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Design of Hovering Flight Controller for a Model Helicopter

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ABSTRACT

This paper describes a procedure to design a hovering flight controller for a model helicopter using LQG theory. Parameters of the model helicopter in hover are obtained using direct measurements and calculations proposed by other research. A feedback controller is designed by using digital LQG theory. First, a full state feedback controller is designed to the discretized system taking desirable transient response and other assumptions into account. Then a full-state estimator is designed and revised until desirable response is obtained while global stability is maintained. Performance of the controller is tested by computer simulations. Experiments have been performed using a 3-degree-of-freedom gimbal that holds the model helicopter, and the controller exhibited stable hover capability.

Nomenclature

A = $n \times n$ system matrix
A_1 = lateral cyclic pitch angle
B = $n \times r$ input matrix
B_1 = longitudinal cyclic pitch angle
C = $m \times n$ output matrix
I = identity matrix
J = quadratic performance index
K_c = controller feedback gain matrix
K_f = estimator feedback gain matrix
m = number of system output
N = process noise covariance matrix
n = order of system (9th)
Q = $n \times n$ weighting matrix
P_c = solution of algebraic Riccati equation for controller
P_f = solution of algebraic Riccati equation for estimator
p,q,r = roll, pitch, yaw rate
R = $r \times r$ weighting matrix
r = order of the control vector
t = time
V = process noise covariance matrix
X = n-dimensional state vector
ϕ,θ,ψ = angle of roll, pitch, yaw
Γ = square root of measurement noise covariance matrix
θ_M = main rotor collective pitch angle
θ_T = tail rotor collective pitch angle
ω_{cf} = corner frequency of servo

Subscripts

F = fuselage
H = horizontal stabilizer
M = main rotor
T = tail rotor
V = vertical stabilizer
c = controller
f = estimator
k = discrete time index
x,y,z = x,y,z direction

1. Introduction

Since 1940s, helicopters have been widely used due to their superior movement ability such as vertical take-off and hovering. In unmanned applications, some helicopter-type RPV's (Remote Pilot Vehicles) have been implemented and they have been proven to be superior to fixed-wing RPV's in versatile flight ability.

Helicopters show inherently unstable control characteristics without any contoller. So most helicopters are equipped with SCAS (Stability and Control Augmentation System) so that they might satisfy the required handling quality[1].

As for the model helicopters, they show almost the same flight capabilities as the real one's; however their response is faster due to their small inertial mass. To compensate for this, most model helicopters are equipped with a rate gyro to assist yaw-direction motion and mechanical stabilization system such as Bell-Hiller stabilizer to increase stability and response time. With these aids, however, hovering is still a difficult thing to do and considerable time and efforts are consumed to get used to it.

Since the longitudinal motion and lateral motion of a helicopter are quite significantly coupled with each other compared with fixed-wing aircrafts, controllers designed by the classical control theory for SISO systems to control each directions independently cannot be applied to helicopters. So, modern multivariable control theory was introduced early. In 1960s, Murphy and Narendra[2] applied LQR theory to design a hover controller. Since 1980, multivariable robust control theories have been developed and applied to helicopter flight controller to enhance the handling qualites[3,4,5]. Studies on model helicopters also have been carried out in recent years. In Purdue University[6], variable structure control theory, neural nets and nonlinear control theory are applied to control the vertical flight mode alone. Furuta[7] designed a hover controller using LQR model-following theory.

In this paper, state equation of the model helicopter is developed and a LQG controller is designed and implemented on a digital computer. The performance of the controller is verified and then applied to experiments on a model helicopter installed on 3-DOF gimbal.

2. Mathematical Model

The flight motion of a helicopter is 6 degrees of freedom. In this research, body axes system attached to the center of mass of the helicopter is used to describe the 6 DOF motion. A free body diagram of flying helicopter using this scheme is shown in Figure 1. 6 equations of motion is derived when writing down the force and momentum factors according to each directions as Eq. (1)[8].

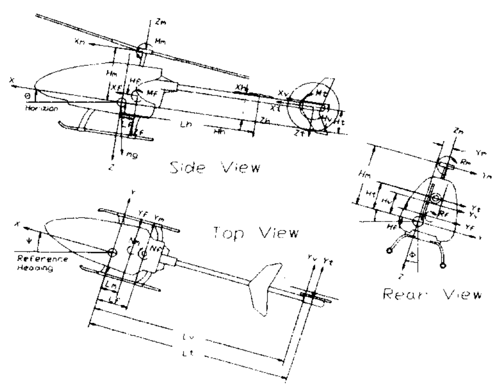

Figure 1. Flight motion axes system and Free-body diagram

$$X_M+X_T+X_H+X_V+X_V=mg\sin\theta+m(\ddot{x}-\dot{y}r+\dot{z}q)$$

$$Y_M+Y_T+Y_V+Y_F=-mg\sin\phi+m(\ddot{y}+\dot{x}r-\dot{z}p) \qquad (1)$$

$$Z_M+Z_T+Z_H+Z_V+Z_F=-mg\cos\theta+m(\ddot{z}-\dot{x}q+\dot{y}p)$$

$$R_M+Y_Mh_M+Z_My_M+Y_Th_T+Y_Vh_V+Y_Fh_F+R_F=I_{xx}\dot{p}-qr(I_{yy}-I_{zz})$$

$$M_M-X_Mh_M+Z_Ml_M+M_T-X_Th_T+Z_Tl_T-X_Hh_H+Z_Hl_H-X_Vh_V+M_F$$
$$+Z_Hl_H-X_Vh_V+M_F=I_{yy}\dot{q}-pr(I_{zz}-I_{xx})$$

$$N_M-Y_Ml_M-Y_Tl_T-Y_Vl_V+N_F-Y_Fl_F=I_{zz}\dot{r}-pq(I_{xx}-I_{yy})$$

In Eq. (1), only rigid body motions are considered while the periodic motions, such as flapping or coning effects that occur during main rotor's rotation, are ignored. In fact, the main rotor blades which flap in every rotation cannot be regarded as rigid bodies, but their dynamic characteristic is much faster than system's, which justifies the assumption of rigidity.

To apply the linear time-invariant(LTI) control theory, nonlinear equations in Eq. (1) are expanded in Taylor's series. Since the motion equations represent the hover, initial velocities and mean velocities are set to zero. Under the assumption that the roll and pitch angles are not large, the trigonometric terms can be reduced to linear ones.

Applying the above assumptions, 6 linearized equations are obtained as Eq. (2). From these, a set of state equations with 8 state variables and 4 control input variables are established. In this equation, engine throttle input is not included as a control input variable, for the engine speed is assumed to be controlled at a constant speed by a independent governor. If the engine speed varies radically, all of the parameters in the state equation varies along and the system cannot be considered as a time invariant system any more. Therefore, it is difficult to apply the LTI control theory.

$$\dot{X}=AX+BU$$

$$X=[V_x\ V_y\ V_z\ \phi\ p\ \theta\ q\ r]^T \qquad (2)$$

$$U=[\theta_M\ \theta_T\ A_1\ B_1]^T$$

$$A=\begin{bmatrix} \frac{1}{m}\frac{\partial X}{\partial \dot{x}} & \frac{1}{m}\frac{\partial X}{\partial \dot{y}} & 0 & 0 & \frac{1}{m}\frac{\partial X}{\partial p} & -g & \frac{1}{m}\frac{\partial X}{\partial q} & 0 \\ \frac{1}{m}\frac{\partial Y}{\partial \dot{X}} & \frac{1}{m}\frac{\partial Y}{\partial \dot{y}} & 0 & g & \frac{1}{m}\frac{\partial Y}{\partial p} & 0 & \frac{1}{m}\frac{\partial Y}{\partial q} & \frac{1}{m}\frac{\partial Y}{\partial r} \\ 0 & 0 & \frac{1}{m}\frac{\partial Z}{\partial \dot{z}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \frac{1}{I_{xx}}\frac{\partial R}{\partial \dot{x}} & \frac{1}{I_{xx}}\frac{\partial R}{\partial \dot{y}} & \frac{1}{I_{xx}}\frac{\partial R}{\partial \dot{z}} & 0 & \frac{1}{I_{xx}}\frac{\partial R}{\partial p} & 0 & \frac{1}{I_{xx}}\frac{\partial R}{\partial q} & \frac{1}{I_{xx}}\frac{\partial R}{\partial r} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ \frac{1}{I_{yy}}\frac{\partial M}{\partial \dot{x}} & \frac{1}{I_{yy}}\frac{\partial M}{\partial \dot{y}} & \frac{1}{I_{yy}}\frac{\partial M}{\partial \dot{z}} & 0 & \frac{1}{I_{yy}}\frac{\partial M}{\partial p} & 0 & \frac{1}{I_{yy}}\frac{\partial M}{\partial q} & \frac{1}{I_{yy}}\frac{\partial M}{\partial r} \\ 0 & \frac{1}{I_{zz}}\frac{\partial N}{\partial \dot{y}} & \frac{1}{I_{zz}}\frac{\partial N}{\partial \dot{z}} & 0 & \frac{1}{I_{zz}}\frac{\partial N}{\partial p} & 0 & 0 & \frac{1}{I_{zz}}\frac{\partial N}{\partial r} \end{bmatrix}$$

$$B=\begin{bmatrix} \frac{1}{m}\frac{\partial X}{\partial \theta_M} & 0 & \frac{1}{m}\frac{\partial X}{\partial A_1} & \frac{1}{m}\frac{\partial X}{\partial B_1} \\ \frac{1}{m}\frac{\partial Y}{\partial \theta_M} & \frac{1}{m}\frac{\partial Y}{\partial \theta_T} & \frac{1}{m}\frac{\partial Y}{\partial A_1} & \frac{1}{m}\frac{\partial Y}{\partial B_1} \\ \frac{1}{m}\frac{\partial Z}{\partial \theta_M} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{I_{xx}}\frac{\partial R}{\partial \theta_M} & \frac{1}{I_{xx}}\frac{\partial R}{\partial \theta_T} & \frac{1}{I_{xx}}\frac{\partial R}{\partial A_1} & \frac{1}{I_{xx}}\frac{\partial R}{\partial B_1} \\ 0 & 0 & 0 & 0 \\ \frac{1}{I_{yy}}\frac{\partial M}{\partial \theta_M} & \frac{1}{I_{yy}}\frac{\partial M}{\partial \theta_T} & \frac{1}{I_{yy}}\frac{\partial M}{\partial A_1} & \frac{1}{I_{yy}}\frac{\partial M}{\partial B_1} \\ \frac{1}{I_{zz}}\frac{\partial N}{\partial \theta_M} & \frac{1}{I_{zz}}\frac{\partial N}{\partial \theta_T} & 0 & 0 \end{bmatrix}$$

To compute the physical values of the partial derivative terms shown in Eq. (2), actual values such as mass, mass moment of inertia, aerodynamic properties of main and tail blades should be known. For this purpose, experiments and calculations based on aerodynamic characteristics charts were performed. With the results from above, the derivatives in Eq. (2). were calculated using the formulas sugguested by Prouty[8].

In addition, servo dynamics and kinematic constraints of linkages are augmented to Eq. (2). The servo system equipped in the model helicopter can be modeled as the 1st order system in Eq. (3).

$$\theta=\frac{K}{s+\omega_{cf}}u \qquad (3)$$

The parameters in Eq. (3) are determined from experimental results such as step response, frequency response, and reference input vs. serve horn angle relationship. Also the kinematics of each linkage is measured or calculated in order to determine the value of K in Eq. (3). Hence, the control input variables in Eq. (2) are included in state vector, and the servo control input u in Eq. (3) becomes the control input of the augmented system.

In this study, experiments were performed on a 5 degree-of-freedom flight test bed. The test bed consists of 3 DOF gimbals for roll, pitch, yaw motions, an arm for vertical motion and a post for horizontal rotation. Since the 5 DOF test bed cannot support true free motion in air (6 DOF) due to the coupled linear motions (vertical & radial, radial & horizontal), the state equation suggested in Eq. (2) cannot be applied directly. So the linear velocity terms in the state vector are eliminated. The terms related to roll, pitch, yaw motion remains. Finally, main rotor collective pitch is excluded because it is not a essential control input in the case of controlling only 3 DOF hover and it is known that the loop transfer function of the LQG conroller-plant is recovered definitely to that of full-state feedback controller-plant when the number of inputs equals to the number of output of given system.

It is necessary to normalize the state equation in order to make it convenient to determine the weighting matrices in the LQR theory. Normalization is achieved by dividing each terms in state equation by appropriate maximum values of each state variables. The maximum values are as follows : attitude angle : 20 deg, angular velocity : 20 deg/s, tail rotor collective pitch : 25 deg, lateral cyclic pitch : 20 deg, longitudinal cyclic pitch : 25 deg, servo reference input : 800 microsec. The final state equation is given in Table 1.

Table 1. The system matrices

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4.9127 & 0 & 0.8181 & 0 & 0.0219 & -4.4855 & 119.8564 & 20.6571 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.4295 & 0 & 2.5807 & 0 & 0.0015 & -0.4566 & 8.6762 & -78.6580 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0.0115 & 0 & 0 & 0 & 2.6196 & 17.7692 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -6.2832 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -6.2832 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -6.2832 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 12.5224 & 0 & 0 \\ 0 & 12.0367 & 0 \\ 0 & 0 & 10.1477 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

3. Controller Design

In this paper, digital LQG theory is employed to design the controller implemented on a digital computer. The LQG theory is known as a well-formulated, convenient and reliable design tool even for the multivariable systems[9,10,11]. LQ Regulator is famous for its strong robustness. When using the LQ-controller, the full-state vector must be available. In practice, however, it is almost impossible in many cases. So a state estimator must be introduced to estimate the whole state variables. To apply the digital LQ theory, the state equation should be expressed in discrete-time form. So the state equation in Table 1 is transformed to discrete form using the bilinear transform[10,11]. Then the digital LQR full state feedback gain is calculated first and the state estimator is designed taking the global loop stability and performance into account through digital simulation in turn.

Full state feedback controller design

A linear, time-invariant system exposed to process and measurement noise is given as Eq. (4),

$$\begin{aligned} X_{k+1} &= AX_k + BU_k + V_k \\ Y_k &= CX_k + W_k \end{aligned} \tag{4}$$

where [A,B] is stabilizable and [A,C] is detectable. In this system, all noise denoted as V_k and W_k are assumed to be zero-mean Gaussian white noise, uncorrelated with each other. In Digital LQG theory, the feedback input is given as following;

$$U_k = -K_c X_k \tag{5}$$

In Eq. (5), the feedback gain Kc is the matrix minimizing the performance index in Eq. (6)

$$J = E\{\sum_{k=0}^{\infty}(X_k^T Q X_k + U_k^T R U_k)\} \tag{6}$$

where Q is positive semi-definite and R is positive definite weighting matrices. The feedback gain Kc is defined as Eq. (7),

$$K_c = (R + B^T P B)^{-1} B^T P A \tag{7}$$

where P is positive semi-definite matrix and satisfies the standard digital algebraic Riccati equation in Eq. (8);

$$A^T P_c A - P_c - A^T P_c B(R + B^T P_c B)^{-1} B^T P_c A + Q = 0 \tag{8}$$

In LQR problem, it is very important to select the best weighting matrices that produce the most desirable response. In general, it is a good choice to set Q to $C^T C$ and R to be identical matrix as the initial choice. When Q is set to $C^T C$, the attitude angle terms in state vector are penalized and as angle terms reduce to zero, the remains automatically go to zero. The weighting matrices should be tuned to produce rapid decay of the transient motion. Another thing to be considered is the saturation of feedback input. By iterating under these constraints, mainly on Q, satisfactory feedback gain is obtained. The final choice of weighting matrices and feedback gain are shown in Table 2.

Table 2. Weighting matrices Q, R and feedback gain of controller

$$Q=\begin{bmatrix} 2&0&0&0&0&0&0&0&0\\ 0&2&0&0&0&0&0&0&0\\ 0&0&2&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0 \end{bmatrix} \qquad R=\begin{bmatrix} 1&0&0\\ 0&1&0\\ 0&0&1 \end{bmatrix}$$

$$K_c=\begin{bmatrix} -0.042 & -0.006 & -0.006 & -0.001 & 1.252 & 0.593 & 0.856 & -0.028 & -0.001\\ 1.087 & 0.258 & 0.126 & 0.049 & 0.048 & 0.024 & -0.027 & 1.645 & 0.063\\ 0.111 & 0.039 & -1.167 & -0.295 & -0.001 & -0.001 & -0.001 & 0.051 & 1.532 \end{bmatrix}$$

Full state feedback estimator design

Since the full-state feedback must be used in LQR theory, and only a part of the state is available, a state estimator should be introduced in a LQG design. The state estimator has the form in Eq. (9).

$$\begin{aligned} \hat{X}_{k+1} &= A\hat{X}_k + BU_k + K_f(Y_k - \hat{Y}_k) \\ \hat{Y}_k &= C\hat{X}_k \end{aligned} \tag{9}$$

In Eq. (9), selecting the gain matrix K_f is the main issue of the estimator design problem. It is well known that the calculation of K_f can be done by solving the algebraic Riccati equation in dual form, which is called duality[9]. K_f can be calculated from the following equations :

$$K_f = AP_f C^T(\mu N + CP_f C^T)^{-1} \tag{10}$$

In Eq. (10), P_f is the solution of Eq. (11).

$$AP_f A^T - P_f - AP_f C^T(\mu N + CP_f C^T)^{-1} CP_f A^T + \Gamma\Gamma^T = 0 \tag{11}$$

Eq. (11) is the dual form of Eq. (8) and the two equations can be solved together without interfering each other, which is known as 'Separation theorem'[9]. One thing that should be noted, however, is that the augmentation of estimator might corrupt the global performance even though both of full-state LQ controller and estimator exhibit robust performances. To avoid such a trouble, careful selection of the weighting matrices in Eq. (11) is very important. With the estimator alone, the optimal choice of Γ and μN is to amke the error covariances of process noise and measurement noise[13] to function as Kalman filter. But when introducing the estimator, the weighting matrices should be selected differently. In the case of analog design, by setting the $\Gamma = B$, $N = I$ and decreasing μ or increasing Γ, the closed loop transfer function asymptotically converges to that of the full-state feedback case when proper conditions (such as minimum phase system) are satisfied[14,15]. The same phenomenon occurs in the digital cases[16], but it is more sophisticated due to the oscillation in higher frequency. By changing the value of μ taking both of LTR effect and the stability of closed loop of discrete sytem into account, the following results are obtained.

Table 3. The weighting matrices and feedback gain of estimator

$$\Gamma = B$$

$$\mu N=\begin{bmatrix} 0.005 & 0 & 0\\ 0 & 0.005 & 0\\ 0 & 0 & 0.005 \end{bmatrix}$$

$$K_f=\begin{bmatrix} 3.021 & 0.009 & -0.006\\ 129.8 & 0.903 & -0.391\\ -0.013 & 2.639 & -0.002\\ -1.063 & 103.9 & -0.093\\ -0.006 & -0.002 & 1.904\\ -0.351 & -0.094 & 60.82\\ -0.478 & -0.111 & 42.54\\ 17.46 & 3.374 & 1.525\\ 1.891 & -20.82 & -0.052 \end{bmatrix}$$

A simulation program was written and implemented on a IBM-PC/386 based on a nonlinear helicopter model in hover. The nonlinear simulation was made by using the Runge-Kutta numerical method. The minimum sampling time of digital system was 20 ms due to the fixed cycle of the PWM signal of mini-servo. A simulation result of the transient response is shown in Figure 2 with the structure of the controller in Figure 3.

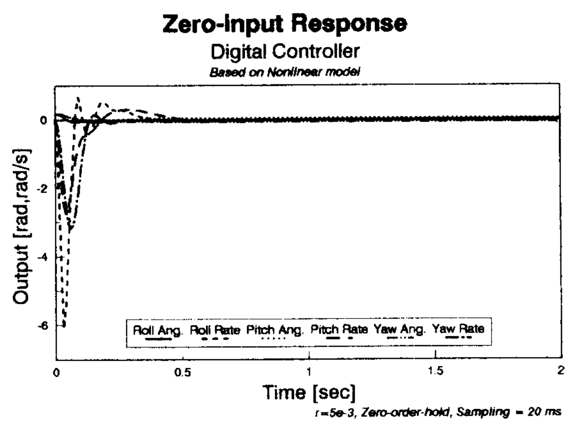

Figure 2. Transient response based on nonlinear simulation

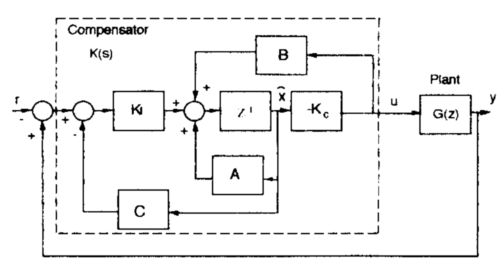

Figure 3. The structure of digital LQG controller

4. Experiment Results

In this study, an experiment device shown in Figure 4 has been designed. The model helicopter used in this study has a 1.5 m-diameter main rotor driven by a 2 stroke-cylce engine. It is mounted on a 3-DOF gimbal which allows motions in roll-pitch-yaw. Each axis of the gimbal is connected to potentiqmeters by flexible couplings. The potentiometers generate the signal for attitude angle and the signals are received and converted by a LAB-PC data acquisition board of 12 bit precistion plugged in an IBM-PC/386. The controller output signals are sent to a PWM signal generator board using 3 8253 timer IC's. The servos equipped on the helicopter are controlled by 20 ms-PWM reference signals. The main rotor RPM is measured by a proximity sensor. Engine speed controller was implemented by a separate digital PI controller, which governs the 2-stroke-cycle engine within 3% error. The whole controller program was written in C.

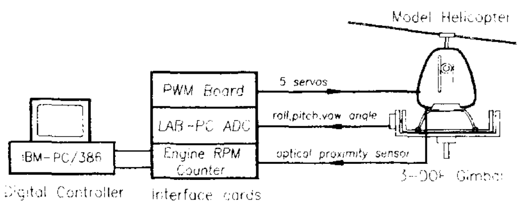

Figure 4. Schematic diagram for experiment apparatus

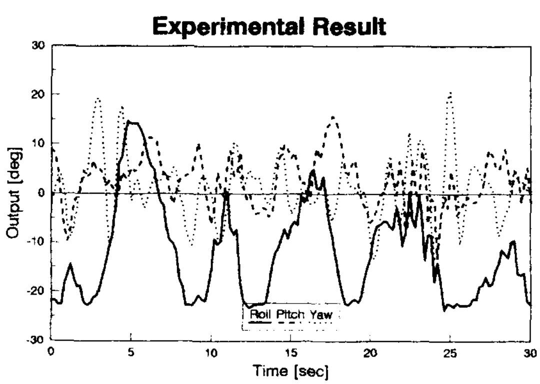

Figure 5. Experimental result

5. Conclusion

So far, the state equation of the model helicopter in hover is derived from its free-body diagram and nonlinear mathematical model which is linearized using Taylor's series expansion. Parameters in the state equation were either measured, or calculated by using the results from other experiments. Servo dynamics and kinematic constraints of each linkage were measured or calculated and augmented into the state equation. Controller was designed by the applying digital LQG theory, and by the iteration of computer simulation stable and optimal controller was obtained. Performance of the controller was tested by the digital computer simulation. Digital controller was implemented on a PC. With the 3 DOF gimbal which constrains the model's motion and also generates the attitude signals, experiments were performed and the controller showed stable controlling capability.

As for future research, a more delicate tuning of the digital LQG controller is required. And other robust multivariable control theory such as H_∞ theory shall be applied for a better performance.

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