d\right)\dot{q}_d \\ &+\left\{\xi(\|\dot{x}_k\|)J_x^T(q_d)J_x(q_d)+S(q_d,\dot{q}_d)\right\}Q_d\dot{q}_d+g(q_d) \\ &= J_d^Tf_d+Q_du_d+J_d^T\bar{\sigma}_d, \end{aligned} \tag{51}$$

$$\bar{\sigma}_d = J_d(u_d+J_d^T\sigma_d) \tag{52}$$

where $Q_d = Q(q_d)$. Subtracting these from eq.(50), we obtain the residual error dynamics

$$\begin{aligned} H(q_k)\Delta\ddot{q}_k+&\left(BQ_k+\frac{1}{2}\dot{H}(q_k)Q_k+H(q_k)\dot{Q}_k\right)\Delta\dot{q}_k \\ &+\left\{\xi(\|\dot{x}_k\|)J_x^T(q_k)J_x(q_k)+S(q_k,\dot{q}_k)\right\}Q_k\Delta\dot{q}_k \end{aligned}$$

$$+A\Delta q_k + h_k = \Delta u_k + J_k^T(\Delta\sigma_k + \Delta f_k) \quad (53)$$

where $\Delta u_k = u_k - Q_d u_d$, $Q_k = Q(q_k)$, $\Delta\sigma_k = \sigma_k - \bar{\sigma}_d$, and h_k is a remaining nonlinear term of $\Delta\dot{q}_k$ and Δq_k. By virtue of the orthogonalization principle, it is possible to prove (see [7]) that the residual error dynamics of eq.(53) is passive in the sense of exponential weighting for the output $y_k = Q_k\Delta\dot{q}_k$ versus residual input Δu_k. In detail it is shown (see [7]) that

$$\Delta\dot{q}_k^T Q_k \Delta u_k \geq \frac{1}{2}\frac{d}{dt}\left\{\Delta\dot{q}_k^T Q_k H(q_k) Q_k \dot{q}_k + \Delta q_k^T A \Delta q_k\right\} + \Delta\dot{q}_k^T Q_k (B - \rho_1 I) Q_k \Delta\dot{q}_k - \Delta q_k^T(\rho_0 + \gamma_0) I \Delta q_k \quad (54)$$

where ρ_0, ρ_1, and γ_0 are some positive constants. From eq.(54) it follows directly that

$$\int_0^t e^{-\lambda\tau}\Delta\dot{q}_k^T Q_k \Delta u_k d\tau \geq e^{-\lambda t}V(t) - V(0) + \int_0^t e^{-\lambda\tau}\left[\Delta\dot{q}_k^T\left\{\lambda Q_k H(q_k) Q_k + Q_k(B - \rho_1 I)Q_k\right\}\Delta\dot{q}_k + \Delta q_k^T\left\{\lambda A - (\rho_0 + \gamma_0)I\right\}\Delta q_k\right] d\tau. \quad (55)$$

where

$$V(t) = \Delta\dot{q}^T Q_k H(q_k) Q_k \Delta\dot{q}_k + \Delta q_k^T A \Delta q_k. \quad (56)$$

If $\lambda > 0$ is chosen so that $\lambda A \geq (\rho_0 + \gamma_0)I$, it holds that

$$\int_0^t e^{-\lambda\tau}\Delta\dot{q}^T Q_k \Delta u_k d\tau \geq -V(0) \quad (57)$$

which means the exponential passivity of error dynamics with respect to output $Q_k\Delta\dot{q}_k$ and input Δu_k. The convergence of $\Delta\dot{q}_k$ and Δq_k to zero as k increases follows immediately from inequality (55) (see [7]).

6. Conclusions

This paper proposes the so called "Orthogonalization Principle" for hybrid control of robot manipulators under geometric endpoint constraint. It is shown that by means of the concept of orthogonalization principle the design of control input for hybrid (position/force) control is reduced to a tractable method of signal decomposition and integration. In detail, firstly the feedback signal is decomposed into positional error signals (position and velocity) and force signals (force and momentum). The positional error signals are projected onto the plane tangent to the surface at the contact point by a projection matrix. The projection matrix can be computed in real time by referring only to the knowledge of present position of the manipulator and the equation of the surface. Hence, projection is executed in real time without use of further computation (no need of inertia-matrix conversion) and projected positional signals are naturally perpendicular to the normal force vector at the contact point. Finally the control input is generated by integration of these projected positional error signals and force and momentum error signals. This paper shows crucial roles of this orthogonalization principle in three basic control problems, 1) hybrid trajectory tracking problem by means of a modified hybrid computed torque method, 2) model-based adaptive control problem under geometric endpoint constraint, and 3) iterative learning control problem under the same situation.

Appendix A (Proof of Theorem 1)

First note that eq.(21) implies the existence of a constant $\gamma_0 \geq 1$ such that $\|s_0(t)\| \leq \|s(t)\| \leq \gamma_0\|s(0)\|\exp(-ct)$. It is also important to note that the inner product of s_0 and Δq yields

$$\frac{1}{2}\frac{d}{dt}\|\Delta q\|^2 = -\alpha\left\{\|\Delta q\|^2 - \|J_\varphi\Delta q\|^2\right\} + \Delta q^T J_\varphi^T(q)\left\{J_\varphi(q) - J_\varphi(q_d)\right\}\dot{q}_d + \Delta q^T s_0. \quad \text{(A-1)}$$

Since $\varphi(q)$ is smooth in the vicinity of $q_d(t)$, it is possible to assume reasonably as shown in Fig.A-1 that there is positive numbers $\eta_0 > 0$ and $c_0 > 0$ such that at any $t > 0$ the inequality $\|\Delta q(t)\| \leq \eta_0$ implies $|J_\varphi(q(t))\Delta q(t)| < \frac{1}{2}\|\Delta q(t)\|$ and $\|J_\varphi(q) - J_\varphi(q_d)\| \leq c_0\|\Delta q\|$. Substitution of this into eq.(A-1) yields

$$\frac{d}{dt}\|\Delta q\| \leq -\left(\frac{3}{4}\alpha - \frac{c_0 c_1}{2}\right)\|\Delta q\| + \|s_0\| \quad \text{(A-2)}$$

where $c_1 = \sup_{t\geq 0}\|\dot{q}_d\|$. If $\alpha \geq 2c_0c_1$, then it follows from eq.(A-2) that

$$\begin{aligned}\|\Delta q(t)\| &\leq \|\Delta q(0)\|e^{-\frac{\alpha}{2}t} + \int_0^t \gamma_0\left\{\|\dot{q}(0)\| + \alpha\|\Delta q(0)\|\right\}e^{\frac{\alpha}{2}(t-\tau)-c\tau}d\tau \\ &\leq \frac{2\gamma_0}{\alpha - 2c}e^{-ct}\left\{\|\Delta\dot{q}(0)\| + \alpha\|\Delta q(0)\|\right\} \quad \text{(A-3)}\end{aligned}$$

Since $\|s_0(t)\| \leq \gamma_0\|s_0(0)\|\exp(-\bar{c}t)$ if $0 \leq \bar{c} \leq c$, eq.(A-3) is valid even if c is replaced by any $\bar{c}$ satisfying $0 \leq \bar{c} \leq c$. Hence, by letting $c = 0$ in eq.(A-3) we obtain

$$\|\Delta q(t)\| \leq 2\gamma_0\left\{\|\Delta\dot{q}(0)\|/\alpha + \|\Delta q(0)\|\right\}. \quad \text{(A-4)}$$

Thus, by taking $\eta = \eta_0/2\gamma_0$, it follows that if $\|\Delta\dot{q}(0)\|/\alpha + \|\Delta q(0)\| < \eta$ then $\|\Delta q(t)\| \leq \eta_0$ for any $t \geq t_0$. Therefore, eq.(A-3) is valid for any $t \geq 0$, which proves the convergence $\Delta q(t) \to 0$ as $t \to \infty$ and, at the same time, $\Delta\dot{q}(t) \to 0$ as $t \to \infty$ from eq.(A-2).

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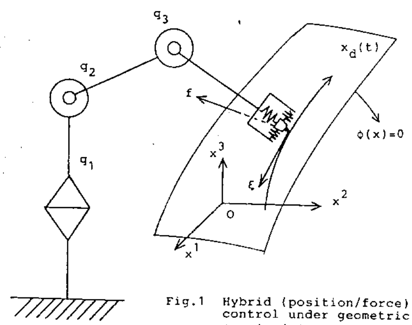

Fig.1 Hybrid (position/force) control under geometric constraint

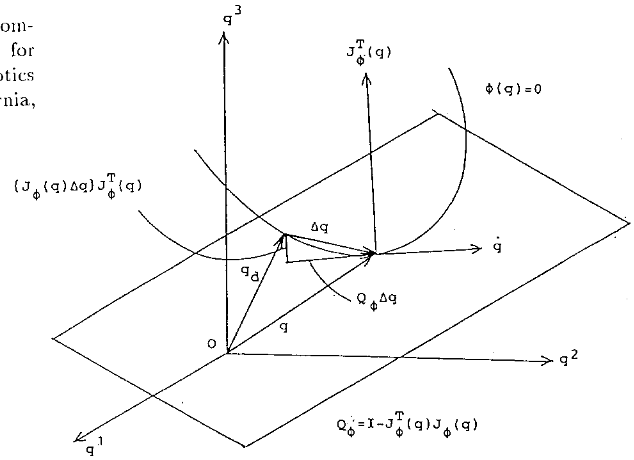

Fig.2 The vector $Q_\phi \Delta q$ is a projection of the vector Δq onto the plane perpendicular to the gradient vector $J_\phi^T(q)$ in joint space.

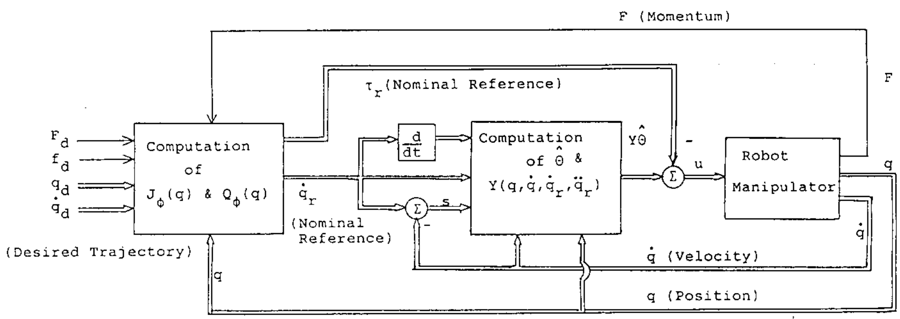

Fig.3 Model-based adaptive hybrid control

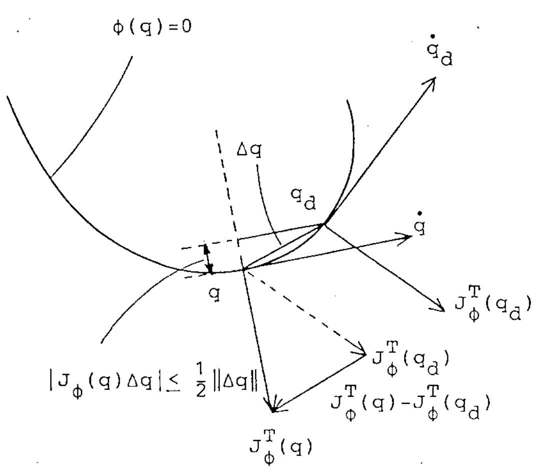

Fig. A-1