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Design of Fuzzy Logic Controller Based on Conflict-Inconsistent Rules

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ABSTRACT

Conflicting or inconsistent rules sometimes help us to represent the control actions of an expert more freely. Also, uncertainties about the control actions of the expert may render rules with conclusions whose membership functions have different width in their shapes. Conventional inference methods for FLC may not effectively handle such inconsistencies and/or rules containing such conclusions. In this paper, an effective inference method dealing with such If-Then rules is proposed.

I. Introduction

In representing control actions of an expert for a plant with fuzzy *If-Then rules*, there can be a situation in which rules are conflicting with each other[1][2]. For example, when the rule base originates from different sources of evidence or when we want to satisfy multiple objectives but can only obtain groups of rules with each group being designed to satisfy only one objective, inconsistent rules can be present[3].

Recall that, even if there are conflicting or inconsistent rules, the inference engine can run to fire rules at the same time with maximum compatibility if such inference method of Mamdani[4] is used. If two rules are fired as in Figure 1, the output value of the Mamdani's FLC is biased near the *more or less about three*. The *quite about one* can be considered to be more certain term than the *more or less about three*. We call this odd situation as a *fat shape domonant phenomenon*. Note that many conventional FLCs[5][6] contains such characteristics. It is our common understanding that certain conclusions should influence more on the output of the FLC than uncetain conclusions. In this paper, we first explain inconsistent rules, and then we propose an inference method that in which the above phenomenon does not occur.

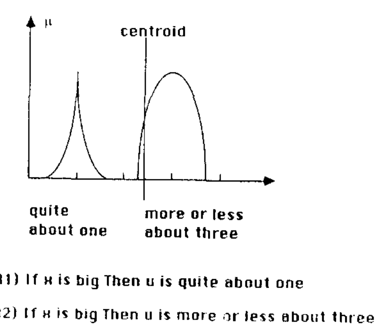


Fig.1. Fat shape dominant phenomenon.

II. Inconsistent rules

We define the inconsistent rules rather "fuzzily" as multiple rules which have "considerably" different consequents(Then-parts) but have "considerably" overlapped domains partitioned by the antecedents(If-part) of the rules.

In the If-Then rules, suppose the antecedents contains n variables while the consequents have only one variable. Then the If-Then rules can be considered as a kind of *look-up table* representing a function $f: R^n \to R$. Classical

If-Then rules can be thought as a look up table that have precise boundaries, whereas fuzzy if-Then rules can be thought as a look-up table that has ambiguous boundaries(see Figure 2). If we add a rule to the four consistent rules in Figure 2 as shown in Figure 3, the five rules can be considered to be inconsistent rules because their partitions by the antecedents are ovelapped. Therfore, inconsistent rules can be thought as a look up table whose partitions are laid to overlap with each other.

Inconsistent rules can be used as an alternative means to represent a control action. For example, the inconsistent rules of the R1 and R2 can be considered to yield a rule approximately equal to the R3.

R1) If (error is big) Then output is big

R2) If (error is big) Then output is small

R3) If (error is big) Then output is medium

As another example, the inconsistent rules in group 1 can be thought to be approximately equal to the rules in group 2 as illustrated in Figure 4.

(group 1)

if x is big Then u is big

if x is small Then u is small

if y is big Then u is big

if y is small Then u is small

(group 2)

if (x is big) and (y is big) Then u is big

if (x is big) and (y is small) Then u is medium

if (x is small) and (y is big) Then u is medium

if (x is small) and (y is small) Then u is big

There are situations in which the control actions of an expert are more easily described by the inconsistent rules. An example will be given in the simulation result in this paper.

III. Design of FLC

We discuss a design method for FLC considering the certainty of consequents of rules so that it is suitable for inconsistent rules.

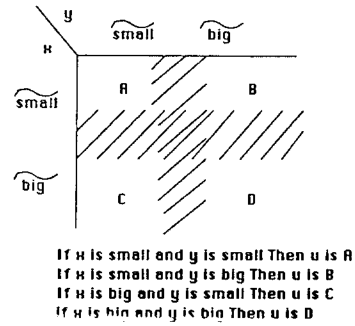


Fig.2. Fuzzy If-Then rules can be thought as a look up table with ambiguous boundaries.

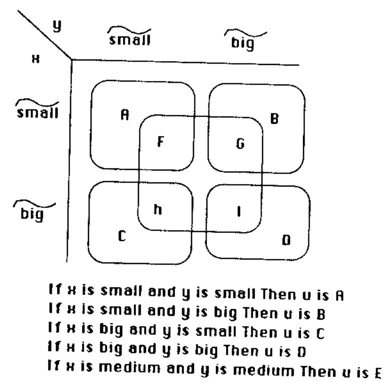


Fig.3. Inconsistent rules can be thought as a look up table whose partitions are laid to overlap each other.

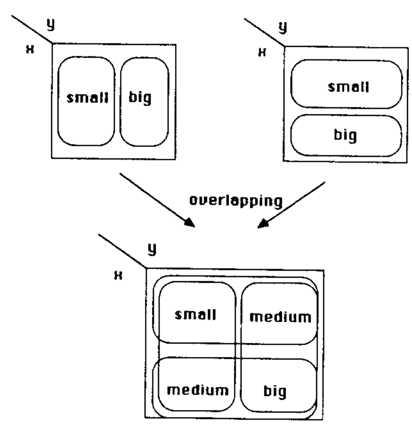


Fig.4. Inconsistent rules can be used as an alternative means to represent the control action.

First, we define a certainty measure in order to consider the certainty of consequents of rules. There are universes of discourse representing conclusions in which only one element should be chosen ultimately. FLC can be an example. Fat shaped fuzzy sets can be considered to be more uncertain than slim shaped fuzzy sets. Many works[7] discussed the method for measuring the fuzziness or the certainty of given fuzzy sets. But no attention is paid to whether the shapes of the fuzzy sets are fat or slim.

Definition 1: Measure of certainty

Let $X = [a, b] \subset R^1$ be the universe of discourse and F_x be the family of all fuzzy sets on X. Let the normalized size function $S_n : F_x \to [0, 1]$ be

$S_n(\tilde{A})=0$, if $\max\{\mu_{\tilde{A}}(x)\} = 0$

$S_n(\tilde{A}) = \int_a^b \mu_{\tilde{A}}(x)dx \;/\; \max\{\mu_{\tilde{A}}(x)\}\cdot(b - a)$ otherwise

The measure of certainty $m_c(\tilde{A})$ is defined as:

$$m_c(\tilde{A}) = \min\{\max\{\mu_{\tilde{A}}(x)\}, 1 - S_n(\tilde{A})\}$$

where $\mu_{\tilde{A}}(x)$ is the membership function of $\tilde{A} \in F_x$.

We find from the above definition that the normalized size of the membership function also influences on the certainty of the fuzzy set.

Let us define a metric on the fuzzy sets to use for the problem of combining conclusions. It may be desirable to define the distance between two fuzzy sets in such a manner that if the set of features of the two fuzzy sets, such as the centroid of the membership function and the measure of fuzziness, are close to each other, the distance between the two fuzzy sets are short.

Definition 2: Feature metric

Let F_x be the family of all fuzzy sets on which feature vectors $P_{\tilde{A}} = [p_A^1, p_A^2, \cdots, p_A^n]$, $P_{\tilde{B}} = [p_B^1, p_B^2, \cdots, p_B^n]$, can be defined. The feature metric $\rho_f(\tilde{A}, \tilde{B})$ is defined on F_x as follows:

$$\rho_p(\tilde{A}, \tilde{B}) = (\sum_{i=0}^{n} (p_A^i - p_B^i)^2)^{1/2}$$

where $\tilde{A}, \tilde{B} \in F_x$.

In case that conclusion alternatives described by fuzzy sets are issued by many conclusion sources, we discuss the problem of combining conclusions.

Problem of combining conclusions:

Let

X : universe of discourse of conclusion of finite interval in R^1.

F_x : family of all fuzzy sets on X.

m_c (certainty measure) : $F_x \to [0, 1]$ in R^1

ρ : feature metric defined on F_x. $F_x \to R^1$.

Given the family of conclusion alternatives from n conclusion sources

$C_a = \{c_1, c_2, \cdots, c_n\}$, $c_i \in F_x$, for $1 \le i \le n$,

the problem of combining conclusions is to find $c_f \in F_x$ which minimizes

$$J = \sum_{i=0}^{n} m_i(c_i)\cdot\rho^2(c_i, c_f) \qquad (1)$$

Here we find the solution of the problem of combining conclusions. Introducing the definition of feature metric into Equation (1), we obtain

$$J = \sum_{i=0}^{n} m_i(c_i)\cdot(\sum_{i=0}^{n} (p_{ci}^i - p_{cf}^i)^2)^{1/2} \qquad (2)$$

Let us find all the elements of the feature vector c_f which minimize (2). Because J is a quadratic function of p_{cf}^k, the solution of $\partial J/\partial p_{cf}^k = 0$ renders the value for a minimum J. Since

$$\partial J/\partial p_{cf}^k = \sum_{i=1}^{n} m_c(c_i)\cdot(\partial/\partial p_{cf}^k)(\rho_f^2(c_i, c_f))$$

$$= \sum_{i=1}^{n} m_c(c_i)\cdot(\partial/\partial p_{cf}^k)(p_{ci}^k - p_{cf}^k)^2$$

$$= -2\cdot\sum_{i=1}^{n} m_c(c_i)\cdot(p_{ci}^k - p_{cf}^k)$$

$$= -2\cdot\sum_{i=1}^{n} m_c(c_i)\cdot p_{ci}^k + 2\cdot\sum_{i=1}^{n} m_c(c_i)\cdot p_{cf}^k = 0$$

we find

$$p_{cf}^k = \sum_{i=1}^{n} p_{ci}^k\cdot w_i\cdot f_b(m_c(c_i)) \;/\; \sum_{i=1}^{n} w_i\cdot f_b(m_c(c_i)) \qquad (3)$$

The Equation (3) shows that the fuzzy set which minimizes the weghted sum of the squared feature metric to given fuzzy sets has the set of features whose members are the weighted algebraic means of the members of the feature vectors of given fuzzy sets.

Finally, we propose the *minimum distant inference method* to overcome the difficulty of *fat shape domonant* *phenomenon* in conventional FLCs. Considering the Mamdani's FLC, the difficulties arise in the operation applying the max-union to each deduced conclusion from each fired rule. Therefore, we replace the operation of max-unuon with the method of combining conclusions.

We apply the Equation (3) to FLC. For this, first, we simply choose only the centroid of membership function for a member of feature vector. By the definition of the features metric, $\rho_f(\tilde{A}, \tilde{B})$ is chosen as

$$\rho_f(\tilde{A}, \tilde{B}) = ((p_A - p_B)^2)^{1/2} = p_A - p_B$$

where p_A and p_B are the centroid of the membership functions of $\tilde{A}$ and $\tilde{B}$, respectively.

Consider the FLC with n inputs and one output. Suppose there are m rules. If the i'th rule with k antecedent conditions is given as

IF (u_1 is $\tilde{A}_1$) and (u_2 is $\tilde{A}_2$) $\cdot$ (u_k is $\tilde{A}_k$) THEN (Y is O_i)

where $u_1 \sim u_k$ are k inputs considered in the i'th rule and $A_1 \sim A_k$ are linguistic terms represented by fuzzy sets and Y is the output of the FLC and O_i is the consequent linguistic term of the i'th rule.

The compatibility of the i'th rule, J_i is defined as

$$J_i = \min\{\mu_{\tilde{A}1}(u_1), \mu_{\tilde{A}2}(u_2), \cdots \mu_{\tilde{A}k}(u_k)\}, \quad k \leq n .$$

The membership function μ_{ci} of the conclusion alternative from the i'th rule c_i is defined as

$$\mu_{ci} = \mu_{oi} \quad \text{if } \mu_{oi} \leq J_i, \quad \mu_{ci} = J_i \quad \text{if } \mu_{oi} > J_i .$$

Let p_{ci} be the centroid of c_i By the result of the method of combining conclusions, the feature (centroid) p_{cf} of the final conclusion c_f, that is to be the output of the FLC, can be written as

$$p_{cf} = \sum_{i=1}^{m} p_{ci} \cdot m_c(c_i) \,/\, \sum_{i=0}^{m} m_c(c_i)$$

Figure 5 illustrates an example of how the FLC with the *minimum distant inference method* calculates the output.

A noteworthy property of the *minimum distant inference method* compared to that of the Min-Max inference method is that if two inconsistent rules are given with maximum compatibility as in Figure 6, the Min-Max inference method with centroid defuzzification yields the output which is close to the centroid of the fat shaped fuzzy set. The *minimum distant inference method* yields output which is close to the centroid of the slim shaped one which can be said to be more certain.

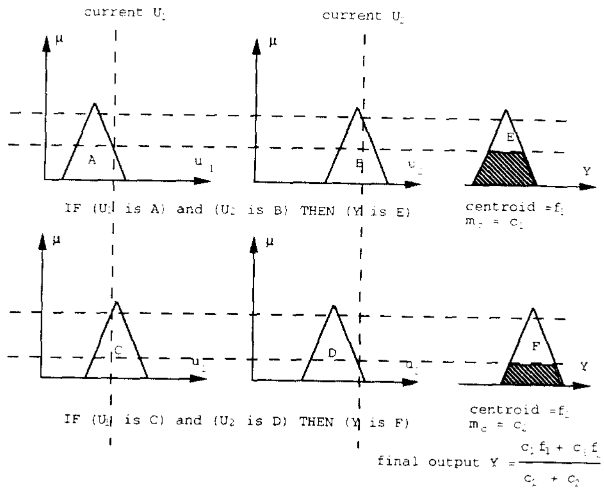


Fig.5. Minimum distant inference procedure.

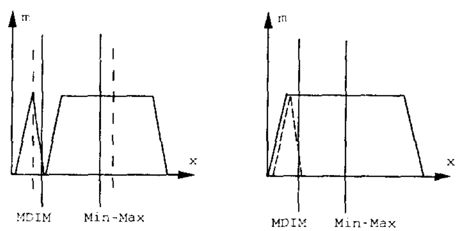


Fig.6. Comparison of the final outputs.

VI. Simulation Results

The overhead crane system shown in Figure 7 is used for transporting a load to a target position. This system can be described as

$$\ddot{\theta} = -(g/l)\cdot(\sin\theta + (a/g)\cdot\cos\theta), \quad a = \ddot{x} = f/M$$

f : input force (N)

a : acceleration of the trolley (m/sec^2)

θ : angle of the load (rad)

x : trolley position

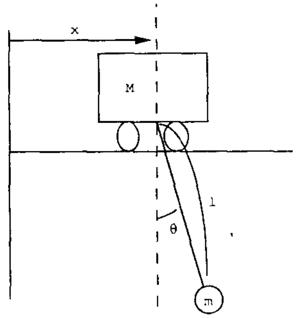


Fig.7. Over head crane system.

	NB	NM	NS	ZE	PS	PM	PB
NB	AZ	AZ	AZ	NB	AZ	AZ	AZ
NM	AZ	AZ	AZ	NM	AZ	AZ	AZ
NS	AZ	AZ	AZ	NS	AZ	AZ	AZ
ZE	AZ	AZ	AZ	ZE	AZ	AZ	AZ
PS	AZ	AZ	AZ	PS	AZ	AZ	AZ
PM	AZ	AZ	AZ	PM	AZ	AZ	AZ
PB	AZ	AZ	AZ	PB	AZ	AZ	AZ

(a) Rule table for reducing swing

	NB	NM	NS	ZE	PS	PM	PB
NB	PM	PLS	PLS	PM	AZ	AZ	AZ
NM	PLS	PLS	PNB	PS	AZ	AZ	AZ
NS	PLS	PNB	PNB	PVS	AZ	AZ	AZ
ZE	PB	PLS	PNB	ZE	NNB	MNS	NB
PS	PS	AZ	AZ	NVS	NNB	NNB	MNS
PM	AZ	AZ	AZ	NS	NNB	MNS	MNS
PB	AZ	AZ	AZ	NB	NNB	MNS	NB

(b) Rule table for positioning trolley

Fig.8. Inconsistent rules.

M : weight of the trolley

g : gravity constant (m/sec^2)

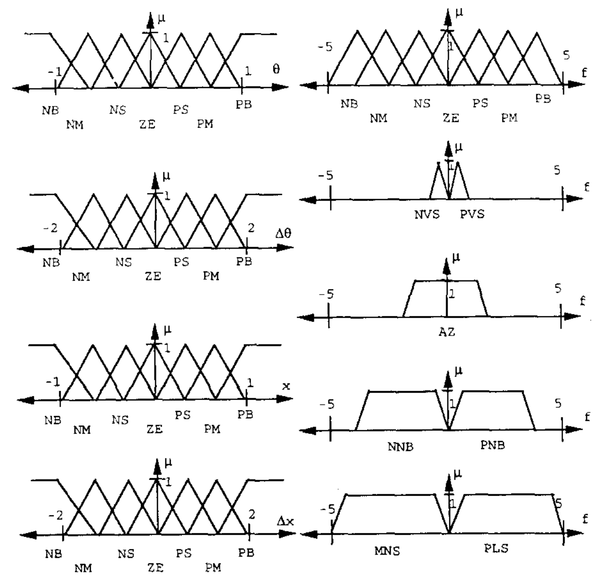
Assume $M = 1$, $l = 1$ and $g = 9.8$ for this case. The control objectives are two fold:

(i) Maintain the position error within a given bound at stop state.

(ii) Maintain swing as small as possible.

For this, we can utilize four variables such as angle of load, change of angle, trolley position, change of trolley position for control. Note that the control action for the objective (i) can cause the swing of the load and the control action for the objective (ii) can cause positioning error. It is very difficult for even skilled experts to describe the control rules satisfying the both objectives. However, obtaining rules for only one objective may be more easy. We obtain two groups of rules designed to satisfy only one objective(Figure 8). The two groups of rules can be said to be inconsistent rules, because the rules are laid to overlap each other in four variable look up table. Also, because of the uncertainty in describing the control action, the fuzzy sets in the consequents of the rules have different width in their shapes as shown in Figure 9 (b). When we could not describe the control action precisely, we use the fuzzy sets with fat shapes such as *MNS*, *PLS* and when we could describe the control action precisely, we use the fuzzy sets with slim shapes such as *NVS*, *PVS*.

The Min-Max inference method is chosen to be compared with the *minimum distant inference method*. In case



(a) Fuzzy sets for antecedents. (b) Fuzzy sets for consequents.

Fig.9. Membership functions for linguistic terms.

that one group of rules is applied to both of inference methods, both the *minimum distant inference method* and the Min-Max inference method satisfy the objective. When two groups of rules are simultaneousely applied, the Min-Max inference method satisfies neither of the two control objectives of anti swing and exact positioning, while the *minimum distant inference method* satisfies all the control objectives(see Figure 10 thru 12).

V. Conclusions

In order to overcome the *fat shap domonant phenomenon* in the conventional inference methods, we have proposed the method of combing conclusions. Also, we have

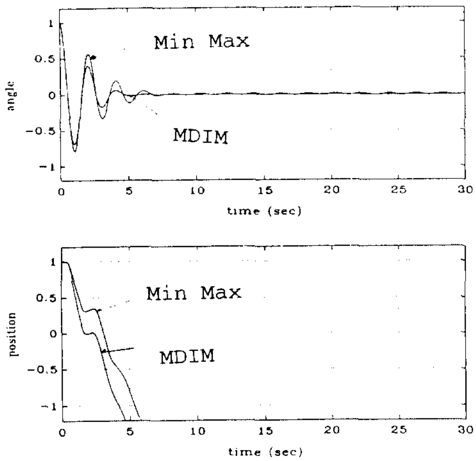


Fig.10. Responses for antiswing rules.

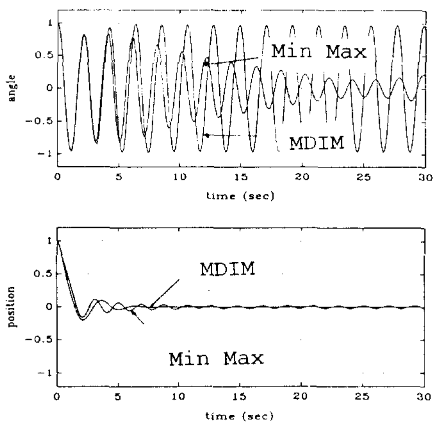


Fig.11. Responses for positioning rules.

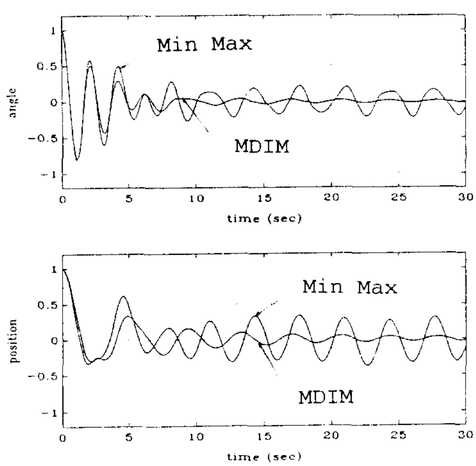


Fig.12. Responses for both groups of rules.

applied it for the *minimum distant inference method* for FLC with success.

The conventional inference methods may not effectively deal with situations in which there are rules having fuzzy sets with different width in their consequents, and more worse when there are also inconsistent rules. By contrast, the *minimum distant inference method* can cope with these situations well. We find that the *minimum distant inference method* can be better in many situations than conventional inference methods, specially in an uncertain environment where inconsistent rules and rules having fuzzy sets with different width in their consequents exist.

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