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Trajectory Control of a Flexible Manipulator with a Prismatic Joint

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Abstract

The tracking control problem of a flexible manipulator with a prismatic joint along a given path is discussed. The nondimensionalization of the elastic part of the manipulator makes it possible to model such a flexible manipulator. For a discontinuous velocity trajectory, an optimal control theory has been applied to formulate the problem. The optimal scheme is given to find the input commands(e.g., joint torques) necessary to produce a specified end effector motion. Simulated results show the potential use of this scheme for a discontinuous velocity trajectory control.

1 Introduction

Tracking accuracy of the end effector of flexible manipulators and the avoidance of its oscillations is one of the major open problems in the new generation of very fast, light and high precision robots. Feedfoward control strategy is important in reducing the tracking error of the end effector of the flexible manipulator along a given trajectory. The fundamental point of the problem is how to solve the inverse dynamics which calculates theoretically the input commands necessary to track a given trajectory [1] [2]. As to the trajectory tracking problem, the existence of its solution has been discussed [3].

In order to formulate the inverse dynamics problem, it is necessary that the prescribed trajectory is continuous. The desired velocity may often be planned as a discontinuous velocity trajectory when we want to make the arm start, stop and change its direction in motion. In such a case, it is theoretically impossible to get the input command which guarantees the zero tracking error along such a discontinuous velocity trajectory. Therefore it is required to make a new approach which tracks the given trajectory and approximates the desired velocity as closely as possible.

Generally speaking, we discuss seperately two control modes in the positioning control problem, those are, 1) access control mode which moves speedily a control object from a present position around a desired position, and 2) positioning control mode which keeps a control object at the desired position. The tracking control problem of a flexible manipulator is essentially the same as the access control mode. In order to speed up the positioning control, it is necessary to consider the oscillatory characteristic of the positioning mechanism and it is important to minimize the input energy for positioning. For the access mode in a positioning control, Yamada et al. formulated the positioning control problem as an optimal regulator problem with conditions of making the load cease to oscillate at the desired position and time, and of minimizing the input energy.

In this paper, a tracking control problem of a flexible manipulator with a prismatic joint is considered. Nondimensionalization of the elastic part makes it possible to model such a flexible manipulator with time-varying arm length. We will discuss the input command which guarantees the zero tracking error. An optimal control theory has been used to formulate the above problem.

2 Dynamic Model of the Manipulator

We discuss the flexible manipulator depicted in Fig.1 which consists of two links, one revolute joint and one prismatic joint.

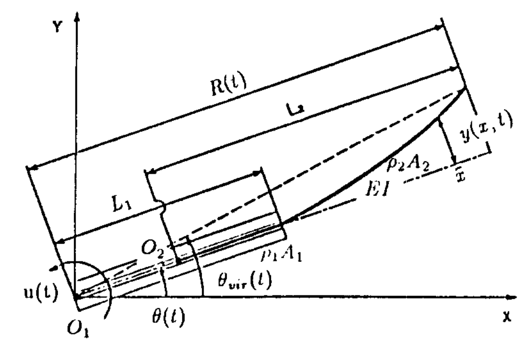


Fig.1 Schematic Drawing of the Flexible Manipulator with a Prismatic Joint

Link 1 is a rigid manipulator which has total length L_1, mass per unit length ρ_1 and area A_1. Link 2 is a flexible manipulator with variable arm length whose total length is L_2, mass per unit length ρ_2, area A_2, moment of inertia I_2 and Young modulus E_2. The origins of link 1 and link 2 are O_1 and O_2 respectively. The distances from origins O_1 and O_2 to any points are expressed by x_1 and x_2, respectively. Link 2 does translation motion about link 1, with a prismatic joint. The elastic part of link 2 is defined by the distance between the tips of link 1 and link 2. An ideal prismatic joint without any undesirable gap in all directions is assumed.

With respect to the inertial coordinate(OXY), the angular displacement of link 1 is represented by $\theta(t)$ (the counter-clock direction is set to be positive). The input commands, which are the torques to be applied at each actuator, are represented by $u(t)$ (the counter-clock direction is set to be positive). The line between the origin of link 1 and the tip of the end effector is called a virtual rigid manipulator. $R_{vir}(t)$ denotes its radial length and $\theta_{vir}(t)$ denotes its angular displacement with respect to the inertial coordinate. We assume that the elastic displacement of link 2, $y(x,t)$, is very small compared to the rigid displacement. The radial displacement of link 2, $R(t)$, is given beforehand.

The potential energy of the system is assumed to be the elastic potential energy of the beams. Therfore, we can write the kinetic energy and the potential energy of the system as follows:

$$2T = \int_0^{L_1} \rho_1 A_1 \dot{r}_1^T \dot{r}_1 dx_1 + \int_0^{L_1+L_2-R(t)} \rho_2 A_2 \dot{r}_2^T \dot{r}_2 dx_1 + \int_{L_1+L_2-R(t)}^{L_2} \rho_2 A_2 \dot{r}_3^T \dot{r}_3 dx_1 \quad (1)$$

$$2V = \int_{L_1+L_2-R(t)}^{L_2} EIy''''(x_1,t)^2 dx_1 \qquad ((L_1+L_2-R(t)) \le x_1 \le L_2) \quad (2)$$

where

$$r_1 = [x_1\cos\theta(t), x_1\sin\theta(t)] \qquad (0 \le x_1 \le L_1)$$

$$r_2 = [(R(t)-L_2+x_2)\cos\theta(t), (R(t)-L_2+x_2)\sin\theta(t)] \qquad (0 \le x_2 \le L_1+L_2-R(t))$$

$$r_3 = [\{(R(t)-L_2+x_2)\cos\theta(t) - y(x_2,t)\sin\theta(t)\}, \{(R(t)-L_2+x_2)\sin\theta(t) + y(x_2,t)\cos\theta\}] \qquad (L_1+L_2-R(t) \le x_2 \le L_2)$$

From Euler-Lagrange equation, we obtain the partial differential equation as follows:

$$\int_{L_1+L_2-R(t)}^{L_2} \rho_2 A_2 \{2y(x_2,t)\dot{y}(x_2,t)\dot{\theta}(t) + y^2(x_2,t)\ddot{\theta}(t) + \frac{1}{3}\rho_2 A_2 \left(3\dot{R}(t)R^2(t) - 3\dot{R}(t)(R(t)-L_2)^2\right)\dot{\theta}(t) - \ddot{R}(t)y(x_2,t) + (R(t)-L_2+x_2)\ddot{y}(x_2,t)\}dx_2 + \frac{1}{3}\rho_2 A_2\left(R^3(t) - (R(t)-L_2)^3\right)\ddot{\theta}(t) + \frac{1}{3}\rho_1 A_1 L_1^3 \ddot{\theta}(t) = u \quad (3)$$

$$\int_{L_1+L_2-R(t)}^{L_2} \{EIy''''(x_2,t) + \rho_2 A_2\{-y(x_2,t)\dot{\theta}^2(t) + 2\dot{R}(t)\dot{\theta}(t) + \ddot{y}(x_2,t) + (R(t)-L_2+x_2)\ddot{\theta}(t)\}\}dx_2 = 0 \quad (4)$$

Using the following relationship, we can nondimensionalize the elastic part of the flexible manipulator.

$$x_2 = (R(t)-L_1)x + (L_1+L_2-R(t)) \quad (5)$$

Therefore, by nondimensionalizing and neglecting the higher order terms, we can approximate motion equations of the flexible manipulator and the rigid manipulator as follows:

The flexible manipulator;

$$\frac{1}{3}\rho_2 A_2\left(R^3(t) - (R(t)-L_2)^3\right)\ddot{\theta}(t) + (R(t)-L_1) \int_0^1 \rho_2 A_2\{-\ddot{R}(t)y(x,t) + (L_1 + (R(t)-L_1)x)\ddot{y}(x,t)\}dx + \frac{1}{3}\rho_1 A_1 L_1^3\ddot{\theta}(t) = u \quad (6)$$

$$(R(t)-L_1)\int_0^1 \{EI\frac{y''''(x,t)}{(R(t)-L_1)^4} + \rho_2 A_2\{\ddot{y}(x,t) + (L_1 + (R(t)-L_1)x)\ddot{\theta}(t)\}\}dx = 0 \quad (7)$$

The rigid manipulator;

$$\frac{1}{3}\rho_2 A_2\left(3\dot{R}(t)R^2(t) - 3\dot{R}(t)(R(t)-L_2)^2\right)\dot{\theta}(t) + \frac{1}{3}\rho_2 A_2\left(R^3(t) - (R(t)-L_2)^3\right)\ddot{\theta}(t) + \frac{1}{3}\rho_1 A_1 L_1^3\ddot{\theta}(t) = u \quad (8)$$

Actuator torques required for a flexible manipulator to track a given trajectory are computed by a simple and efficient method using special coordinate systems, called virtual rigid link coordinates. Nondimensionalization of the elastic part makes it possisble to model such a flexible manipulator with time-varying arm length. To approximate the partial differential equations by ordinary differential equations, the method of modal analysis is used. Finally the dynamics of the flexible manipulator with a prismatic joint are given by [4]:

$$\dot{\mathbf{z}} = \mathbf{A}\cdot\mathbf{z} + \mathbf{b}\cdot\mathbf{u} \qquad (9)$$

$$\theta_{vir} = \mathbf{c}\cdot\mathbf{z} \qquad (10)$$

where

$$\mathbf{z} = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ \vdots \\ z_{2N-1} \\ z_{2N} \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 0 \\ b_1 \\ 0 \\ b_2 \\ \vdots \\ 0 \\ b_N \end{bmatrix}$$

$$\mathbf{c} = \begin{bmatrix} c_1 & 0 & c_2 & 0 & \cdots & c_N & 0 \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & \cdots & \cdots & & & 0 \\ 0 & 0 & 0 & 0 & \cdots & & & \vdots \\ 0 & 0 & 0 & 1 & 0 & & & \vdots \\ \vdots & & -\omega_1^2 & -2\xi\omega_1 & 0 & & & \\ \vdots & & & 0 & \ddots & & & \\ & & & \vdots & \ddots & \ddots & & \\ & & & & & \ddots & 0 & 1 \\ 0 & \cdots & \cdots & 0 & \cdots & & -\omega_N^2 & -2\xi\omega_N \end{bmatrix}$$

coefficients $\omega_1, \cdots, \omega_N, b_1, \cdots, b_N, c_1, \cdots, b_N$ are the functions of $R(t)$ and/or $\ddot{R}(t)$ which denotes 2nd time derivative of $R(t)$.

3 Optimal Input Command Planning

The problem which makes the end effector track along the given trajectory is the same as the problem which makes $R(t)$ and $\theta(t)$ track along the desired radial displacement function $R_d(t)$ and the angular displacement function $\theta_d(t)$, respectively. Since it is assumed that the radial displacement function is given beforehand and the trajectory error in radial direction is very small compared to that in rotational direction, we discuss only the tracking control problem along the desired angular trajectory $\theta_d(t)$.

Input command planning problems are different from the positioning control problems in the following points : 1) there exists the deflection of the manipulator, and 2) after that the manipulator reaches the desired velocity, it is required to suppress the tip oscillation and to track the desired trajectory. Therefore, in this section we will discuss input command planning for the discontinuous velocity trajectory by using an optimal control theory. The procedure to calculate the input command $u(t)$ which is necessary to track a desired trajectory is as follows:

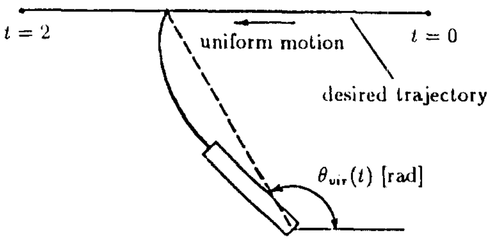


Fig.2 Desired Trajectory of the Flexible Manipulator

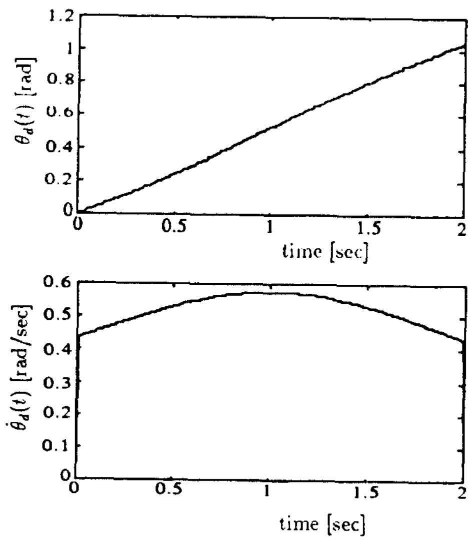


Fig.3 Desired Trajectory of a Rotational Direction

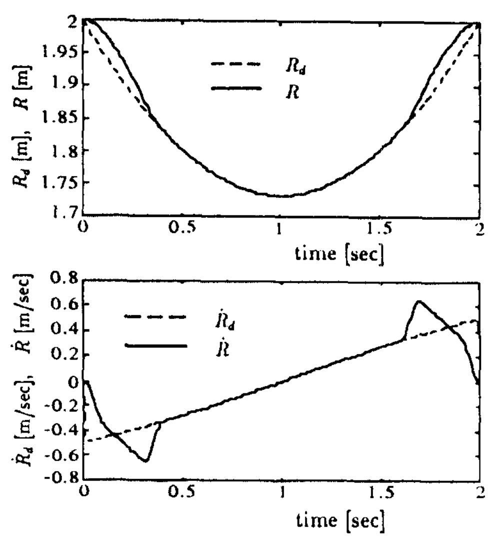


Fig.4 Desired Trajectory of a Radial Direction and the Modified Trajectory by Spline Function

step 1 Fig.2 shows the desired trajectory which moves uniformly along a straight line. We get the desired trajectory $[R_d(t), \theta_d(t)]$ by solving the inverse dynamics of rigid manipulator. Fig.3 and Fig.4 show displacement and velocity of the rotational desired trajectory, $\theta_d(t)$, and the radial desired trajectory, $R_d(t)$, respectively.

step 2 Since it is assumed that a radial displacement function is given beforehand, we modify a radial displacement function by means of a spline function. Fig.4 also shows the modified time history of $R(t)$ and $\dot{R}(t)$, corresponding to the radial displacement and velocity.

step 3 We formulate the problem to determine the optimal input commands necessary to track the angular displacement and velocity of the virtual rigid manipulator along the rotational desired trajectory. That is to say, by introducing the optimal control theory, we obtain the optimal input command, $u_{opt}(t)$, to satisfy a specified objective function. The optimal criterion to be minimized is expressed by

$$2J = \int_0^{t_f} L\{t, \mathbf{z}(t), \mathbf{u}(t)\}dt \qquad (11)$$

where

$$L\{t, \mathbf{z}(t), \mathbf{u}(t)\} = w_{L1}^2\{\theta_d(t) - \theta_{vir}(t)\}^2 + w_{L2}^2\{\dot{\theta}_d(t) - \dot{\theta}_{vir}(t)\}^2 + \left(\frac{\mathbf{u}(t)}{u_{max}}\right)^2$$

$$\theta_{vir}(t) = \theta(t) - tan^{-1}\{y(t)/R(t)\}$$

where, t_f is a final time. w_{L1} and w_{L2} denote weighting factors. u_{max} denotes the maximum input command.

step 4 We get the input commands which satisfy the optimal criterion by using Fletcher-Reeves method [5].

4 Simulation

We have simulated the input command which guarantees the zero tracking error about the desired trajectory shown in Fig.2. The results will be given for the flexible manipulator with the following design parameters:

Link 1;
- length of link 1, L_1 = 1000 (mm)
- mass per unit length, $\rho_1 A_1$ = 0.504 (Kg/m)

Link 2;
- length of link 2, L_2 = 1000 (mm)
- mass per unit length, $\rho_2 A_2$ = 0.504 (Kg/m)
- flexural rigidity of link 2, $EI = 94.76 * 0.05$ (Nm^2)
- damping ratio, $\xi = 0.02$

Other parameters;
- final time, t_f = 2 (sec)
- weighting factors, $w_{L1} = w_{L2} = 1000$
- value of the maximum input command, u_{max} = 1000

Fig.5 shows the elastic deformation of link 2 under the condition:

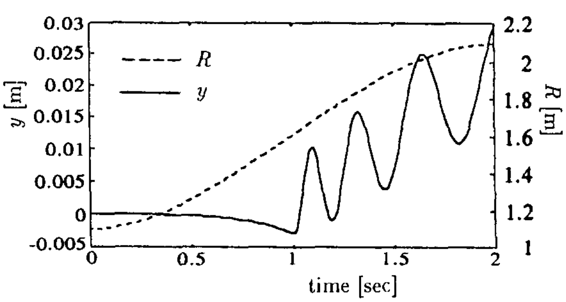
input command ;

$$u(t) = -sgn(t-1)$$

radial trajectory ;

$$R(t) = -0.25 * t^3 + 0.75 * t^2 + 1.1 \quad (0 \le t \le 2)$$

From Fig.5, we know that the more the length of flexible part is, the larger frequency of oscillation is. Fig.6 shows the optimal input commands $u(t)$ which are computed in cases of rigid manipulator and of flexible manipulator by using an optimal control theory, respectively. In case of rigid manipulator, we computed the optimal input command, assuming that the elastic displacement of Link 2, $y(x,t)$, is zero. Fig.7 and Fig.8 show respectively the angular displacement and velocity of the virtual rigid manipulator when the input commands $u(t)$ are applied to the manipulator. From Fig.7, we know that mean error of angular displacement is reduced in magnitude from 0.168(m) to 0.014(m). The result shown in Fig.8 is that mean error of angular velocity is reduced in magnitude from 0.636(m/s) to 0.098(m/s). From the above results, we may conclude that the scheme presented in this paper is effective in tracking the desired trajectory which is discontinuous in velocity.



$R(t) = -0.25 * t^3 + 0.75 * t^2 + 1.1$

Fig.5 Elastic Deflection of the End Effector of Link 2 and the Trajectory of a Radial Direction

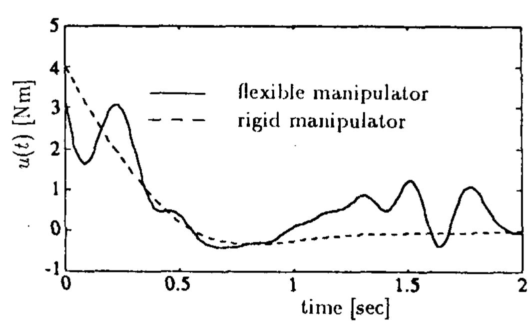


Fig.6 Optimal Input Commands Computed by Optimal Control Theory

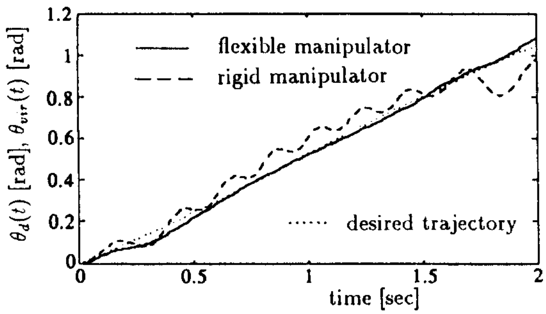


Fig.7 Trajectory Responses under the Optimal Input Commands (Angular Displacement)

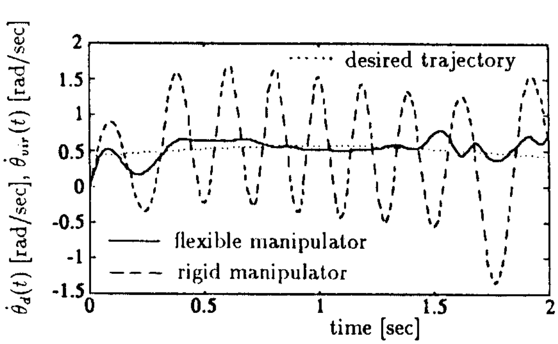


Fig.8 Trajectory Responses under the Optimal Input Commands (Angular Velocity)

5 Conclusion

One of the important issues on the tracking control of a flexible manipulator is how to determine a suitable control input command and to reduce the tracking error of the end effector along a discontinuous velocity trajectory. A procedure has been presented to obtain the input commands which are required to track the end effector of the flexible manipulator with a prismatic joint along a discontinuous velocity trajectory.

A simulation study was carried out with respect to the desired trajectory which moves uniformly along a straight line. The optimal trajectory responses obtained by the optimal input commands which considered elastic motion do effectively reduce tracking error than those which did not consider any elastic motion.

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