AN INTERACTIVE MULTICRITERIA SIMULATION OPTIMIZATION METHOD

Wan Seon Shin
Department of Industrial Engineering
Sung Kyun Kwan University

and

Carolyn R. Boyle
Department of Industrial Engineering
Mississippi State University, USA

This study proposes a new interactive multicriteria method for determining the best levels of the decision variables needed to optimize a stochastic computer simulation with multiple response variables. The method, called the Pairwise Comparison Stochastic Cutting Plane (PCSCP) method, combines good features from interactive multiple objective mathematical programming methods and response surface methodology. The major characteristics of the PCSCP algorithm are: (1) it interacts progressively with the decision maker (DM) to obtain his preferences, (2) it uses good experimental design to adequately explore the decision space while reducing the burden on the DM, and (3) it uses the preference information provided by the DM and the sampling error in the responses to reduce the decision space. This paper presents the basic concepts of the PCSCP method along with its performance for solving randomly selected test problems.

I. INTRODUCTION

Although simulation research is often presented in textbooks as a collection of isolated self-contained simulation experiments, in reality it is more often an iterative process consisting of formulating a research question (hypothesis), designing and conducting a simulation experiment to test the hypothesis, analyzing the results of the experiment, and deciding if the hypothesis is supported, if it needs to be reformulated, or if further experimentation is needed. This paradigm is particularly evident in simulation optimization experiments, where the objective of the research is to determine those values of the decision variables that will produce an optimal response. Such research is typically conducted as a series of simulation experiments, each one providing information that is used in designing succeeding ones. The process is terminated when the DM finds those values of the decision variables that yield an optimal or near optimal response.

Numerous approaches to the problem of optimizing computer simulation experiments have been proposed as evidenced in the review articles that have appeared on this subject [2][10]. Most research, however, has concentrated on single-response optimization or situations where the multiple responses can be combined in some way into a single response [1][3][14]. Although some research has attempted to address directly the multivariate nature of the multiple-response optimization problem [5][6][8], they fail to address the importance of man-machine interaction during the decision making process when multiple criteria are involved. The motivation of this study is reflected by the fact that many papers that propose modifications or extensions of interactive algorithms illustrate the new method by applying it to real-world problems [11].

The primary objective of this research is to develop a new interactive method for conducting multiple-response simulation optimization experiments. This research combines interactive methods from multiple objective mathematical programming (MOMP), statistical and experimental design techniques from response surface methodology (RSM), and the special features of stochastic simulation experiments. The goal of this research is to develop an approach that: (1) interacts progressively with the DM to obtain local information about his preferences, (2) uses good experimental design to adequately explore the decision space while reducing the burden on the DM by limiting the number of preference judgements needed, and (3) uses the preference information provided by the DM and the sampling error in the response variables to reduce the decision space.

II. MULTIPLE-RESPONSE COMPUTER SIMULATION EXPERIMENTS

A simulation experiment can be regarded as a mapping, called a response function, from the decision space (input space) to the response space (output space). Symbolically, this relationship can be expressed as

$$\underline{\eta} = (f_1(\underline{x}),\ f_2(\underline{x}),\ \ldots,\ f_k(\underline{x}))'$$

where

$\underline{\eta}$ is a k-dimensional true response vector,
$\underline{x}$ is an n-dimensional decision vector, and
f_j, j = 1, ..., k, are the response functions of $\underline{x}$.

In general, not all values of the decision variables are feasible or even of any interest to the DM. Thus the experimental region will only be a subset, X, of the decision space that takes the form

$$X = \{\underline{x} \mid g_i(\underline{x}) \le 0,\ i = 1, \ldots, m\}$$

where the $g_i(\underline{x})$ are constraints on $\underline{x}$.

In a stochastic simulation experiment, the measurements of the response variables are subject to random sampling error, i.e., the observed response is

$$\underline{y} = \underline{\eta} + \underline{\epsilon} = (f_1(\underline{x}),\ f_2(\underline{x}),\ \ldots,\ f_k(\underline{x}))' + \underline{\epsilon}$$

where $\underline{y}$ is a k-dimensional observed response vector and $\underline{\epsilon}$ is a k-dimensional vector of random sampling errors.

When there are multiple response functions, in order to impose an ordering on the response vectors, it is convenient to assume that the DM has a single-valued unknown preference function. Using this preference function, the DM can express preferences among the response variables to determine his best compromise solution.

Best Compromise Solution:

A decision vector $\underline{x}$ is the best compromise solution if it is efficient and the response vector, $(f_1(\underline{x}), f_2(\underline{x}), \ldots, f_k(\underline{x}))'$, maximizes the DM's preference function.

Within this framework, the objective of the multicriteria simulation optimization is to find the best compromise solution.

In this study, a number of statistical or functional assumptions were made. The assumptions are:

1. Objective functions are concave.
2. System constraints are linear.
3. The computer model has already been developed and validated and the DM is ready to perform experiments using the model to make inferences about the real system.
4. The individual components of the random sampling error vector are each assumed to have mean zero and jointly to have a variance-covariance matrix $\underline{\Sigma}$.
5. The vectors of random sampling errors are mutually statistically independent.
6. The vectors of random sampling errors have a multivariate normal distribution.

III. THE PAIRWISE COMPARISON STOCHASTIC CUTTING PLANE METHOD

It is evident that RSM, with its emphasis on experimental design and concern with random variation in the responses, and MOMP, with its emphasis on interactive techniques, can each be profitably applied to computer simulation optimization experiments. In this research, a new strategy, called the Pairwise Comparison Stochastic Cutting Plane (PCSCP) method, is developed by combining features of both techniques. The PCSCP method consists of four stages:

1. Find the center of the current feasible region in decision space.
2. Perform a designed experiment centered at that point.
3. Determine the most preferred experimental point by eliminating all the less preferred points. This is accomplished by:
   i) eliminating points that are dominated by other experimental points,
   ii) interacting with the DM using paired comparison questions, and
   iii) using a cutting plane based on the estimated gradient of the preference function in response space.
4. Reduce the feasible region in decision space by formulating a new constraint based on the estimated gradient of the preference function at the most preferred experimental point.

In the first stage, the center of the current feasible region in decision space is found using the Modified Method of Centers as in the TCP algorithm [7][12]. In the second stage, a designed experiment is conducted at the center that will allow good coverage of the experimental region and assessment of random variation in the responses. Because the designed experiment is necessarily performed at only a finite number of points, in the third stage the problem can be treated as a discrete MOMP. By interacting with the DM, the best of these discrete points can be found using a gradient-based approach [4]. After asking the DM a series of paired comparison questions, his preferences can be used to estimate the local gradient of the preference function in response space at the current decision point, taking into account the variation in the observed responses. This gradient is used to construct a cutting plane that may eliminate additional less-preferred points. The process of alternately interacting with the DM and eliminating points with the cutting plane continues until the most preferred experimental point is found. In the fourth stage, the problem is again considered as continuous. The estimated gradient at the most preferred experimental point is used to reduce the feasible region.

The detailed explanation of the method and illustration with a numerical example are referred to Shin and Carolyn [13].

IV. COMPUTATIONAL STUDY WITH THE PCSCP METHOD

In order to evaluate the performance of the PCSCP algorithm, ten problems selected from the literature [9] were tested under various conditions. Six evaluation criteria were used to assess the effectiveness of the method and they are: Number of iterations(NIT), Total number of questions (TQUEST), Number of questions per iteration (NQUEST), Proportion of increments, Average preference difference, and Difference from the theoretical optimum.

Analysis of Correlation and the size of experimental region

To investigate the impact of the level of correlation among the elements of the response vector and the size of experimental region on the performance of the PCSCP method, the test problems were solved under a number of conditions. These conditions included five levels of correlation among the elements of the response vector (0, 0.25, 0.50, 0.75, 0.90), four sizes of experimental region (100%, 75%, 50%, or 25% of the distance to the nearest constraint), and two estimation rules for calculating local gradients (calculation after the first round of comparisons or calculation after determining the best point in the current experiment). Each of ten test problems were solved five times, using independent random number streams, and the results on each of the evaluation criteria were averaged over the five repetitions. The computational results of this analysis can be found in Shin and Carolyn [13].

From this computational study, the following appear to be characteristics of the algorithm:

1. Positive correlation among the responses does not appreciably affect performance.
2. Larger experimental regions improve performance.
3. The rule for estimating local gradients does not appreciably affect performance.
4. The default criterion for termination based on the radius of the experimental region is probably too large.

Although TQUEST and NQUEST appear to be reduced in the presence of high positive correlation between the responses, the amount of correlation is generally unknown for a given simulation experiment. Zero correlation seems to represent the worst case situation.

Because there is not much difference between the two rules for estimating local gradients, the second rule is preferable because it reduces the number of interactions with the DM and of nonlinear programming problems that must be solved.

Analysis of Variation and Replications

The same ten test problems were tested under another 40 sets of conditions, using zero correlation, using the maximum size for the experimental region, and estimating local gradients after the first round of questioning. The new conditions included five levels of variation in the elements of the response vector (1.00, 0.05, 0.10, 0.05, 0.01), four numbers of replications at the comparison points (1, 3, 5, or 7 replications), and two stopping rules (stopping if the radius of the design region is less than 1 or stopping after 20 iterations). The computational results are referred to Shin and Carolyn [13].

This additional computational study of the algorithm suggests the following characteristics:

1. Moderate to low variation in the responses does not appreciably affect performance.
2. Greater replication at the comparison points may improve performance if the responses vary greatly.
3. Using stopping rule 1 increases the proportion of iterations in which the utility was improved from the previous iteration.
4. Using stopping rule 2 decreases the average difference from the optimum, but the total number of questions becomes unacceptably large.

Because the amount of variation in the responses is generally unknown for a given simulation experiment, a modification of the algorithm might involve using the variances calculated from the first experiment to decide on the number of replications at the comparison points. The stopping rule based on the radius of the design space is better than the stopping rule based only on the number of iterations. The default radius could be reduced to increase the precision of the estimate of the optimum. The total number of iterations should be limited to about 5 iterations so that the number of interactions with the DM does not become excessive.

V. CONCLUSION

This research developed a new interactive multicriteria method for determining the best levels of the decision variables needed to optimize a stochastic computer simulation with multiple response variables. The PCSCP method combines good features of interactive methods developed for MOMP problems and response surface methodologies. Initial computational studies verify the validity of the method and its robustness in solving problems under various situations.

VI. REFERENCES

[1] Biles, W. E. (1979), "Experimental Design in Computer Simulation," in Proceedings of the 1979 Winter Simulation Conference, pp. 3-9.

[2] Farrell, W. (1977), "Literature Review and Bibliography of Simulation Optimization," in Proceedings of the 1977 Winter Simulation Conference, pp. 117-124.

[3] Garcia-Diaz, A., Hogg, G. L., and Tari, F. G. (1981), "Combining Simulation and Optimization to Solve the Multimachine Interference Problem," Simulation, 28, 193-201.

[4] Malakooti, B. and Deviprasad, J. (1989), "An Interactive Multiple Criteria Approach for Parameter Selection in Metal Cutting," Operations Research, 37, 805-818.

[5] Matwiczak, K. M. (1990), "AMOS - Automated Multiple-Response Optimum- Seeker for Simulations," unpublished Ph.D. dissertation, Texas A&M University, Dept. of Industrial Engineering.

[6] Montgomery, D. C., and Bettencourt, Jr., V. M. (1977), "Multiple Response Surface Methods in Computer Simulation," Simulation, 29, 113-121.

[7] Musselman, K. J. and Talavage, J. J. (1980), "A Tradeoff Cut Approach to Multiple Objective Optimizations," Operations Research, 28, 1424-1435.

[8] Rees, L. P., Clayton, E. R., and Taylor, III, B. W. (1985), Solving Multiple Response Simulation Models Using Modified Response Surface Methodology Within a Lexicographic Goal Programming Framework," IIE Transactions, 17, 47-57.

[9] Ringuest, J. L., and Gulledge, T. R. (1985), "Interactive Multiobjective Complex Search," European Journal of Operational Research, 19, 362-371.

[10] Safizadeh, M. H. (1990), "Optimization in Simulation: Current Issues and Future Outlook," Naval Research Logistics, 37, 807-825.

[11] Shin, W. S., and Ravindran, A. (1991), "Interactive Multiple Objective Optimization: Survey I - Continuous Case," Computers and Operations Research, 18, 97-114.

[12] Shin, W. S., and Ravindran, A. (1992), "A Comparative Study of Interactive Tradeoff Cutting Plane Methods for MOMP," Forthcoming in European Journal of Operational Research.

[13] Shin, W. S., and Carolyn, R. B. (1992), "A Pairwise Comparison Stochastic Cutting Plane Method for Multicriteria Simulation Optimization," An Unpublished Manuscript (Submitted to Management Science).

[14] Smith, D. E., and Mauro, C. A. (1982), "Factor Screening in Computer Simulation," Simulation, 37, 49-54.