Modeling and Parameter Estimation of a Fish-Drying Control System

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Abstract

The major purpose here is to estimate the drying time required in the fish-drying process employed. The basic element of the prediction of the drying time is the model or the equation, which governs the change in weight. By an intuitive consideration on the mechanism of dehydration, a mathematical model of the fish-drying process is built, which is described by a system of linear differential equations. Further, a modified system of linear differential equations for a model of drying is also proposed for more accurate estimation. The parameter estimation of this system of equations provides the prediction of necessary drying time.

Introduction

This paper deals with the estimation for the fish-drying control system described in the companion paper[1]. Estimation here is to provide necessary information about drying fish of particular size and fatness. So far this issue is accepted as a skill-requiring profession. And as is mentioned in the companion paper, the purpose here is to supplement skilled operators in skill-requiring fields. An aspect of this purpose is to provide the necessary training for the operators working in this system. In order to get along without skilled operators, just automatization is not sufficient. Required is the education of novice people in the field they are concerned with and let them be interested in their field and be settled. Combining with the content of Reference [1], this paper attempts to meet this need of the modern society.

Mathematical Modeling of the Fish Drying Process

Intuitive consideration gives the schematic described in Figs.1(a) to 1(d) for the stages of the drying process. Fig. 1(a) is the original status of humidity inside a fish body before drying. A fish body is assumed to be a cylinder of finite length and hence a cross section has a circular shape everywhere. Humidity is equal at everywhere before starting drying. The abscissa denotes radius from the center of the body and the ordinate shows the normalized humidity. The beginning of drying gives a gradual change in humidity distribution in the fish body. This is the first stage of drying. Fig. 1(b) is for this situation, in which the humidity starts to show some decrease from the fish surface. As the drying proceeds, the drying gets into the second stage or the middle stage. It is for more or less steady drying. And the decrease in humidity is seen almost everywhere in the body. Something to be noticed is the possibility of a thin film of air at the surface, as is shown in Fig. 1(c). This is just a hypothesis from the analogous consideration to the heat transfer. The existence of this film causes a small moisture gap at the surface, and this prevents the exact balance in humidity between the part of the fish just under the

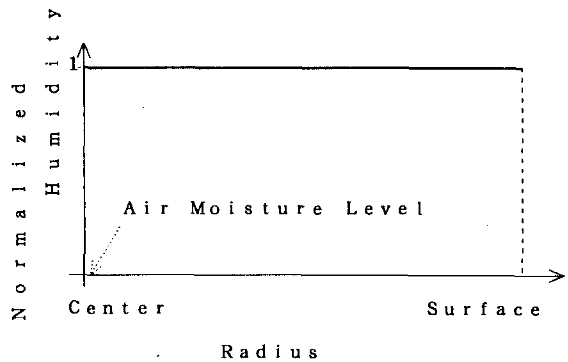

Fig. 1(a) Water distribution before drying

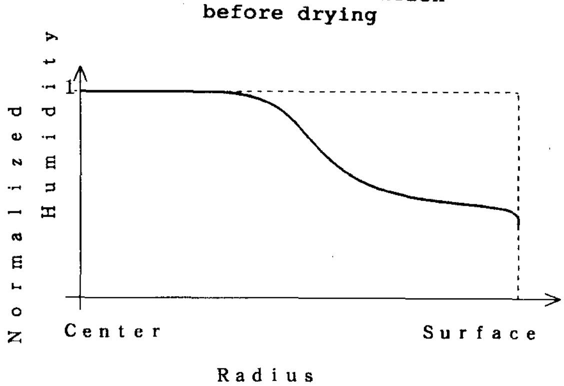

Fig. 1(b) Beginning stage of drying

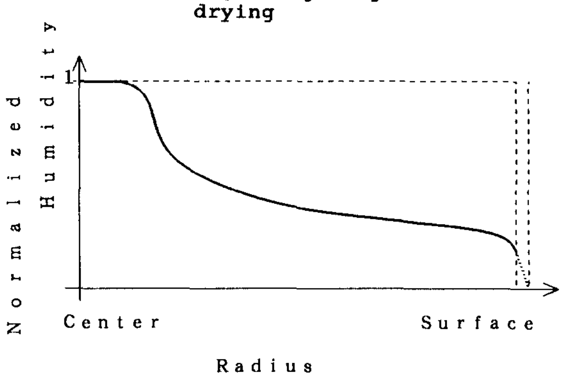

Fig. 1(c) Middle stage of drying

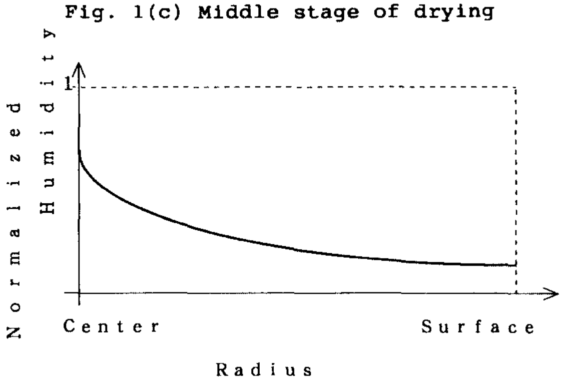

Fig. 1(d) Final stage of drying

Fig. 1 Humidity change in a fish body during drying drocess

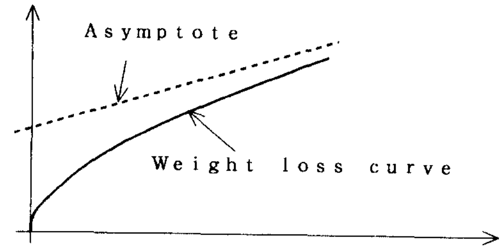

Fig. 2 Behavior of Drying (increase in weight loss with time)

surface and the surrounding air. Anyway, the drying is not exactly steady, because, as is discussed in what follows, the situation changes throughout the drying process. The third stage is the final situation of the drying where the moisture in the fish body almost balances with that of the surrounding atmosphere. (See Fig. 1(d)) Dehydration has almost completed this stage. But the humidity does not come to the exact balance with the surrounding air as already mentioned.

The intuitive consideration given in the above, together with the graphs of the outcomes from the real drying system as in what follows or in Reference [1], provides the following mathematical model. The extent of dryness at an elapsed drying time t is measured by the normalized water loss (of a particular fish body) at the time t; i.e.,

loss of water
= original weight before drying
- weight at time t.

normalized loss of water x(t)

$$= \frac{\text{loss of water}}{\text{original weight}}$$

There are the following features in the drying behavior:

1. A rather large initial drying speed rapidly decreases with time.
2. After the first stage, the behavior seems to have an asymptote.

Fig. 2 shows a schematic for these features. These features imply the following system of differential equations:

$$\frac{dx(t)}{dt} = u + v(t)$$

$$\frac{dv(t)}{dt} = -\alpha * v(t) \qquad \cdots(1)$$

where u is a constant speed which corresponds to the asymptote and v(t) is the speed with a constant rate of decrease. Equations (1) fits any one of the experimental results very well as will be described later. In this sense, the phenomenon seems to be well explained by Equations (1). But returning to the preceding intuitive discussion, it can easily be understood that there must be another factor in the behavior of dehydration; i.e., there is always some limit in dehydration.

Class(mm)	Number of Fish	Class(mm)			Decrease in Derecentage in Drying (%)									
		Maximum	Minimum	Average	0(H)	4(H)	8(H)	12(H)	16(H)	20(H)	24(H)	28(H)	32(H)	44(H)
150	3	153	151	152.0	0.000	12.40	11.80	21.80	25.20	28.00	30.70	32.70	34.60	39.90
155	8	159	155	156.6	0.000	14.10	19.90	24.20	27.80	31.00	33.90	36.30	38.30	43.50
160	24	164	160	162.0	0.000	11.80	11.70	20.60	23.70	26.40	29.10	31.10	33.00	38.10
165	21	169	165	167.1	0.000	10.50	14.80	18.10	20.80	23.20	25.60	27.40	29.10	33.60
170	39	174	170	172.1	0.000	10.30	14.60	17.90	20.60	23.00	25.40	27.30	28.50	33.20
175	32	178	175	176.3	0.000	9.60	13.40	16.40	18.90	21.10	23.40	25.10	26.20	30.70
180	25	184	180	182.1	0.000	9.20	12.90	15.70	18.10	20.30	22.40	24.00	25.60	30.50
185	5	187	185	185.6	0.000	10.10	14.10	17.20	19.70	22.00	24.40	26.00	27.80	33.20
Average	157			171.6	0.000	10.50	14.70	18.00	20.80	23.20	25.60	27.40	29.00	33.90

Table 1 Fish weight change in drying

Equations (1) does not include this factor. Only a single asymptote is assumed with a constant inclination u. This results in the infinite dehydration after the infinite drying time which contradicts the above consideration and can not happen in the real situation. A solution to this issue is to let the inclination u decrease with time as is the case in v(t). Incorporating this factor yields the following system of equations:

$$\frac{dx(t)}{dt} = u(t) + v(t)$$

$$\frac{dv(t)}{dt} = -\alpha * v(t)$$

$$\frac{du(t)}{dt} = -\beta * u(t) \qquad \cdots(2)$$

Coincidence of the Solution of Equation (1) and Equation (2) with the Experimental Results

We will start with how to estimate the unknown parameters of Equations (1) and Equations (2).

The parameters α, v(0) and u are unknown in Equations (1). As these parameters can not be estimated directly, the method of least squares is combined with the method of golden section (refer to [2]) to estimate parameters. The outline of this method is as follows:

Step1: Assuming some value for the optimal α, estimate the parameters v(0) and u by solving the normal equations of least squares.

Step2: Search for the optimum estimate of α by using the golden section.

Search the optimum estimates α, v(0) and u by using step1 to step2 iteratively.

In the same manner, the optimum estimates α, β, v(0) and u(0) in Equations (2) are obtained by using the golden section which is extended for two-dimensional search.

Of these two methods, the terms "α-solution" and "(α,β)-solution" are used to refer to the solutions of Equations (1) and (2), respectively in what follows. Let us compare these solutions with the experimental results to judge whether the present theory is good or not. The experimental results in the fish-drying process are shown in Table 1.

For the experimental details refer to Reference[1].

Table 2 and Table 3 show the comparison of the α-solution and the (α,β)-solution, respectively, with the experimental results for the case of the 150mm class. The validity of these solutions can be seen more clearly when the results are shown graphically as in Fig. 3 and Fig. 4. As these figures indicate, each of the α-solution and the (α,β)-solution coincides with the experimental results. The (α,β)-solution is, however, better than the α-solution. This is the case in all other classes, too. Thus only the (α,β)-solution will be used in what follows. Another example is given in Table 4 and Fig. 5 applying the (α,β)-solution to the averaged data at the bottom of Table 1, which also shows the validity of the (α,β)-solution. This fact implies the possibility of that the state of fish-drying can be known by measuring the weight of fish without directly inspecting the fish in the drying room. The fact that the averaged data is approximately equal to the change in the 170mm class which is most frequent also supports this possibility, as is evident from Table 1.

prediction of fish drying will be described briefly, employing the (α,β)-solution. Table 5 shows the (α,β)-solution solved for only the first 4 measurements at every 4 hours of drying from 4 hours to 16 hours of drying. Those parameters estimated and the magnitudes of errors of solutions to the measured values are not much different from those in Table 4 which are obtained for all the measurements from the beginning till the end of drying. The (α,β)-solution which takes into account the existence of change in the asymptote u is applicable to the future state prediction during drying.

Conclusion

The major points obtained here will be restated as follows:

1. A fish drying process can be expressed by Equations (1) or Equations (2).

2. The unknown parameters in the above equations are estimated by the method as

Time (H)	0.00	4.00	8.00	12.00	16.00	20.00	24.00	28.00	32.00	44.00
Weight Decrease (%)	0.00	12.40	17.80	21.80	25.20	28.00	30.70	32.70	34.60	39.90
Calculated Value (%)	0.00	11.85	18.35	22.42	25.39	27.86	30.10	32.23	34.32	40.50
Error	0.00	0.55	-0.55	-0.62	-0.19	0.14	0.60	0.47	0.28	-0.60
Optimal Results	α = 0.197					β = 0.513		u(0) = 3.540		

Table 2 Comparison between the α-solution and the measurements for the 150 mm class in Table 1

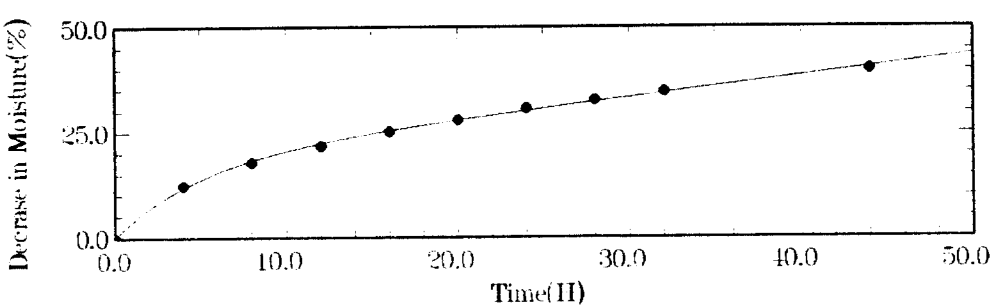

Fig. 3 Graph of the results shown in Table 2

Time (H)	0.00	4.00	8.00	12.00	16.00	20.00	24.00	28.00	32.00	44.00
Weight Decrease (%)	0.00	12.40	17.80	21.80	25.20	28.00	30.70	32.70	34.60	39.90
Calculated Value (%)	0.00	12.35	17.93	21.82	25.08	27.95	30.51	32.80	34.85	39.79
Error	0.00	0.05	-0.13	-0.02	0.12	0.05	0.19	-0.10	-0.25	0.11
Optimal Results	α = 0.400			β = 0.028		$a(0)$ = 1.177		$\vartheta(0)$ = 3.953		

Table 3 the (α,β)-solution for the same data as the one used in Table 2 (150 mm class in Table 1)

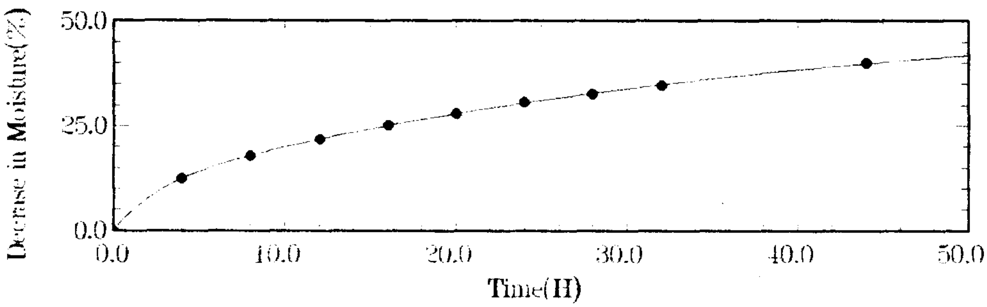

Fig. 4 Graph of the results shown in Table 3

Time (H)	0.00	4.00	8.00	12.00	16.00	20.00	24.00	28.00	32.00	44.00
Weight Decrease (%)	0.00	10.50	14.70	18.10	20.80	23.20	25.60	27.40	29.10	33.90
Calculated Value (%)	0.00	10.43	14.77	18.06	20.79	23.25	25.46	27.44	29.31	33.77
Error	0.00	0.07	-0.07	0.04	0.01	-0.05	0.14	-0.04	-0.21	0.13
Optimal Results	α = 0.457			β = 0.025		$a(0)$ = 0.959		$\vartheta(0)$ = 3.691		

Table 4 (α,β)-solution for the class "Average" shown in Table 1

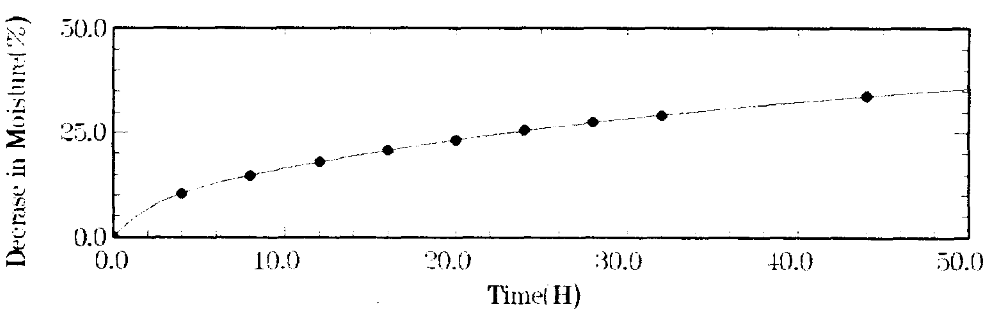

Fig. 5 Graph of the results shown in Table 4

Time (H)	0.00	4.00	8.00	12.00	16.00	20.00	24.00	28.00	32.00	44.00
Weight Decrease (%)	0.00	10.50	14.70	18.10	20.80	23.20	25.60	27.40	29.10	33.90
Calculated Value (%))	0.00	10.45	14.79	18.07	20.79	23.24	25.45	27.42	29.28	33.73
Error (Prediction)	0.00	0.05	-0.09	0.03	0.01	-0.04	0.15	-0.02	-0.18	0.17
Optimal Results	$\alpha = 0.444$			$\beta = 0.025$		[illegible]$(0) = 0.956$		[illegible]$(0) = 3.606$		

Table 5 Application of the (α,β)-solution to drying state prediction

the combination of the least squares and the golden section.

3. Although either system of equations gives good estimates, the (α,β)-solution is more effective than the α-solution.

4. It is also effective that predicting the state of fish-drying by applying the (α,β)-solution to the first some measurements of fish weight.

In addition to the above, it can also be shown that Equations (1) and (2) hold in any other drying processes applying these to the data for other processes shown in References [1] and [2], although the discussion on this fact is omitted in this paper.
Further study is necessary, but the present method makes it possible to construct the fish-drying control system.

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