A Knowledge-Based DSS for the Decision Making under Multi-Objectives

崔 容銑 , 金 聖曦
韓國科學技術院 産業工學科

Abstract

본 논문은 多目的線形計劃法을 위한 Decision Support System ASEOV-VIM을 소개하고 있다. ASEOV-VIM에는 1) efficient solution set전체에 대한 개괄과 search direction을 제시하는 ASEOV; 2) decision maker의 preference information을 도출해내는 VIM; 3) 위 두 부분간의 inteface로 활용되는 Mediator 등 3 개의 하부시스템이 있다. ASEOV-VIM은 TURBO-C를 이용하여 personal computer에 구현되었다.

I. Introduction

Multiple objective linear programming (MOLP) has been of much research interest and resulted remarkable development over the past two decades [Hwang et. al., 1980; Evans, 1984; White, 1990; Shin and Ravindran, 1991].

The most efficient and widely employed MOLP methods take the two component structure. The first, *the analyzer*, is concerned with the identification of a possible or a set of possible nondominated solutions of the MOLP problem. And the second, *the preference extractor*, tries to induce the preference structure of the decision maker (DM) from the interaction with the solutions obtained by the analyzer.

While the tasks involved with the analyzer are well–posed both in analytical and in applied aspects, the same does not occur with the preference extractor. This is essentially due to the lack of a general agreement on how to employ the mathematical programming theory in order to allow the DM to express his preferences in a proper way. Considering the criticality of the DM's response in solving MOLP, a DSS approach is rather essential to help DM in expressing his preference and solving the problem.

Multiple Criteria Decision Support System (MCDSS) are considered as a specific type of system within the broad family of DSS [Jelassi et al., 1985]. Even though MCDSS include much the same components as traditional DSS, MCDSS have special characteristics that distinguish them from other DSS : 1) allow analysis of multiple criteria; 2) use a variety of multiple criteria decision methods to compute efficient solutions; and 3) incorporate user's input in various phases of problem solving.

In this paper, we introduce a MCDSS ASEOV–VIM which has the following three components: 1) **ASEOV** presents the overall structure of the efficient solutions and the candidate goal for the search direction; 2) **VIM** presents the efficient trajectories along the search direction and acquires DM's preference information; and 3) **Mediator** plays the role of knowledge based interface between the two components, to enhance the role of each component and to guide DM through the decision making process. Menus and interactive uses of computer graphics play a central role in ASEOV–VIM. ASEOV–VIM is written in TURBO–C and implemented on an personal computer. An illustrative Example is provided.

II. Approximation of the Set of Efficient Objective Values

In this section, we introduce subsystem ASEOV (Approximation of the Set of Efficient Objective Values) to systematically approximate the set of efficient objective values, let it be denoted by N, in general p–dimensional objective linear problems. With the initial approximation error allowance, which can be under the control of the DM, ASEOV can present DM with the overall structure of N without DM's any burden. And this insight over N will make DM easier in assessing his own preference to find out the final best compromise solution, rather than given with just some subset which does not represent the whole N. A new way of determining the maximum approximation error of a given approximate of N, using the *Tchebycheff metric*, is introduced. By handling on the linearly transformed objective set Y directly, the unnecessary computational efforts at the extreme points of X which are transformed to nonextreme points of Y are reduced, increasing the computational efficiency as a result.

The MOLP can be stated as

$$\max_{x \in X} \; f(x) = (f_1(x), \ldots , f_p(x))$$

where $X = \{ x : x \in R^n,\ A x \leq b,\ x \geq 0,$ where A is a $m \times n$, and b is a $m \times 1$ matrix $\}$.

Let,

$Y = f(X) = \{ f(x) : f(x) \in R^p,\ x \in X \}$,

$N = \{ f(x) : f(x) \in Y,\ x \text{ is efficient} \}$,

and $N_{ex} = \{ f(x) : f(x) \in N \text{ and } f(x) \text{ is an extreme point of } Y \}$.

Since $f(x)$ is a linear transformation from R^n to R^p, $Y = f(X)$ is a polyhedral set in R^p for every polyhedral set $X \subset R^n$. And every polyhedral set is the intersection of finite collection of hyperplanes, there should exist G and g, such that $Y = f(X) = \{ y : y \in R^p, G y \leq g$, where G is a $m' \times p$ and g is a $m' \times 1$ matrix $\}$.

Let $IdE(y^0)$ denote the indices of the face hyperplanes of Y which contain at least one of the *efficient* edges incident to any $y^0 = f(x^0) \in N_{ex}$.

Definition linear manifold $ME(y^0)$

For any $y^0 \in N_{ex}$, let the linear manifold $ME(y^0)$ denote the set of $\{ y \in R^p : G_i y = g_i, i \in IdE(y^0) \}$, where G_i (or g_i) represents the i th row vector (element) of $G(g)$.

That is the linear manifold $ME(y^0)$ denotes the intersection of the face hyperplanes, of Y which contain at least one of the efficient edges incident to $y^0 \in N_{ex}$. Then every efficient edge of Y incident to y^0 is contained in $ME(y^0)$ by the definition. This assures that every efficient extreme point and edge of Y are contained in $\cap_i \{ ME(y^i)$, where $y^i \in N_{ex} \}$.

For any $y^0 = f(x^0) \in N_{ex}$, an adjacent extreme point of y^0 need not be the image of an extreme point of X that is adjacent to $x^0 \in X$. This collapsing effect [Dauer, 1987; Dauer and Liu, 1990] shows that handling directly on objective space can save some computational effort in determining the efficient reduced cost coefficient vector for any y^0 at the given efficient multiobjective simplex tableau $T(x^0)$. That is, it can reduce the number of extreme points of X which need to be analyzed.

Theorem [Dauer and Liu, 1990] – ***Edge Test***

Let $y^0 = f(x^0) \in N_{ex}$ and let $R = (r^1, .., r^h)$ be the reduced cost coefficient matrix in the corresponding multiobjective simplex tableau $T(x^0)$. Let E^j be the edge of X determined by column j in $T(x^0)$. The image of E^j under f is contained in an edge of Y if and only if r^j is in a frame of cone (R).

Remark

For any $y^0 = f(x^0) \in N_{ex}$, an edge of Y incident to y^0 can be represented by $y^0 + d \cdot r^j$, $0 \leq d \leq \Delta_j$, where Δ_j is the corresponding minimum ratio scalar at $T(x^0)$ [Bazaraa and Jarvis, 1977] and if r^j is not redundant. By the edge test, the collapsing extreme points of X adjacent to x^0 are identified. And this will reduce the unnecessary computational effort of further analyzing at those points.

And, in order to determine the efficiency status of any r^j which is in a frame of cone(R) at a given efficient multiobjective simplex tableau $T(x^0)$, we use the *Efficiency Test.*

Theorem [Ecker and Kouada, 1978] – ***Efficiency Test***

The edge E^j of X determined by the j th column of an efficient and nondegenerate tableau $T(x^0)$ is efficient iff there is a $\lambda > 0$ such that $\lambda^t R \leq 0$ and $\lambda^t r^j = 0$.

This method requires neither linear programming subproblem to test the efficiency of each incident edge, nor bounded X. And even the degenerate X can be handled [Ecker and Kouada, 1978; Dauer and Liu, 1990].

When the frame of cone(R) and efficient r^j's of $y^0 = f(x^0) \in N_{ex}$ are determined, we can get a manifold $ME(y^0)$, which intersectively comprises N. ASEOV approaches to determine the minimal subset of N_{ex}, let it be denoted by IN_{ex}, and the corresponding manifold $ME(y^i)$ for each $y^i \in IN_{ex}$. And the intersection of these manifolds, $\cap_i \{ ME(y^i)$, where $y^i \in IN_{ex} \}$, which is an upperbound of N, is used as an approximate of N. This approximate contains the sparse representative subset of N under certain allowance. As the set IN_{ex} grows, the approximate of N will get closer to real N and the approximation error will decrease. The algorithm ASEOV determines minimal IN_{ex} of which corresponding approximation error is under given allowance.

The Tchebycheff Metric and the Approximation Error

Some of the extreme points of this intersected manifold, i.e. the approximate of N, for any $IN_{ex} \subset N_{ex}$ are infeasible to X. These *infeasible–to–X* extreme points of given approximate of N are used as the reference points, where each approximation error is measured. And the maximum approximation error at each approximation stage is determined as the maximum of the approximation error computed at every reference point.

We use the Tchebycheff metric to measure the approximation error at each reference point. That is,

$$\begin{aligned} &\min\ \alpha \\ &\text{s.t. } \alpha \geq \lambda_i^{rf}(rf_i - f_i(x)), \qquad 1 \leq i \leq p, \quad x \in X, \end{aligned}$$

where rf_i represents the i th objective value of a reference point rf. This problem finds out the closest point on the face of Y directly, from the reference point in the direction of λ^{rf}.

The direction vector λ^{rf} used in the above problem is given as the average of the outer normal vectors of the face hyperplanes of Y where the reference point rf lies. Let $Idr(rf)$ denote the indices of those face hyperplanes, for any reference point rf. That is, $G_i \cdot rf = g_i$ iff $i \in Idr(rf)$. Then the above problem can be considered as determining the averaged deviation for each criterion, i.e. $Avg(\Delta f(x))$, between the reference point rf and every point y which lies on any currently unknown face hyperplanes of Y whose outer normal vector is represented by a convex combination of G_i's, $i \in Idr(rf)$. Let the solution of the above problem be $x^{\sim}$. Then, our approximation error estimate is computed as $rf - f(x^{\sim})$. For easy comparison, these values will be converted into the % deviation for each criterion as

$$\frac{rf_i - f_i(x^{\sim})}{\max_{x \in X} f_i(x) - \min_{x \in X} f_i(x)}.$$

When the DM is more concerned about certain objective criteria, he can assign different allowance for each criterion.

III. Visual Interactive Method

Visual interactive method (VIM) is one of the recent popular approaches in solving MOLP. VIM is employed for the easy elicitation of DM's preference by displaying the efficient frontier of the improving direction [Winkels and Meika, 1984; Korhonen and Laakso, 1986].

There are several techniques for generating efficient solutions depending on the different methods to scalarize the multiobjective decision making (MODM) problem. The *achievement scalarizing function* is one of them [Wierzbicki, 1980]. The achievement scalarizing function used in this study is

$$S(q, f(x), \lambda) = \max_i \{\lambda_i (q_i - f_i(x))\} - \epsilon \, \Sigma_{i=1}^{p} f_i(x)$$

where ϵ is a very small positive scalar, q is a given reference point in objective space, and λ is a given weighting vector.

The reference point may or may not be feasible. An achievement scalarizing function $S(q, f(x), \lambda)$ is one that projects the reference point q onto N. The projected point in N is defined by the objective vector that lies on the lowest valued frontier of $S(q, f(x), \lambda)$ that intersects Y. Such an objective vector is obtained by solving the *achievement scalarizing program*

$$\min \{ S(q, f(x), \lambda) \mid x \in X \}$$

Now, MOLP can be converted to the following achievement scalarizing program.

$$\begin{aligned} \min \quad & \{\alpha - \epsilon \Sigma_{i=1}^{p} f_i(x)\} \\ \text{s.t.} \quad & f_i(x) + \alpha/\lambda_i \geq q_i, \quad i = 1, .., p \\ & x \in X \end{aligned}$$

in which $\alpha \in R$ is, in general, unrestricted in sign.

To project onto N an unbounded line segment $\mu(q, d)$ that emanates from q in the direction d, we solve the *achievement scalarizing parametric program*

$$\begin{aligned} \min \quad & \{\alpha - \epsilon \Sigma_{i=1}^{p} f_i(x)\} \\ \text{s.t.} \quad & f_i(x) + \alpha/\lambda_i \geq q_i + \theta \cdot d_i, \quad i = 1, .., p \\ & x \in X \end{aligned}$$

for θ going from 0 until the solutions of the above program exist on the efficient facets. The step–size $\theta*$ is determined by selecting a preferred solution from the projected trajectories of efficient solutions. The weighting vector λ takes as the vector $(F* (x) - (q + g)/2)$.

IV. A Knowledge–Based Interface *Mediator*

One of the evolving problems of the interactive MOLP methodologies is the inconsistency from the DM's replies. The inconsistencies of DM may cause much problem in interactive methods, since whether the procedure is progressive and convergent depends mainly on the capabilities of DM. However there is not any clear solutions to this drawback yet.

The opposite case of inconsistency is when the number of alternatives, candidate reference goals or efficient extreme points to investigate, presented to DM are too large. From the human psychological viewpoint, the number of proper capacity (alternatives) to process (selection) is 7 ± 2 [Miller, 1968].

In this section we propose a knowledge–based interface between **ASEOV** and **VIM**, called *Mediator*. The Mediator will act as a multi–criteria decision analyst. The role of Mediator is to interface between these two components, enhancing the role of each component and finally deduce properly satisfied solution.

The aim of *Mediator* is to maximally utilize the DM's preference information provided from his replies, with minimum burden to DM. In the context of extended DSS, it is required to develop a system based not only on a set of methods but also on knowledge about usage of the methods and their possibilities. From an operational point of view we tried to build a type of intelligent (or so–called 'knowledge–based') interface between the

MCDA methods and a user who is not familiar with these methods. The peculiarity of this interface is that its knowledge base does not contain knowledge about a specific problem domain, but consists of the elements of the methodological knowledge used in multicriteria decision aid.

Determination of Search Direction

The search direction $d^{(k)}$ (k is the stage count) to find out the next better solution is determined as follows. The currently determined elements of N_{ex} and the *infeasible–to–X* reference points will be used as the candidates for the reference goal $g^{(k)}$. As the approximation proceeds the number of the candidates will grow. To reduce the burden of DM and to maximally utilize the DM's preference information, the dominance cone [Gal, T. and Kim, S.H., 1992] is used to prescreen these candidate criterion vectors.

The dominance cone is constructed based on the DM's preference information given in selecting his preferred solution from the efficient trajectories by VIM. Let's say there are 5 breaking points y^i in the efficient trajectories. If the DM has selected y^* not any one of them, then it means $U(y^*) > U(y^i)$, $i = 1, .., 5$. These will be transformed as $w\,y^* - w\,y^i \geq \epsilon$, $i = 1, .., 5$.

This constraint set constitutes a dominance cone on objective space, and is used to pre–screen the candidate criterion vectors. And the goal $g^{(k)}$ is determined as the ideal point of those which passes screening. Then the search direction is determined as $d^{(k)} = g^{(k)} - q^{(k)}$, where $q^{(k)}$ is the current criterion vector.

V. A MCDSS *ASEOV–VIM*

The overall structure of the proposed MCDSS *ASEOV–VIM* is presented in figure 1.

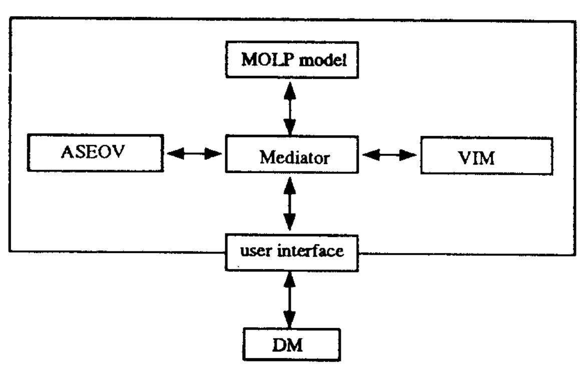

Figure 1. The Structure of ASEOV–VIM

The role of each component are as follows:

1) **ASEOV** plays the role of the analyzer. It presents the subset of efficient solutions and candidate goals for the next search direction around where DM is interested based on his preference information. Another characteristic of **ASEOV** is that it gives the overall structure of *N* under given alowance which helps DM in assessing his preferences.

2) **VIM** plays the role of the preference extractor. It presents the efficient trajectories along the search direction starting from the current solution and acquires DM's preference. The computer graphics play a central role in **VIM**.

3) **Mediator** acts like a multiple criteria decision analyst. The role of mediaor is to interface among the above components and finaly deduce the properly satisfied solution.

And the user interface offers various menus and graphics to help the system user who is not familiar with MOLP. Each component will be explained briefly with corresponding result by the following example.

Consider the MOLP of table 1. The problem and the convergence of the solutions are shown in figure 2.

Table 1. An Example Problem

$max\ f_1(x)$	1				
$max\ f_2(x)$		1			
$max\ f_3(x)$			1		
	1.7	1.8	9.7		100
	4.3	2.3	8.7		100
	2.4	4.3	8.5		100
	6.5	2.5	7.1		100
	5.0	5.0	7.2		100
	2.7	6.4	7.1		100
	8.2	2.7	4.8		100
	6.9	5.1	5.1		100
	5.1	7.1	5.2	≤	100
	2.7	8.0	2.4		100
	9.3	2.5	2.6		100
	8.4	4.8	2.5		100
	6.8	6.8	2.6		100
	4.8	8.4	2.6		100
	2.6	9.2	2.5		100
	-4.4	-8.0	6.0		42.0
	-5.3	7.9	-5.1		51.0
	5.8	-3.2	-7.5		35.0

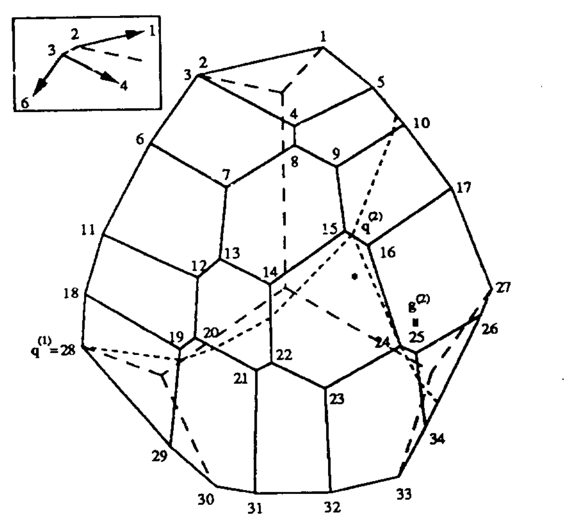

Figure 2. The Problem and the Solutions

The ideal point of this problem is $F^*(x) = (\ 9.9139,\ 9.5619,\ 9.9050\ )$. Let's say the utility function of the DM is a Tchebycheff function $min\ max\ \lambda_i\ (\ f_i^*(x) - f_i(x)\ )$ with $\lambda = (\ .2,\ .5,\ .3\ )$. Then the optimum solution is (3.7810, 7.1087, 5.8164).

1) ASEOV : The initial approximate of N after ASEOV is shown in figure 3 (a). There are 14 currently determined elements of N_{ex} and 12 reference points.

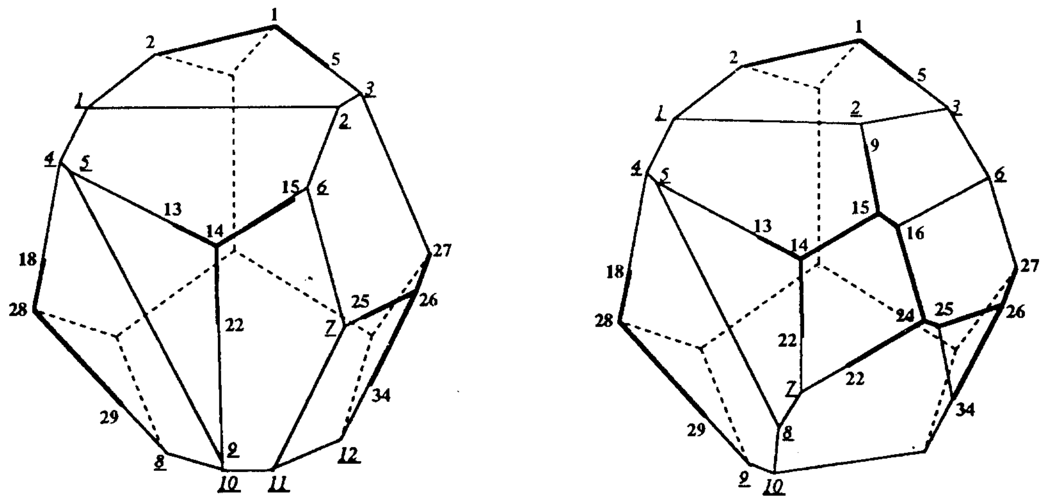

Figure 3. Approximate of N with Reference Points at (a) stage 1 and (2) stage 2

2) Mediator : As there is no preference information, Mediator sets $q^{(1)} = F^{1*}(x) =$ (9.9139, 0.0000, 3.0001), $g^{(1)} = F^{*}(x)$, $d^{(1)} = g^{(1)} - q^{(1)} =$ (0.0000, 9.5619, 3.9047), and $\lambda^{(1)} = F^{*}(x) - (q^{(1)} + g^{(1)})/2 =$ (0.0000, 4.7810, 1.9530).

2) VIM : The efficient contour presented to DM by VIM is shown in figure 4, an example screen of ASEOV–VIM.

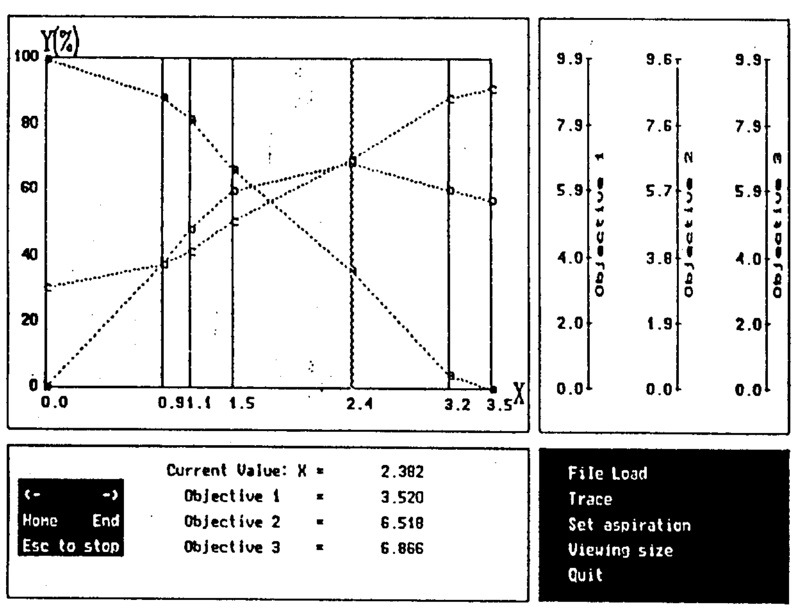

Figure 4. An Example Screen of ASEOV–VIM

There are four windows shown. The window (a) shows the efficient trajectories along the search direction with the relative achievement (in %) of each criterion. The window (b) shows the real range of each objective value (*min* $f_i(x)$, *max* $f_i(x)$, and some middle point values). The window (c) shows the value of each criterion at a selected point. And the window (d) shows the available menus at the given circumstance. By selecting the menu "Trace", the user can follow the given efficient trajectories by controlling the left and right arrows. The vertical dotted line moves along the trajectory and the user can see value of each criterion at a selected point. The menu "Set Aspiration Level" is used to assign the aspiration level for a criterion or more if DM wants. The assigned aspiration level with the selected objective criterion will be returned to the original problem as additional constraints, like $f_i(x) \geq$ certain value. And the "Viewing Size" menu will be used when DM want to see some portion of the given trajectories if he is interested and want to see more carefully some specific region. Let DM has selected point (3.5305, 6.5224, 6.8626) as the best of them.

4) Mediator : Based on the DM's preference information, dominance cone is sonstructed as $w\ (\ y^* - y^i\ ) \geq 0$. After screening point 2,4,25,29 remain. It informs ASEOV to investigate around these points.

5) ASEOV : Figure 3 (b) shows the approximate N at stage 2 with more elements of N_{ex} and some new reference points.

By this way the solution is improved. Table 2 shows the solutions and the converging rate.

Table 2. The Convergency of the Solutions

iteration		f (x) (% deviation from optimum)		
0	$q^{(1)}$	9.9139 (+162%)	7.1087 (-100%)	5.8164 (-48.4%)
1	$q^{(2)}$	3.5305 (-6.7%)	6.5224 (-8.3%)	6.8629 (+18.0%)
2	$q^{(3)}$	3.5474 (-6.2%)	7.3905 (+4.0%)	5.6607 (-2.7%)
	optimum	3.7810	7.1087	5.8104

VI. Conclusion

A multi–criteria decision support system ASEOV–VIM has been developed. By integrating the three components, ASEOV–VIM can be a good decision aid for the large scale complex MOLPs. The menus with the graphic interface helps DM who is not familiar with MOLP. The subsystem which helps the modelling of MOLP is not implemented yet, which can be of interest for the further research.

REFERENCES

1. Bazaraa, M. S., and Jarvis, J. J., *Linear Programming and Network Flows*, John Wiley and Sons, New York (1977).
2. Dauer, J. P., and Liu, Y. –H., "Solving Multiple Objective Linear Programs in Objective Space", *European Journal of Operational Research* 46 pp. 350—357 (1990).
3. Dauer, J. P., and Saleh, O. A., "Constructing the Set of Efficient Objective Values in Multiple Objective Linear Programs", *European Journal of Operational Research* 46 pp. 358—365 (1990).
4. Dauer, J. P., "Analysis of the Objective Space in Multiple Objective Linear Programming", *Journal of Mathematical Analysis and Applications* 126 pp. 579—593 (1987).
5. Ecker, J. G., Hegner, N. S., and Kouada, I. A., "Generating All Maximal Efficient Faces for Multiple Objective Linear Programs", *Journal of Optimization Theory and Applications* 30 pp. 353—381 (1980).
6. Evans, G. W., "An Overview of Techniques for Solving Multiobjective Mathematical Programs", *Management Science 30* pp. 1268—1282 (1984).
7. Gal, T. and Kim, S.H., "A New Interactive algorithm for Multicriteria Linear Programming Using Maximally Changeable Dominance Cone", *European Journal of Operational Research* to be published, (1992).
8. Hwang. C. L., Paidy, S. R., and Yoon, K., "Mathematical Programming with Multiple Objectives: A Tutorial", *Computers and Operations Research* 7 pp. 5—31 (1980).
9. Jelassi, M. T., jarke, M., and Stohr, E., "Designing a Generalized Multiple Criteria Decision Support System", *J. of Management Information System I*, pp. 24—43 (1985).
10. Korhonen, P. and Laakso, J., "A Visual Interactive method for solving the Multiple Criteria Problem", *European Journal of Operational Research* 24 pp. 277—287 (1986).
11. Shin, W. S. and Ravindran, A., "Interactive Multiple Objective Optimization: Survey I—Continuous Case", *Computers and Operations Research* 18 pp. 97—114 (1991).
12. Miller, G. A., "Magical Number Seven, Plus or Minus Two: Some Limits on Our Capacity for Processing Information", *Psychological Review* 63 pp.140—156 (1956)
13. White, D. J., "A Bibliography on the Applications of Mathematical programming Multiple—Objective Methods" *J. of Operational Research Society* 41 pp. 669—691 (1990).
14. Wierzbicki, A., "The use of Reference Objectives in Multiobjective Optimization", In *Multiple Criteria Decision Making, Theory and Applications* (Fandel, G. and Gal, T. Eds.), Springer, New York (1980).
15. Winkels, H.—M. and Meika, M., "An Integration of Efficiency Projections into the Geoffrion Approach for Multiobjective Linear Programming", *European Journal of Operational Research* 16 pp. 113—127 (1984).