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Tree Search Approach to the Control of a Pendulum

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Abstract

This paper presents a tree search technique to solve the dynamic control problems. To illustrate the proposed procedure, the swinging control of a pendulum carried by a motor-driven cart is discussed as an example. Since the control system is of two degrees and the control problem is a nonlinear one, it is difficult to determine a swinging control rule analytically. However, by means of the proposed tree search approach, the problem can be solved in a relatively easy way. Some numerical calculations are performed to verify the methodology. The result of the study shows that the proposed tree search technique is suitable for the dynamic control problems, in particular, for the complicated nonlinear dynamic control problem.

1 Introduction

As is well known, the tree search is widely and effectively used to solve static system problems as one of the fundamental problem-solving methods[1,2,3]. However, whether the tree search technique is still effective for solving dynamic system problems is not yet clear. Therefore, this paper aims at studying the possibility and feasibility of the application of tree search to the dynamic control problems.

Generally speaking, many control problems can be classified as a kind of problem which is to transfer the system from its initial state into a goal state by providing proper control inputs. For these problems, if the system is transformed into a discrete-time system and meanwhile the control inputs take only discrete values[4], then the problem can be treated as a state-space-search problem[5] in which the tree search approach is considered to be applied.

In the conventional approaches of tree search, the heuristic search is most efficient for solving complicated problems. Since a heuristic function that maps the problem state descriptions to measures of desirability can be used to guide the search process efficiently toward a solution, the efficiency of the search process can be greatly improved and the computation time can be remarkably saved, by only sacrificing a little of claims of optimum possibly[3]. Hence, the heuristic approach is adopted to solve the dynamic control problem in this paper.

As a demonstration example, the tree search approach is applied to solving a typical nonlinear dynamic problem, namely the swinging control of a pendulum carried by a motor-driven cart. The control purpose is to make the pendulum stop at the bottom as soon as possible while it swings down from an initial angle, or to swing the pendulum up from the bottom to the top to an upside-down position, by giving proper control inputs to the motor-driven cart in which the pendulum is carried. By means of the proposed approach, the control rules are determined easily as shown in this paper.

2 Heuristic search approach

The problem solving process based on the heuristic search can be described as follows. As shown in Fig.1, beginning at the start state, the system is moved from one state to another state one by one until it has arrived at the goal state, according to every decision of the inputs. In the search process, only the state in which the heuristic function is the smallest will be selected as the start state for further search at each step[3]. Therefore the heuristic function plays a very important role in heuristic search process. The more accurately the heuristic function estimates the true merits of the each node in the search tree, the more direct will be the solution process[1]. If we can find a heuristic function which could guide the search process in the most profitable direction by suggesting which path to follow first among all the available paths, an optimal or an supoptimal solution will be obtained. Usually, the heuristic function is represented in some simple forms in solving static problems[3]. However, for a dynamic control problem the heuristic function is a state function which could be represented in several forms. For example, the square function or the energy function are generally taken as a heuristic function in solving a dynamic control problem.

Up to now, there have existed several algorithms concerning heuristic search such as A^* algorithm[6], AO^* algorithm[7, 8] and Graph Traverser[9,10]. Among those algorithms, the G.T.(Graph Traverser) is recommended for solving a dynamic control problem because it is simple and the number of nodes generated during the search process is relatively small. Therefore, based on the G.T. algorithm the heuristic search algorithm used for dynamic problems is established in this paper, which is summarized briefly as follows.

The following notations will be used in the algorithm.:

S : Start state $X(t_0)$
G : Goal state $X(t_m)$
$H(X)_n$: Heuristic function at the node n
$C(n)$: Set of CLOSED nodes
$O(n)$: Set of OPEN nodes
$\Gamma(n)$: Set of nodes which are connected to the node n
N_{max} : Maximum limitation of the generated nodes

Heuristic search algorithm

step1. Let S into $C(n)$; put $\Gamma(S)$ to $O(n)$.

step2. Find $n \in O(n)$ such that $H(X)_n = \min_{k \in O(n)} H(X)_k$.

step3. Generate $\Gamma(n)$.

step4. If $G \in \Gamma(n)$, then the search procedure results in success.

step5. If $G \notin \Gamma(n)$, then move n from $O(n)$ to $C(n)$. Add $\Gamma(n)$ to $O(n)$.

step6. If the number of generated nodes exceeds N_{max}, the procedure falls in failure; otherwise go back to step2.

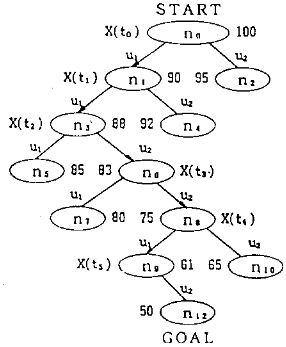


Fig.1. The tree search example

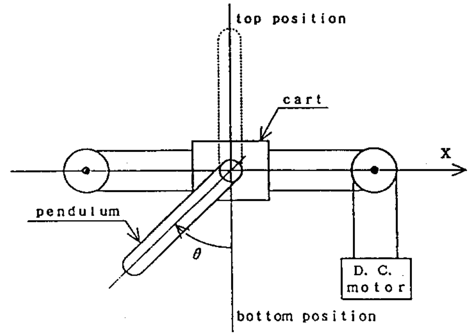


Fig.2. Model of the cart-pendulum system

3 Swinging control of a pendulum

3.1 Pendulum-cart system description

The pendulum-cart system is shown in Fig.2, in which the pendulum is hinged to a cart and the cart is driven by a D.C motor moving along a line of limited length so as to make the pendulum swing. Assume that the driving force is proportional to the voltage $u(t)$ which is added to the amplifier without any delay. The voltage $u(t)$ is taken as the control input. Assume that the friction of the cart is proportional to the velocity of the cart and the friction around the pivot axis is proportional to the angular velocity of the pendulum. Consequently, the nondimensional motion equations of the system are expressed as

$$\begin{cases} \xi_1 \ddot{x}(t) + \xi_2 \dot{x} - \ddot{\theta}(t)\cos\theta(t) + \dot{\theta}^2(t)\sin\theta(t) = \xi_3 u(t) \\ \ddot{x}(t)\cos\theta(t) - \ddot{\theta}(t) - \xi_4\dot{\theta}(t) - \sin\theta(t) = 0 \end{cases} \tag{1}$$

where
$\theta(t)$: Angle of the pendulum
$x(t)$: Position of the cart
$u(t)$: Control input
ξ_1: Nondimensional mass constant
ξ_2: Nondimensional friction constant of the cart
ξ_3: Nondimensional voltage constant
ξ_4: Nondimensional friction constant of the pendulum
All of the parameters used in the state equations are obtained from a experiment[11], namely
$\xi_1 = 3.82$; $\xi_2 = 4.0$; $\xi_3 = 5.36$; $\xi_4 = 0.0348$.

3.2 Swinging-up control

In this section the methodology of the proposed approach will be described in detail, through solving the swinging-up control problem. The purpose of the swinging-up control is to drive the pendulum from the equilibrium point of the bottom to the top to an upside-down position by moving the cart appropriately.

Firstly, transform the motion equation (1) as follows.

$$\begin{cases} \dot{x}(t) = v(t) \\ \dot{\theta}(t) = \phi(t) \\ \dot{v}(t) = F_1(X) + G_1(X)u(t) \\ \dot{\phi}(t) = F_2(X) + G_2(X)u(t) \end{cases} \quad (2)$$

where, X is assumed to be the state vector $\{x, \theta, v, \phi\}$, and

$$F_1(X) = \frac{\phi(t)[\phi(t)\sin\theta(t) + \xi_4\cos\theta(t)] + \cos\theta(t)\sin\theta(t) + \xi_2 v(t)}{\cos^2\theta(t) - \xi_1} \quad (3)$$

$$G_1(X) = -\frac{\xi_3}{\cos^2\theta(t) - \xi_1} \quad (4)$$

$$F_2(X) = \frac{\phi(t)(\phi(t)\sin\theta(t)\cos\theta(t) + \xi_1\xi_4) + \xi_1\sin\theta(t) + \xi_2 v(t)\cos\theta(t)}{\cos^2\theta(t) - \xi_1} \quad (5)$$

$$G_2(X) = -\frac{\xi_3\cos\theta(t)}{\cos^2\theta(t) - \xi_1} \quad (6)$$

The initial conditions can be expressed as

$$\phi(0) = 0, \qquad \theta(0) = 0 \quad (7)$$

$$v(0) = 0, \qquad x(0) = 0 \quad (8)$$

The goal conditions can be expressed as

$$| \phi(t_g) | \leq 0.052, \qquad \pi - | \theta(t_g) | \leq 0.07 \quad (9)$$

According to Runge-Kutta integration the next recurrence equation of the state vector can be obtained.

$$X(t + \Delta t_s) = X(t) + Fx(X(t), u(t)) \quad (10)$$

Where, Δt_s is the sampling time in the calculation of Runge-Kutta integration, and Fx is decided according to Eq.(2).

Then, the pendulum-cart system is treated as a discrete-time system in which the sampling time is assumed as Δt and the control input is taken as the discrete values which is $u(t_i) = \{-\gamma, 0, \gamma\}$. Where, $\Delta t = n\Delta t_s$,n is an integer.

Therefore, by means of the formulas of Eq.(10) the state vector $X(t_{i+1})$ could be calculated recursively according to the values of $X(t_i)$ and $u(t_i)$. However, for each of the input patterns, the state vector takes different values as expressed in Fig.3. Therefore, it is the key point of the problem to chose an appropriate one among $X_1(t_{i+1})$, $X_2(t_{i+1})$ and $X_3(t_{i+1})$ on the purpose of attaining the given goal state. In other wards, if all of the input patterns could be selected correctly for every state, it means the control problem has been solved. Here, we used the heuristic search procedure to search the set of the control inputs which can make the pendulum reach the goal state. According to the heuristic search algorithm, one state corresponds to one value of the heuristic function, and the state in which the value of the heuristic function is the smallest is chose in every decision of the search process.

In the concerned swinging control, the heuristic function is constructed based on the relation of the energy transformation of the pendulum as follows.

$$H(\phi, \theta) = | 0.5\phi^2(t) - k[1 + \cos\theta(t)] | \quad (11)$$

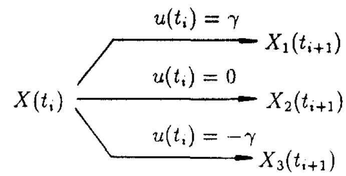


Fig.3. The recurrence of the state vector

To characterize the given heuristic function, a contour map of the function is shown in Fig.4. According to the map, the function H takes the largest value at the beginning of the search, and then decreases until it reaches zero with the increase of angle and speed. After reaching line zero, the heuristic function keeps its minimum value so that the angle and speed couldn't escape from line zero, but vary along line zero until the goal range has been attained, according to the heuristic search algorithm. Hence, it is obvious that the given function is suitable to be a heuristic function in the concerned problem.

To demonstrate the proposed procedure, the numerical calculation is performed, and the solutions are obtained as follows. Fig.5 shows the number of generated nodes and the trajectory length while the magnitude of control input γ is varied. According to Fig.5, the value of γ has a remarkable effect on the swinging motion. The swinging time is relatively short while $1.0 \leq \gamma \leq 2.0$. When γ becomes large, the swinging time and the generated nodes increase abruptly because the angular velocity of the pendulum increases so large that it passes over the goal range by one step. Conversely, if γ is very small, the pendulum does not have enough angular velocity to reach the top position and it has to be swung up again and again. For the case when $\gamma = 1.4$, the search trajectory, the time response of the input and the time response of the angle of the pendulum are illustrated in Fig.6, respectively.

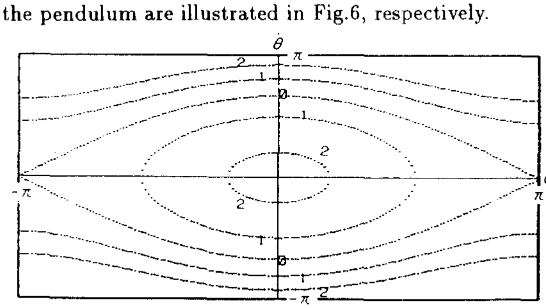


Fig.4. The contour map of heuristic function H

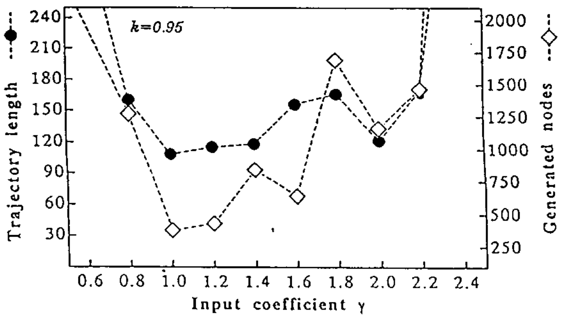


Fig.5. The solutions under different coefficient γ

Besides, it is found that if the sampling-time is very large, then the goal state could be possibly passed over by one search step when the range of the goal is not large enough. On the other hand, if the sampling-time is very small, then the number of the generated nodes will be become very large and will cost much computation time. In the concerned problem, the nondimensional sampling-time Δt is chosen as 0.0713, which is obtained from $\Delta t = \omega \Delta t_r$, where $\omega = 6.261$ $(1/sec)$ is natural angular frequency and $\Delta t_r = 1/88$ (sec) is the sampling-time in a real time system.

3.3 Swinging-down control

If the pendulum shown in Fig.2 swings down from an initial position, it could stop at the bottom position after taking a number of damping swinging motions without adding any control input to the cart. However, the time taken may be too long to satisfy given conditions or requirements. Therefore, we try to make the pendulum stop at the bottom position from its initial angle as soon as possible, by adding proper control input to the cart, in this section.

The initial conditions are given by

$$\phi(0) = 0, \qquad \theta(0) = \theta_0 = \pi/2 \qquad (12)$$

$$v(0) = 0, \qquad x(0) = 0 \qquad (13)$$

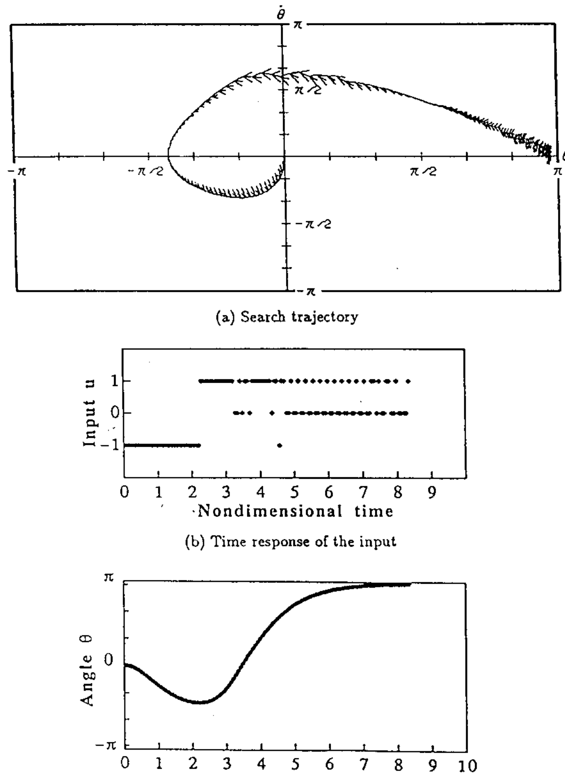
The goal conditions are defined as

$$|\phi(t_g)| \leq 0.052, \qquad |\theta(t_g)| \leq 0.07 \qquad (14)$$

As the same as the swinging-up control case, the sampling time is taken to be 0.0713 and the control input is taken to be $\{-\gamma, 0, \gamma\}$. A square function is used as the heuristic function.

$$H(\phi, \theta) = | k\phi^2(t) + \theta^2(t) | \qquad (15)$$

It is obvious that the heuristic function takes large value in the initial state and minimum value in the goal state. By using the heuristic search algorithm with the above heuristic function, the solutions are obtained as follows. Figure.7



(a) Search trajectory

(b) Time response of the input

(c) Time response of the angle of the pendulum

Fig.6. The result when $\gamma = 1.4$

shows that the solutions are obtained within a large range ($0.0 \leq \gamma \leq 7.0$), however, the influence of the magnitude of control input on the swinging-down motion is remarkable. The smaller the γ is, the more the swinging motion time takes. Conversely, if γ is very large the direction of the velocity may turn to the opposite direction when the pendulum closes to the goal range, so that the stopping time becomes longer as well. The search trajectory in a better case when $\gamma = 3.0$ can be seen in Fig.8.

3.4 Swinging-up control under the goal limitation for the cart

Up to now, the swinging-up and swinging-down control has been discussed without considering any goal constraint on the cart. Assume a constraint in which the cart should return to the start position with minimum velocity when the pendulum reaches the goal position. With this constraint, the control problem will become very complex, especially for the swinging-up control case. Next, we will apply the proposed heuristic search approach as well to determine the control rule of the swinging-up control under the goal constraint for the cart.

Comparing to the case described in section 3.2, the goal condition is changed to :

$$| \phi(t_g) | \leq 0.052, \qquad \pi - | \theta(t_g) | \leq 0.07 \qquad (16)$$

$$| v(t_g) | \leq 0.1, \qquad | x(t_g) | \leq 0.1 \qquad (17)$$

Due to the additional constraints on the cart, it is difficult to find a proper heuristic function because the control objectives contain not only the movement of the pendulum but also the cart.

The fundamental idea to establish the heuristic function for the complex system is to constitute heuristic functions for the pendulum and the cart respectively, and then to combine the two functions into one heuristic function. Based on the idea, the heuristic function for this cart-pendulum system is established as follows.

$$f_p = | 0.5\phi^2(t) - k[1 + cos\theta(t)] | \qquad (18)$$

$$f_c = | \pi - [v^2(t) - (x(t) + \sqrt{|\theta(t)|})^2] | \qquad (19)$$

$$H(X) = f_p + f_c \qquad (20)$$

namely,

$$H(x, \theta, v, \phi) = | 0.5\phi^2(t) - k[1 + cos\theta(t)] | + k_c | \pi - [v^2(t) - (x(t) + \sqrt{|\theta(t)|})^2] | \qquad (21)$$

The function f_p is established as the same as the first case. The function f_c is established for the cart based on a square function in which the heuristic function takes the largest value at the start state and the minimum at the goal state.

In addition, it has been found that the pattern of the control input also has an influence on the swinging control; in which especially the difference among the values of the input pattern greatly affects the variation of the position and the velocity of the cart. For example, if we use the control input pattern $\{-\gamma, 0, \gamma\}$ for the problem, no feasible solution can be found although this pattern is effective for the previous case. In this case, the control input pattern and a feasible solution are determined simultaneously. Let input $u(t_i) = \gamma\{-n, 0, m\}$ ($m = 1, 2, 3, ...k$; $n = 1, 2, 3, ...k$). Then the heuristic search is performed for each input pattern. The calculation starts when $n = 1$ and $m = 1$, and does not finish until a feasible solution is found. In the calculation, the set of input pattern is determined as $\{-3\gamma, 0, 5\gamma\}$. It should be noted that the process of determining the input pattern requires only some computation time, not extra computation memory. Note also that it is possible to consider a more general case where the input pattern is $u(t_i) = \gamma\{-(n+1), -n, 0, m, (m+1)\}$.

Some feasible solutions are found within the range $0.29 \leq \gamma \leq 0.41$. When γ exceeds this range, it is very difficult to make the pendulum-cart system reach the goal state. If γ is small, the pendulum has not enough angular velocity to reach the top position. Conversely, if γ is large, the cart cannot stop at the goal position when the pendulum reaches top position. The result when $\gamma = 0.29$ is shown in Fig.9 where the search trajectory, the time response of the control, the time response of the angle of the pendulum and the time response of the position and the velocity are illustrated. The solution shows that not only the swinging time but also the maximum horizontal moving distance of the cart is short. Besides, it should be noted that the computation memory used in the search process is very small because the search trajectory length is only 108 and the number of generated nodes is only 402. If we use the breadth-first search or the depth-first search in the same case the number of generated nodes will be over $(3^1 + 3^2 + 3^3 + \cdots + 3^{108})$. Therefore, it is impossible to perform the computation with such a larger number.

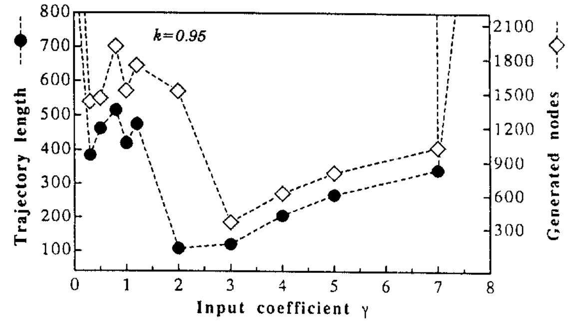


Fig.7. The solutions under different coefficient γ

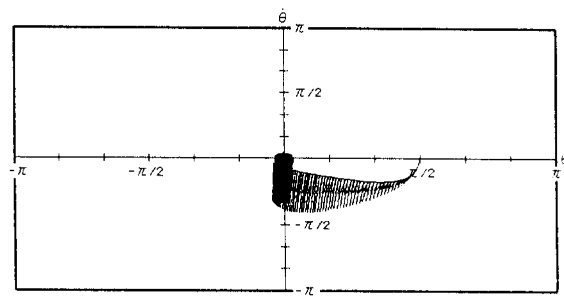
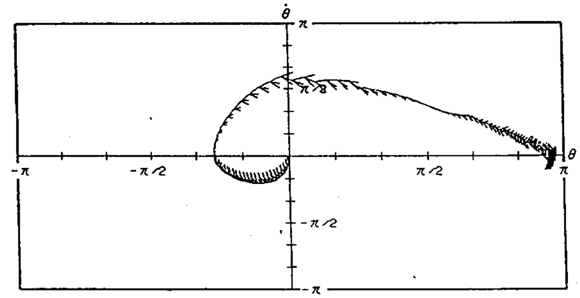


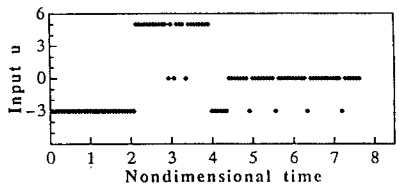
Fig.8. The search trajectory when $\gamma = 3.0$

4 Concluding remarks

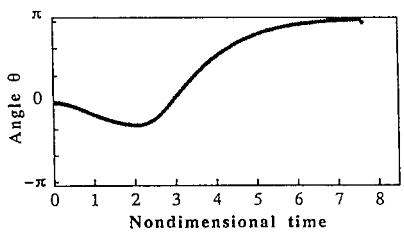
This paper proposed a heuristic search approach for solving the dynamic control problems. The approach is proven to be effective by successfully solving the swinging control of a pendulum. Since the concerning motion is governed by a forth-order nonlinear differential equation, it is difficult to find an analytical approach to solve the problem. Furthermore, if a constraint is added to the cart movement, the problem becomes even more difficult. Instead of finding an analytical solution, we apply the heuristic technique to solve the problem. Namely, the cart-pendulum system is firstly transformed into a discrete-time system, and the



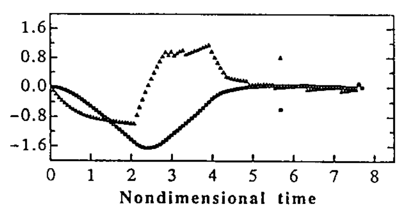
(a) Search trajectory



(b) Time response of the input



(c) Time response of the input



(d) Time response of the velocity and the position when $\gamma = 0.29$

Fig.9. The results when $\gamma = 0.29$, $k = 0.81$ and $k_c = 0.15$

control input is also taken as the discrete values. Then, by using the proposed heuristic search algorithm combining with the Runge-Kutta integration, feasible solutions are obtained for the three case of swinging control problems easily; the computation memory used in the search is not much. Therefore, we can conclude that the proposed tree search approach is a powerful technique for solving the dynamic control problems, especially for solving the nonlinear dynamic control problems which are difficult to be solved analytically.

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