THE GENERALIZED METHODOLOGY OF SIGNAL DETECTION IN NOISE

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It is reported on the methodology of signal detection in noise which is based on a comparision of statistical parameters of observation sample from region of frequency-time noise space where a signal may be present and observation sample from region of this noise space and it is known a priori about the latter that the signal is absent in this region.

It is suggest that it is exist the frequency-time noise space Z where a signal may be present. The classical task of signal detection is considered in the following manner. It is necessary to control the hypothesis H_0 that the observing process is normal with zero mean value versus the alternative H_1 that the same process is also normal but with a time-variant mean value a(t). In statistical terms the problem is solved in the following manner: noncorrelated with each other coordinates $X_i=\sqrt{\lambda_i}\int_0^T X(t)\Phi_i(t)dt$ are taken as the elements of observing statistic, where X(t) is the realization of a adopted process on the observation interval (0,T), λ_i and $\Phi_i(t)$ are eigennumbers and eigenfunctions of integral equation $\Phi(t)=\lambda\int_0^T R(v-t)\Phi(v)dv$, $0<t<T$, and $R(\tau)$ represents a known correlation function of the noise. Normally, only the N first coordinates are taken. Then for the observing sample $X_1,\ldots,X_N$, likelihood function of random values X_i having a zero mean could be written (here and in the further writing, the variance is assumed to be to one) as

$$f_{X|H_0}(X|H_0)=\frac{1}{(2\pi)^{N/2}}\exp\left(-0.5\sum_{i=1}^{N}X_i^2\right), \quad (1)$$

what corresponds to the absence of signal in the observing sample. For the sample with nonzero mean value a(t) defined as

$$X_i=\xi_i+a_i=\sqrt{\lambda_i}\int_0^T[\xi(t)+a(t)]\Phi_i(t)dt, \quad (2)$$

where $\xi(t)$ is the normally distributed noise with zero mean value. Then likelihood function in the presence of signal in the sample X_i will be

$$f_{X|H_1}(X|H_1)=\frac{1}{(2\pi)^{N/2}}\exp\left[-0.5\sum_{i=1}^{N}(X_i-a_i)^2\right]. \quad (3)$$

Using (1) and (3), we can write a ratio of likelihood functions

$$\frac{f_{X|H_1}(X|H_1)}{f_{X|H_0}(X|H_0)}=\frac{\exp\left[-0.5\sum_{i=1}^{N}(X_i-a_i)^2\right]}{\exp\left(-0.5\sum_{i=1}^{N}X_i^2\right)}= \dots$$

$$= \exp\left(\sum_{i=1}^{N} X_i a_i - 0.5\sum_{i=1}^{N} a_i^2\right) = \ell(X_1, \dots, X_N) = C. \quad (4)$$

Taking the logarithm of (4), we obtain

$$\sum_{i=1}^{N} X_i a_i \underset{H_0}{\overset{H_1}{\gtrless}} K_N, \quad K_N = \ln C + 0.5\sum_{i=1}^{N} a_i^2 . \quad (5)$$

Making N approaching to ∞ and passing to an integral form of writing, we obtain [1]

$$\int_0^T X(t)a(t)dt > K_N = \ln C + 0.5E_a . \quad (6)$$

where $E_a = \int_0^T a^2(t)dt$ is the signal energy. Therefore, algorithms of signal detection are reduced to the calculation of $\sum_{i=1}^{N} X_i a_i$ or $\int_0^T X(t)a(t)dt$ and a comparision with the threshold K_N. This kind of signal detection is supposed to be optimal by the following selected quality criteria: Bayesian criterion, including as particular cases of maximizing the posterior probability and the likelihood; Neyman-Pirson criterion; minimax criterion [1,2].

By analysing the detection algoritm (6), an indicator determing the noise immunity is obtained. The essence of this analysis comes from the substitution of real value $X(t)=\xi(t)+a(t)$ (hypothesis H_1) and $X(t)=\xi(t)$ (hypothesis H_0) in (6):

$$\int_0^T X(t)a(t)dt = \int_0^T a^2(t)dt + \int_0^T a(t)\xi(t) : H_1,$$

$$\int_0^T X(t)a(t)dt = \int_0^T a(t)\xi(t)dt : H_0.$$

where the $\int_0^T a^2(t)dt$ is the signal energy and $\int_0^T a(t)\xi(t)dt$ is the noise component with zero expectation and finite variance defined as $(T \to \infty)$

$$M\left[\int_0^{\infty} a(t)\xi(t)dt\right]^2 = \int_0^{\infty} dt \int_0^{\infty} a(t)a(s)M[\xi(t)\xi(s)]ds = \frac{E_a N_0}{2}. \quad (7)$$

A signal-to-noise ratio is considered as a qualitative indicator of algorithm (6) for the determinated signal mixed additively with the noise:

$$q = \sqrt{2E_a/N_0} . \quad (8)$$

This indncator is one of the significant definition of the detector precision and, therefore, together with another factors, determines the noise immunity.

It is known that sufficient statistic of mathematical expectation $\sum_{i=1}^{N} X_i$ and sufficient statistic of variance $\sum_{i=1}^{N} X_i^2$ are the jointly sufficient statistics, defining a normal distribution of random value X. In (4) the sufficient statistic $0.5\sum_{i=1}^{N} X_i^2$ of likelihood functions $f_{X|H_1}(X|H_1)$ and $f_{X|H_0}(X|H_0)$ are cacelled. It is really so in the form of writing and according to the assumptions of probability theory. But in a physical sense this causes some tactlessness. The fact is that signal presence is implied in the numerator (4) (mean value a_i of observing sample X_i is not equal to zero), and its absence is implied in denominator of (4), while observing the same coordinate value. In the presence of the one sample X_i another approach is hard to assume. A question arises: can the algorithm of signal detection in additive mixture with the noise be synthesized without loss of sufficient variance statistic, defining the distribution? Another factor which picks up author's ears consists in that signal detection is made on the background of noise component $\int_0^T a(t)\xi(t)dt$ (6), raised by interaction of signal (its model, if more strictly) and input noise. Owing to this interaction the variance of noise component is directly proportional to the signal energy (energy of signal model), as it follows from (7). The consequence of this is the fact that qualitative indicator of the algorithm (6), defined by (8), is not directly proportional to the ratio $2E_a/N_0$ but is proportional to the square root of $2E_a/N_0$. Is this good or bad? With $2E_a/N_0 < 1$ it would be good, but for q<1 when the probability of false alarm is equal, for example, 10^{-3}, the probability of correct detection doesn't exceed 0.0027, this is practically a non-working region. With q>1, the probability of correct detection is less than that for the directly proportional dependence E_a/N_0. This conclusion seems to be unusual, but it is real, what will be seen from the latter text.

Analysing (6), it can be noted that this algorithm is adopted as optimal under the following conditions:
1). Likelihood ratio of functions (or functionals) is taken using one sample, where a numerator assumes signal presence and denominator assumes its absence. With the square-law statistic reduced, additional information is lost. The obtained expression provides the calculation only of sufficient statistic of observing process expectation. 2). In a theoretical plan the obtained algorithm is not realizable due to the following reasons: a) the absence of the signal, which we want to detect, in the reception point (in the absence of signal in the observal sample $z(t)=0$ or $z_i=0$, a left part of (5) and (6) becomes zero, and any physical sense is lost); b) calculation of threshold value K_N is not possible because of the absence of signal energy value in the reception point. In a practical plan this algorithm is realizable if the signal structure in the reception point is replaced by its analog (model), and instead of the obtained value it is defined by Neyman-Pirson criterion, for example. 3) In the case of replacement of signal structure by its model in the reception point, a noise component occures, caused by noise and signal model interaction. 4) Variance of the noted noise component is proportional to the energy of signal model. 5) Algorithm (5) or (6) doesn't allow to obtain in a clear form the direct proportional ratio of signal-to-noise energy characteristics. This results in that detection responses are the square root function of signal-to-noise energy characteristics ratio. 6) Algorithm (5) or (6) doesn't support the detection of signal whose structure doesn't correspond to the structure of signal model in the reception point. 7) In a general form, consider that the moment of signal appearance relatively the selected zero on the time axis is unknown, a detector synthesized according to this algorithm, must represent a tracker but not a pure detector.

By force of stated here we assume that there are two independent frequency-time noise spaces Z and Z^* which submit to the same distribution with the same statistical parameters. In the Z space as before there are regions in which signal may be present, in Z^* space signal is absent what is known a priori. In the following this space will be named as reference and, hence, the sample from this space will be also reference. It is required to accept a decision about the signal presence or absence in the sample from Z space by comparing the statistical parameters of this sample distribution with the statistical parameters of reference sample distribution.

The formulated problem must be solved using a statistical decision theory. So, it is necessary to collect the statistical data which determine the distribution statistical parameters of samples from Z and Z^* spaces and to compare them. If the statiatical parameters of two samples are the same or differ from each other with a given precision a decision is made about signal absence in the sample from Z space. If the distribution statistical parameters of sample from the Z space differ from the statistical parameters of the reference sample on a value which exceeds the precision given before then a decision about signal presence in the Z space is made.

Now the whole problem consists in the obtaining a jointly sufficient statistic for definition of distribution statistical parameters. It is known [1,2] that sufficient statistic is defined from the condition of likelihood function extremum. The condition of likelihood function extremum by defining parameter with a given precision in a general case takes the following form $\partial f_N(X_1,\dots,X_N,\theta)/\partial\theta=0$ where N is a sample size which defines a given precision, θ is a defining parameter. However, this equation is not used practically. Simple mathematical technique simplifies the representation of this equation. Since logarithm is a monotonic function the extremums of functions $f_N(X_1,\dots,X_N)$ and $\ln f_N(X_1,\dots,X_N)$ are achieved at the indentical values of θ parameter. Therefore, the likelihood equation is written usually in the following form

$$\partial \ln f_N(X_1,\dots,X_N,\theta)/\partial\theta = 0 \qquad (9)$$

Using (3) and (9) as shown in [2] it is consequent that $\sum_{i=1}^{N} X_i\xi_i$ and $\sum_{i=1}^{N} X_i^2$ are the jointly sufficient statistics of likelihood function (3) parameters for the sample X_i. Likelihood function for the reference sample $\eta_1,\dots,\eta_N$ has a following form

$$f_N(\eta_1,\dots,\eta_N)=\frac{1}{(2\pi)^{N/2}}\exp\left(-\sum_{i=1}^{N}\eta_i^2\right), \qquad (10)$$

where $\sum_{i=1}^{N}\eta_i^2$ is a sufficient statistic of likelihood function parameters for η_i sample.

While definition of sufficient statistics with samples X_i and η_i a problem of their comparision arises. A difference device is used usually for this purpose (Fig.1).

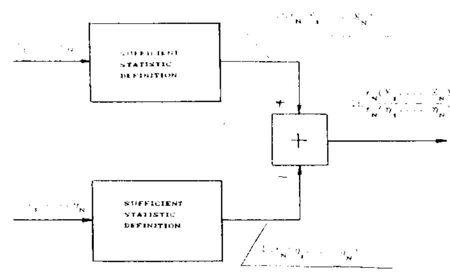

Fig.1.

The resulting sufficient statistic will

$$\ln f_X(X_1,\dots,X_N)-\ln f_\eta(\eta_1,\dots,\eta_N)=$$
$$=0.5\left(\sum_{i=1}^{N}2X_i\xi_i-\sum_{i=1}^{N}X_i^2+\sum_{i=1}^{N}\eta_i^2-\sum_{i=1}^{N}\xi_i^2\right), \qquad (11)$$

be observed on the output of difference device. Last member of the right part of (11) is usually concerned as threshold value which, as in (5), is independent from the sample. Expression (11) obtained while the definition of resulting sufficient statistic represents a logarithm of likelihood function ratio from which an logarithm of signal detection with two independent samples follows one of these samples is reference sample with a priori information about signal absence in it

$$\ln f_X(X_1,\dots,X_N) - \ln f_\eta(\eta_1,\dots,\eta_N) =$$
$$= \ln f_X(X_1,\dots,X_N) - \ln f_\eta(\eta_1,\dots,\eta_N) =$$
$$= 0.5\left(\sum_{i=1}^{N} 2X_i a_i - \sum_{i=1}^{N} X_i^2 + \sum_{i=1}^{N} \eta_i^2 - \sum_{i=1}^{N} a_i^2\right) = \ln C,$$
$$\sum_{i=1}^{N} 2X_i a_i - \sum_{i=1}^{N} X_i^2 + \sum_{i=1}^{N} \eta_i^2 = 2\ln C + \sum_{i=1}^{N} a_i^2 = K_N^*.$$

Hence, proceeding from a generally accepted notions it follows that hypothesis H_1 about signal presence in X_i sample is adopted if the following inequality is satisfied

$$\sum_{i=1}^{N} 2X_i a_i - \sum_{i=1}^{N} X_i^2 + \sum_{i=1}^{N} \eta_i^2 > K_N^*, \qquad (12)$$

and the hypothesis H_0 about signal absence in X_i sample is adopted if the reverse inequality is satisfied. It can be stated that the algorithm (12) is reduced to the collection of resulting jointly sufficient statistic $\sum_{i=1}^{N}(2X_i a_i - X_i^2 + \eta_i^2)$ and its comparision with K_N^* threshold.

Let N→∞ and transiting to an integral form we obtain

$$2\int_0^T X(t)a(t)dt - \int_0^T X^2(t)dt + \int_0^T \eta^2(t)dt > K_N^*. \quad (13)$$

The basis of the algorithm (13) analysis made in the same procedure shows that while substitution in (13) instead of $X(t)$ its value in the condition of signal presence equal to $a(t)+\xi(t)$ we obtain

$$2\int_0^T [a(t)+\xi(t)]a(t)dt - \int_0^T [a(t)+\xi(t)]^2 dt +$$
$$+ \int_0^T \eta^2(t)dt = \int_0^T a^2(t)dt - \int_0^T \xi^2(t)dt + \int_0^T \eta^2(t)dt \to H_1$$

or $\int_0^T \eta^2(t)dt - \int_0^T \xi^2(t)dt \to H_0$, where $\int_0^T a^2(t)dt = E_a$ is the signal energy, $\int_0^T \eta^2(t)dt - \int_0^T \xi^2(t)dt$ is the background noise. It is necessary to note that in statistical sense $\int_0^T \eta^2(t)dt - \int_0^T \xi^2(t)dt \to 0$ (T→∞) since the processes $\xi(t)$ and $\eta(t)$ are uncorrelated between each other and have the same spectral density of power equal to N_0 I (in ...).

It is proved that using the statistical decision theory the algorithm of signal detection both with the sample X_i and with two uncorrelated samples X_i and η_i has the same approach and is defined from the likelihood function ratio. The difference is in that numerator and denominator of the detection algorithm (5) contain the same sample (4) when it is assumed that signal is present in numerator and absent in denominator. In the algorithm (12) numerator contains a sample in which signal presence is possible and denominator contains a sample η_i in which it is known a priori that signal is absent in it. Resulting from the stated here it can be marked that the algorithm (5) uses only sufficient statistic $\sum_{i=1}^{N} X_i$ for the definition of expectation and the algorithm (12) uses jointly sufficient statistic $\sum_{i=1}^{N} 2X_i a_i$ for the definition of expectation and sufficient statistic $\sum_{i=1}^{N}(\eta_i^2 - X_i^2)$ for the definition of likelihood function variance what provides more complete information at decision making comparing with algorithm (5). Heuristic synthesis of the algorithm (12) or (13) was made in [3]. The algorithm (12) or (13) is free from i.1, first part of i.2, ii. 3-6 of conventions which are peculiar to the algorithm (5) or (6) and under it is as optimal.

The physical interpretation of the algorithm (12) or (13) consists in following:

1)A filter F_2 can be a source of sample η_t which is mismatched on central frequency relatively to the carrier signal frequency on a value which can be defined on the basis [4] dependently on a concrete practical situation. 2)A linear input of transducer (filter F_1) is a source of sample ξ_t. A bandwidth of the filter F_1 corresponds to the effective bandwidth of signal (filter F_2 on the bandwidth is matched with filter F_1). 3)The first member of the algorithm (12) or (13) corresponds to the synthesis of correlation channel with doubled gain factor. 4)The second member of the algorithm (12) or (13) corresponds to the synthesis of detector autocorrelation channel related with filter F_1. 5)The third member of the algorithm (12) or (13) corresponds to the synthesis of detector autocorrelation channel related with filter F_2. 6)The statistic of detector autocorrelation channel related with filter F_1 is subtracted from statistic of detector autocorrelation channel related with filter F_2 what results in achieving $\int_0^T \eta^2(t)dt - \int_0^T \xi^2(t)dt$ $(T \to \infty)$ in the statistical sense. 7)Statistic of autocorrelation channel is subtracted from statistic of correlation channel what results in the full compensation of noise component $\int_0^T a(t)\xi(t)dt$ in the statistical sense typical to the algorithm (5) or (6).

On a base of the offered physical interpretation of algorithm (12) or (13)

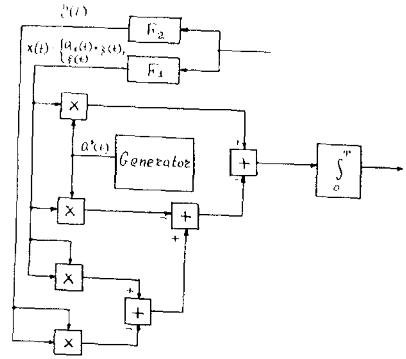

Fig.2.

a functional diagram of detector presented on Fig.2 was synthesized. The experimental investigations of signal detection algorithm (12) or (13) [5-8] are shown that this algorithm is more informated than signal detection algorithm (5) or (6).

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