Determination of a Holdsite of a Curved Object Using Range Data

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ABSTRACT

Curved 3D objects represented by range data contain large amounts of information compared with planar objects, but do not have distinct features for matching to those of object models. This makes it difficult to represent and identify a general 3D curved object. This paper introduces a new approach to representing and finding a holdsite of general 3D curved objects using range data. We develop a three-dimensional generalized Hough transformation which can be also applied to general 3D curved object recognition and which reduces both the computation time and storage requirements. Our approach makes use of the relative geometric differences between particular points on the object surface and some model points which are prespecified arbitrarily and task dependently.

1. INTRODUCTION

To find an exact holdsite of general 3D curved objects is one of the most important topics in robotics area in various industrial and medical applications. Material handling, welding, and bolting are fundamental tasks in manufacturing, and the success of these tasks depend on mostly how to find a holdsite accurately. Usually, 2D intensity images have been used to obtain the states of environment. If the subject of a task is a 2D surface or a simple 3D planar object, and if the task to be performed is simple and does not require high precision, a 2D image may be enough to obtain the sufficient information for determining a holdsite. However, as the object becomes complicated, 2D images have limitation to determine a precise holdsite.

Recently, as fast and efficient range sensors have become commercially available, there has been a tremendous increase in the use of range data for robotics applications. A 3D range image has an identical image format with commonly used intensity data. But, range data of a 3D object presents explicit information about the surface. The 3D shape from range data directly approximates the 3D shape of the corresponding visible object surfaces. These explicit characteristics of range data makes it much easier to represent and identify general curved objects by their geometric shape. A 3D object is usually described by faces of the object[2,3], which are generally segmented into simple surface patches, such as planar, cylindrical or quadric patches. The features of surface patches and the topological relationships between adjacent patches are commonly used for representing and specifying an object. However, it is very difficult to obtain a unique set of curved patches with unique boundaries[4,8] if the object has general curved shapes. This requires a tremendous computation cost to determine a holdsite. If an object has some distinct edges or vertices[9], the complicated time consuming segmentation process can be eliminated. However, curved 3D objects do not have distinct features for matching to those of object models.

We introduced a generalized Hough transformation (GHT) to recognize a general curved object using several model points[10,11]. In this paper, our GHT is modified and applied to find a precise holdsite of a general curved object.

The general assumptions for determining an exact holdsite of a 3D curved object can be summarized as (1) the geometric model of an object, at least the object surface of interest, is known, (2) other unknown objects may exist in the image plane, (3) the object may be partially occluded by other objects, and (4) image data may be contaminated with noise. Considering the aforementioned assumptions in mind, we introduce a view-point independent approach to determining a holdsite of 3D curved objects using a 3D generalized Hough transformation technique. Our approach makes use of the relative geometric differences between particular points on the object surface and some model points which are prespecified arbitrarily and task dependently. A generalized Hough transformation is implemented to estimate the location of those model points using relative geometric differences which are view-point independent. Using this information, the holdsite of an object is determined.

In section 2, we explain how to locate the model points. Section 3 discusses some of the view-point independent attributes available from the range data of a 3D object surface. We introduce the extended and modified version of the 3D Hough transformation in section 4 using view-point independent attributes available. In section 5, we show computer simulation results. Finally, we conclude this paper in section 6 with some discussions of the advantages and disadvantages of this algorithm.

2. SPECIFYING AN OBJECT BY MODEL POINTS

A general 3D curved object represented by range data contains a large amount of information, but no distinct features. This makes it difficult to specify an exact holdsite. A common approach used to reduce the complexity of pattern matching is to compress the data representation into a simple form. An image can be represented by surface patches with its surface boundaries[2]. Volume representation[7] is possible if true 3D data is available. A 3D object is also described by its 3D curved axis(or spine) and cross-sections which are perpendicular to the axis[12]. The simplified representation of an object should make it easier and faster to determine the exact holdsite. In general, methods using range images perform better for object representation at the expense of more complex recognition processes, than methods using intensity images.

The more intuitive way to represent a 3D object is to simplify an object by some number of model points[10,11]. These points are defined task-dependently, and assumed to be attached to (on, inside of, or outside of) the object by an invisible string. Actually, model points are considered as a part of an object and represent the object. With an assumption that the geometric information of all possible object models is known, the required number and the location of model points depend on the tasks and the object complexity. However, they would better be specified easily to determine the orientation of the object. The holdsite of an object and the direction of approaching can be uniquely defined with respect to these model points.

3. VIEW-POINT INDEPENDENT ATTRIBUTES

First, we want to review some fundamental concepts of the parametric differential geometry of surfaces in three dimensional Euclidean space. Let $\boldsymbol{r}$ be a small patch in a three dimensional parametric vector space. And let $\boldsymbol{x} = \boldsymbol{x}(u(t),v(t))$ be a regular curve C on $\boldsymbol{r}$ through a surface point $\boldsymbol{P} \in \boldsymbol{x}$. Since $\boldsymbol{x}_u \times \boldsymbol{x}_v \neq 0$ for all u and v, it follows that $\boldsymbol{x}_u$ and $\boldsymbol{x}_v$ are linearly independent. $\boldsymbol{x}_u$ and $\boldsymbol{x}_v$ denote $\partial \boldsymbol{x}/\partial u$ and $\partial \boldsymbol{x}/\partial u$ respectively. Because $dx/dt = \boldsymbol{x}_u(du/dt) + \boldsymbol{x}_v(dv/dt)$, the tangents to the surface at $\boldsymbol{P}$ are linearly dependent upon the two linearly independent vectors $\boldsymbol{x}_u$ and $\boldsymbol{x}_v$. Therefore, every tangent passing through $\boldsymbol{P}$ lies on the same plane. Let us call this the tangent plane at $\boldsymbol{P}$, which is expanded by two linearly independent vectors x_u and x_v. The tangent plane is perpendicular to a surface normal n which is defined as

$$n = \pm \frac{x_u \times x_v}{\|x_u \times x_v\|} = \text{Unit Normal Vector} \tag{1}$$

The normal curvature k_n of C at P is defined as

$$k_n = \frac{II(du,dv)}{I(du,dv)} \tag{2}$$

where $I(du,dv)$ and $II(du,dv)$ are the first and second fundamental forms of x such that

$$I(du,dv) = dx \cdot dx = E\,du^2 + F\,dudv + G\,dv^2$$

$$II(du,dv) = -dx \cdot dn = L\,du^2 + M\,dudv + N\,dv^2$$

We set $E = x_u \cdot x_u$, $F = x_u \cdot x_v$, $G = x_v \cdot x_v$, $L = x_{uu} \cdot n$, $M = x_{uv} \cdot n$, and $N = x_{vv} \cdot n$. Also, $x_{uv} = \partial^2 x / \partial u \partial v$, and $x_{uv} = x_{vu}$.

A point on the surface where the value of k_n is constant with respect to any direction is called an umbilical point. It is easy to see from equation (2) that a surface point is umbilical if and only if

$$k = \frac{L}{E} = \frac{M}{F} = \frac{N}{G} \tag{3}$$

A point on a plane or a sphere is a good example for an umbilical point. Any point on a plane has $k_n = 0$ for all directions, and $k_n = 1/r^2$ for all directions at a point on a sphere. Both points are umbilical. If a point on the surface is not umbilical, k_n has distinct maximum and minimum values, say k_1 and k_2, in two perpendicular directions on its tangent plane. These two directions are called the principal directions, and the corresponding normal curvatures, k_1 and k_2, are called the principal curvatures. At any point P on a surface which is not umbilical, there exist two principal directions. Moreover, the normal vector n and two principal directions on the tangent plane are perpendicular each other.

Let two unit vectors n_1 and n_2 represent the direction of two principal directions, and n_3 be a unit normal vector n. We can establish a right-handed coordinate system at a surface point P by using three vectors n_1, n_2 and n_3 such that $n_1 \times n_2 = n_3$, $n_2 \times n_3 = n_1$, and $n_3 \times n_1 = n_2$. Let us call this as a surface oriented coordinate system(SOCS). Assume one model point Q. Let $Q^o = (Q_x{}^o, Q_y{}^o, Q_z{}^o)$ denote the coordinate vector of Q with respect to a viewer oriented coordinate system(VOCS, i.e. world coordinate system), and $Q^p = (Q_x{}^p, Q_y{}^p, Q_z{}^p)$ be the coordinate vector of Q with respect to the SOCS. As shown in Fig. 1, Q^p with respect to the SOCS is independent of viewer's observation point but Q^o is view-point dependent. Let the point $P^o = (P_x{}^o, P_y{}^o, P_z{}^o)$ denote the coordinate of P with respect to the VOCS. Then, we can derive the relation between Q^p and Q^o such that

$$Q^p = [n_1 \vdots n_2 \vdots n_3]^T (Q^o - P^o) \tag{4}$$

The Gaussian curvature K is the product of principal curvatures, and the mean curvature H is the mean of principal curvatures, It is well known that Gaussian curvature K and the absolute value of mean curvature H at a surface point are also view-point independent. K and H are obtained such that

$$K = k_1 k_2 = \frac{LN - M^2}{EG - F^2} \tag{5}$$

$$H = \frac{k_1 + k_2}{2} = \frac{EN + GL - 2FM}{2(EG - F^2)} \tag{6}$$

K and H are invariant to arbitrary translations and rotations of a surface. Gaussian curvature is isometric invariant of a surface.

We can extract one more view-point independent attribute from the surface data using a model point Q. Consider an object surface as illustrated in Fig. 1. The predetermined model point is located on a plane perpendicular to the normal vector at the surface point P. Let p be a vector from P to Q, and the angle between n and p be θ. Then, we obtain the scalar product $d = n \cdot p$ and $\cos\theta = \| p \| / d$. The scalar values of d and $\cos\theta$ are also view-point independent.

The aforementioned view-point independent attributes available from the surface can be summarized as follows.

At a surface point P with given model point Q,

1) Gaussian curvature and mean curvature are invariant under rotation and translation.
2) At any surface point P which is not umbilical, the SOCS can be established. A coordinate Q^p of the model point Q with respect to the SOCS is view-point independent.
3) (d, θ) pair at any surface point P is also view-point

independent.

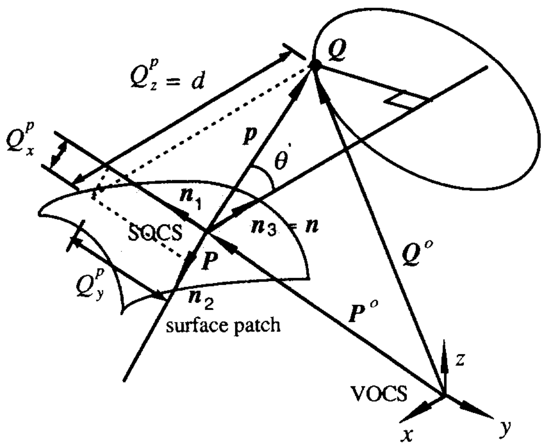

Fig. 1. Relative Locations between Points

4. DETERMINATION OF A HOLDSITE

We introduce a generalized 3D Hough transformation technique to estimate the prespecified model points. A holdsite of an object is then, uniquely determined with respect to these model points.

4.1. Construction of a Constraint Table

If the information stored in a data base is view-point independent, it can be used directly for recognizing an arbitrarily oriented object without any further processing during the recognition phase. Our model data base contains the information of relative locations between predefined model points and surface points of an object model. The information of the relative location is used to identify an object. Actually, this information is a set of constraints on the possible positions of object model points relative to the object's surface points. As such, we call this portion of model data base a *constraint table*. The location of the model points is used to specify the orientation of the identified object in an image.

As discussed in the previous section, Gaussian curvature K and absolute value of mean curvature H at a surface point are view-point independent. A SOCS is defined at any surface point $\boldsymbol{P}$ if $\boldsymbol{P}$ is not umbilical. Given a model point, the position $Q^p = (Q_x^p, Q_y^p, Q_z^p)$ with respect to any SOCS at $\boldsymbol{P}$ is also view-point independent. Values of d and θ are view-point independent, too, regardless of whether $\boldsymbol{P}$ is umbilical or not.

Thus, the constraint table contains sets of data Q^p for a surface point $\boldsymbol{P}$ which is not umbilical, and/or (d,θ) pair for an umbilical surface point indexed with (K, H). In other words, the model point is located either at Q^p with respect to the SOCS if the surface point $\boldsymbol{P}$ is not umbilical, or a certain place on the circle centered at a point which is located at distance d from the surface point to the direction of normal if $\boldsymbol{P}$ is umbilical. The circle is perpendicular to the normal vector, and has a radius $d \tan\theta$. In general, there are many Q^p's and/or (d,θ) pairs under the same index (K, H). Figure 2 shows the structure of a constraint table. The developed constraint table is similar to the R-table introduced by Ballard[1]. To construct a constraint table, collect the Q^p's and (d,θ) pairs at surface points with the same curvature pair (K, H).

4.2. 3D Hough Transformation

A Hough transformation[1,6] is a popular method used for detecting two dimensional shapes described as parametric curves. Its main advantage is that it is relatively ineffected by gaps in curves and is relatively noise immune. A Classical Hough transformation is a good technique for detecting primitive geometries such as straight lines, circles and conics from a two dimensional image. However, as the number of parameters increases, more storage is required for the accumulator array. In general, a Hough transformation is a technique applied in 2D space. The key to generalizing the Hough transformation algorithm for a 3D object recognition problem is the use of the constraint table to estimate the location of prespecified model points. The geometric relationship between estimated model points is used to identify a unknown object.

The general idea of 3D Hough transformation is as follows. At each surface point in a range image, curvature values K and H are first computed. The information of the possible location of model points are obtained by consulting the constraint table using these K and H values as indices. See Figure 2. If a surface point $\boldsymbol{P}$ is not umbilical, a set of coordinates Q^p is obtained from the constraint table. Q^p is the location of possible model points. Equation (4) is evaluated to determine an array entry. If a surface point is umbilical, the SOCS can not be established, and the elements corresponding to the circle based on (d,θ) are updated.

The accumulator array is enumerated for each surface point. Maxima in the accumulator array corresponds to possible model points. Once the locations of prespecified model points are found, the holdsite of an object can be uniquely determined.

5. SIMULATION

Any curved object can be described by the combination of spheres. A sphere is simple to express, and the simulation dealing with a sphere is easy to analyze its results. For this reason, we use spheres with different sizes to examine the performance of our generalized 3D Hough transformation. Five spheres are considered with different radiuses such as 30, 50, 70, 100, and 150 pixels. For each sphere, one model point is considered at the center of a sphere. We also assume that the images are perturbed by noise. Gaussian noise is considered with a variance of 2%, 4%, 6%, 8%, and 10% where % denotes the percentage with respect to the resolution of a pixel. A cell size for the Hough parameter space is considered to be the Z direction.

Our simulations show that cells with maximum votes are obtained distinctly regardless of the presence of noise. The locations of these cells are very close to the center of a sphere. It is also observed that although the maximum votes obtained from simulation using a smaller sphere is smaller than that using a larger sphere, the ratio with respect to the number of pixels belonging to a smaller sphere is higher. The range data between pixels change more in an image of a smaller sphere than a larger sphere. Hence, a small amount of noise introduced to a smaller sphere causes less effect than the noise introduced to a larger sphere.

The curvature is a function of second order differentiation so that it is very sensitive to the noise. However, a curved object in an image consists of a large number of pixels with different curvatures. So, even in a contaminated or occluded image scene, there exist a considerable number of pixels to find prespecified model points. Therefore, once the update of the accumulator array is finished, the cells with relatively

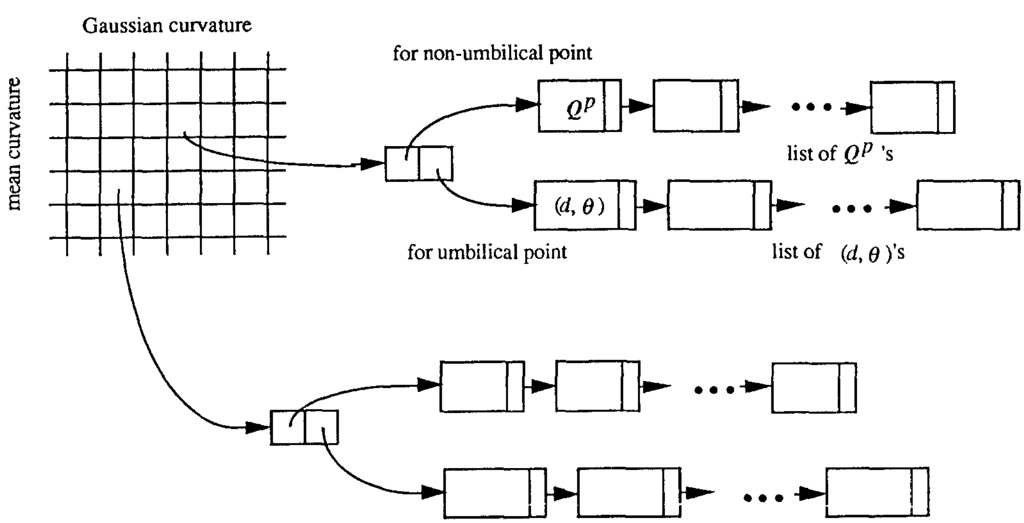

Fig. 2. Format of a Constraint Table

higher votes become clustered around the real model point. In general, the advantage of a Hough transformation is its noise immunity.

6. CONCLUSIONS

This paper introduces a new approach to representing and finding a holdsite of general 3D curved objects using range data. With the assumption that the information of all possible objects are known, our simulations confirm that several well defined model points can be used successfully in object recognition and orientation estimation of a general curved object in a three dimensional space. Since model points are specified arbitrarily and task dependently, further processing can be reduced in application by locating the model points at places which are useful for further operations in the task. Our method has several advantages such as: simple knowledge base with less storage requirement; flexibility of the database design according to the tasks to be performed; and finally, the capability of compensation of the uncertainties of positions estimation due to noise and quantization error. However, our algorithm may not suitable for the object which is purely planar. Since the value of curvatures is zero at every point on a plane, the constraint table has a very long list of (d,θ) pairs with the index $(K, H) = (0,0)$. It will then take long time to update all of these (d,θ) pairs at every point on a plane.

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