DEVELOPMENT OF COMPUTER SOFTWARE FOR CALCULATION OF VOLUMETRIC ERROR MAP IN 3 AXIS CMMs

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ABSTRACT

Verification, calibration, and compensation are becoming more essential elements for manufacture and maintenance of high performance CMMs. A computer module of volumetric error generation has been developed to calculate volumetric errors (random as well as systematic) from measured parametric errors, accepting most types of CMMs in current use. New transformation rules have been derived to transform all the parametric errors with respect to the origin of working volume considered, then incorporated into the module of error calculation. Two cases of practical CMMs are tested with the developed module, and showed good performance.

1.INTRODUCTION

Coordinate measuring machines (CMMs) have given revolutionary effects to manufacturing industry, in that they can provide efficient, economic, and thus more productive means of quality control to the manufacturing processes. Verification, calibration, specification, and compensation are becoming more essential elements for users as well as manufacturers. Volumetric errors, which can be obtained from kinematic combination of parametric errors in working volume,have been one of relevant issues for the CMM calibration/compensation, as all the measurement tasks are directly or indirectly involved within the volumetric errors.

In the assessment of volumetric errors, there were some difficulties:(1) Error propagation mechanics, that is, volumetric error equations are varying with the kinematical configurations of machines. Even in the same kinematic configuration, different arrangements in coordinate axes may give different volumetric error equations;(2) Measured parametric errors at one location may be different from that at the other location within the working volume of CMM, thus the effects of measurement offsets (distance from reference axis) have to be considered (transformation) prior to calculating volumetric errors.

In this paper, a new general purpose computer module has been developed to generate volumetric errors from measured parametric errors, accepting most types/configurations of CMMs, which have been currently used in the industry. Several sets of new transformation rules have been derived to transform all the measured parametric errors with respect to a reference point (origin of the working volume considered), then incorporated into the module of volumetric error generation.

2. MODEL DESCRIPTION AND VOLUMETRIC ERROR EQUATIONS

As various types of CMMs are in current use, the developed module aims to cover those various types and configuration of commercial CMMs. Many national standards ([1],[2],[3]) such as British Standards, ANSI/ASME, and VDI/VDE describe several models. For example, the BS classify 8 models depending on the machine configurations, which can be grouped into 10 kinematically different models, depending on machine kinematic configuration and coordinate axes arrangements. As the parametric error method (synthesis technique) has been used in this module, 21 independent error components are considered for the volumetric error map within the working volume. Also, effective thermal expansion coefficients are considered in the volumetric error equations for thermal errors. The volumetric error equations for the error map have been derived via rigid body kinematics, considering the transformation and rotation matrices of each moving machine element.([4],[5]) All the volumetric error equations are in the appendix.

2.1 Moving Bridge CMM

Moving bridge, cantilever, gantry,L-shaped type CMMs can be grouped into the moving bridge CMM, as the above 4 types have kinematically identical configurations. The moving bridge type has two possible models, MB1, MB2, depending on the coordinate axes arrangement, as shown in fig.1a.

2.1.1 MB1 model

Carriage motion along the bridge is considered as X axis, the bridge motion along the base as Y axis, and the vertical motion along the vertical ram as Z axis.

2.1.2 MB2 model

Bridge motion along the base is considered as X axis, Carriage motion along the bridge as Y axis, and the vertical motion along the vertical ram as Z axis.

2.2 Fixed bridge CMM

Fixed bridge type CMM has horizontal table motion, carriage motion, and vertical ram motion. Two different kinematic models are possible as shown in fig.1b, depending on the coordinate axes configuration.

2.2.1 FB1 model

Carriage motion along fixed bridge is considered as X axis, table motion along CMM bed as Y axis, and vertical ram motion as Z axis.

2.2.2 FB2 model

Table motion along CMM bed is considered as X axis, carriage motion along fixed bridge as Y axis, and vertical ram motion as Z axis.

2.3 Moving horizontal arm CMM

Moving horizontal arm CMM has 3 axis movements:vertical column motion on CMM bed, vertical carriage motion along the vertical column, and horizontal arm motion along the carriage, and it has two possible kinematic models depending on the axes configuration, as shown in fig.1c

2.3.1 MH1 model

Motion of the moving horizontal arm is considered as X axis, motion of the vertical column along CMM bed as Y axis, and motion of the vertical carriage along the vertical column as Z axis.

2.3.2 MH2 model

Motion of the vertical column on CMM bed is considered as X axis, motion of the horizontal arm along the vertical carriage as Y axis, and motion of the vertical carriage along the vertical column as Z axis.

2.4 Fixed horizontal arm CMM

Fixed horizontal arm CMM has 3 axis motions: table motion on CMM bed, motion of vertical column on the CMM bed, and motion of vertical carriage along the vertical column; and it has two possible kinematic models depending on the axes configuration, as shown in fig.1d.

2.4.1 FH1 model

The table motion on CMM bed is considered as X axis, motion of the vertical column on the CMM bed as Y axis, and motion of vertical carriage along the vertical column as Z axis.

2.4.2 FH2 model

Motion of the vertical column on the CMM bed is considered as X axis, motion of the table on the CMM bed as Y axis, and motion of the vertical carriage along the vertical column as Z axis.

2.5 Column type CMM

Column type CMM has slight different machine configuration from the others, in that it has two axes table motions, primary and secondary axis. Let the primary axis be the axis of motion which is generated by lower machine guide way of table; the secondary axis as the axis of motion by upper machine guide way of table. The column type CMM has 3 axis motion:the primary axis motion of table, secondary axis motion of the table, and the vertical ram motion; and it can be divided into two kinematic models, CX1, CY1, depending on the coordinate axes configuration.

2.5.1 CX1 model

Motion of the primary axis is considered as X axis, motion of the secondary axis as Y axis, and motion of the vertical ram as Z axis.

2.5.2 CY1 model

Motion of the secondary axis is considered as Y axis, motion of the primary axis as Y axis, and motion of the vertical ram as Z axis.

3.PARAMETRIC ERROR INPUT

The aim of the parametric input module is to input all the parametric errors, then generate data files, which will be used in the next stages.

3.1 File name structure

As the developed system is designed to work around limited memory environment such as DOS, it is vital to manage a cluster of data files for information transfer between the modules. In view of file management, a specific file name structure is designed, in which file name consists of 3 parts:(1) machine specification name (2) index for systematic and random error (3) extension name for parametric error index.

(1) Machine specification name

The machine specification name can have any alpha numeric characters up to 7 characters, and it is for convenience of sorting the family of parametric errors when various machines calibration data are mixed up. The machine specification name can be manufacturer's name, or laboratory's name,e.g.UMIST1, etc.

(2) Index for systematic and random error

Every parametric error can have both systematic and random portions, where the systematic error indicates highly reproducible systematic characteristics and the random error indicates degree of scattering of error in random pattern. "S" is given for the systematic error; "R" is given for the random error, e.g. UMIST1S for systematic error of UMIST1 machine.

(3) Extension name for parametric error index

For convenience of data file indexing, specific sets of parametric error index are given to the extension name. The indexes are as follows;

Master control file :.CTL

CTL is given to the extension name for the master control file which includes informations on machine specification, and working volume.

Positional error file :.DPX,.DPY,.DPZ

Extension name DPX,DPY,and DPZ are given for the files of positional errors along X,Y, and Z axis, respectively.

Straightness error file : .DX_iY_j ($1<i,j<3$)

DX_iY_j($1<i,j<3$) is given for the files of straightness error, that is,

- DYX,DZX for the files of Y,Z straightness errors along X axis;
- DXY,DZY for the files of X,Z straightness errors along Y axis;
- DXZ,DYZ for the files of X,Y straightness errors along Z axis.

Angular error file : .EX_iY_j ($1<i,j<3$)

EX_iY_j($1<i,j<3$) is given for the files of angular errors, that is,

- EXX,EYX,EZX for X,Y,Z rotational errors along X axis;
- EXY,EYY,EZY for X,Y,Z rotational errors along Y axis;
- EXZ,EYZ,EZZ for X,Y,Z rotational errors along Z axis.

Squareness error file : .ALP,.BT1,.BT2

ALP,BT1,BT2 are given for the files of squareness error in XY,YZ, and ZX plane, respectively.

Thermal error file : .THX,.THY,.THZ

THX,THY,and THZ are given for the file of thermal coefficient in X,Y,and Z axis respectively.

For example, UMIST1S.DYX indicates the file name of Y straightness error (systematic) along X axis of UMIST1 machine.

3.2 Master control file

The developed system begins with data input of machine informations, such as machine name, examiner, environmental conditions, machine type, initial nominal coordinates of X,Y,X axis, step sizes in each axis. The machine model is one of possible configurations mentioned above in section 2. These informations are then saved into an ASCII text file, which is called "master control file", having extension name ".CTL". The master control file will always be referred at each stage of module execution for machine informations.

3.3 Parametric error file

(1) Positional/straightness/angular error file

The files of positional/straightness/angular errors are ASCII text files, which includes the information on respective errors. The contents of the error files are machine type, measurement offset within the working volume, and respective parametric error data. The measurement offsets are significant in case of positional/straightness errors, and it will be discussed in later sections.

(2) Squareness/Thermal error file

The file of squareness errors is also ASCII text files, which include machine specification name, machine type, measurement offsets, and squareness errors. The measurement offsets are used in the stage of error transformation. Thermal error files, ASCII text files, include thermal expansion coefficients along relevant axis. [unit:μm/m/C]

3.4 Input modes of Positional/straightness/angular errors

The input modes for parametric errors are designed to accommodate numerical synthesis data as well as practical calibration data.

(1) Systematic error

In case of systematic error input, three input modes are offered, key board input, curve input, and data file input; key board input mode makes it possible to enter the parametric data by pressing key board, curve input mode is designed to enter numerical synthesis data easily by best fitted polynomial curves. The polynomial curve is up to 10th order by the least squares approach, where few sampling data input are utilised to calculate error data over the whole range. The inputted parametric error data are then saved into each data file with relevant file name, and the data files also can be used for the entry of parametric error data. External data files can also be used as the data file if similar file format is followed.

(2) Random error

Random error input method consists of 3 modes: key board input, uniform bandwidth input, and data file input. Identical explanations are applicable to the case of key board input and data file input modes. In addition,the random error can be assigned as a uniform value over a whole measurement span, because the random portion of a parametric error often can be picked up from repeatability, which is the maximum value of 2σ or 3σ of repeated measurements.

The inputted parametric errors can be plotted on the computer screen,or plotter, then saved into each data file of specific file name as mentioned in 3.1

4.Transformation of parametric errors

In general, measured parametric errors such as positional, straightness, squareness errors are varying with the measurement location(measurement offset). For example, two X positional errors at two different measurement locations may not give identical values, because some angular errors may be associated with the measurement offsets(distance between the two measurement locations).In practice, however, it is unavoidable to allow some extent of measurement offsets due to physical limitations in measurement environment such as optical and mechanical obstacles. Therefore empirical transformation rules are desirable to globalise the measured parametric errors, giving unique values with respect to a reference point, such as machine origin or origin of the working volume considered. Kunzmann et al.([6]) considered the effect of angular roll errors on squareness errors, in order to calculate the roll errors from squareness measurement in 2D objects. In this paper, the empirical transformation rules have been derived for the above 10 kinematically different models, using volumetric error equations and by intuition. Most of positional,straightness, and squareness errors are influenced by the associated angular errors with measurement offsets.

Let (x_o, y_o, z_o) be the reference point or origin of working volume, where all parametric errors have to be transformed; (x,y,z) be the practical measurement location within the working volume. The transformation rules have slight different formulation depending on their kinematic models,and the full formulations are given in the appendix A.2.

5. Volumetric error map

5.1 Construction of volumetric error

As the parametric errors are inputted, and saved into specific files ofparametric errors, volumetric errors are now calculated with respect to a reference point, that is the origin of the concerned working volume, using the volumetric error equations which were mentioned in section 2 and the appendix A.1.

The random portion of the volumetric error can be calculated by the quadrant formula([7],[6]) of error propagation, i.e., random error $R\Delta X_i\,(1<i<3)$ is,

$$R\Delta X_i = \sqrt{\sum_j ((\partial \Delta X_i/\partial E_j \circ \Delta RE_j)(\partial \Delta X_i/\partial E_j \circ \Delta RE_j)) + (\Delta X_p \circ \Delta X_p)}$$

where ΔX_i $(1<i<3)$ is systematic volumetric error

E_j $(1<j<21)$ is systematic parametric error

ΔRE_j $(1<j<21)$ is random parametric error

ΔX_p is random probing error.

The data files of parametric errors are assessed, and the offset coordinate of probe stylus(X_p, Y_p, Z_p) are inputted to complete volumetric error calculation. The temperature deviation from a reference temperature are also inputted to the system for thermal error effects. After the systematic and random volumetric errors are constructed, are then saved into a random accessible data file, enabling direct and quick access to volumetric errors at each node within working volume with saving main memory. Either S.VOL or R.VOL is given for the data files of volumetric errors depending on systematic and random errors, and the files of volumetric error will be frequently used in later stages for display and further applications.In order to complete the volumetric error calculation, the coordinates of probe offset are entered, and temperature deviation data are to be inputted for thermal error consideration.

5.2 Representation of volumetric errors

3D display

The developed package has been programmed so that the volumetric error map can be displayed in 3 dimension. The calculated volumetric error is plotted vectorially at each grid, and original nominal grids being plotted in different colors within the specific working volume. Simulation of length measurement has been performed along 4 space diagonals for quick checks on the magnitude of practical parametric error map. Random errors can be optionally considered for the 3D display.

1D display

It is useful to know the volumetric errors along specific measuring lines parallel to either X,Y, or Z axis. The X,Y,and Z components of the volumetric error at each nominal position are evaluated along the specific measuring line, then displayed on the computer screen. The resultant error is calculated as root mean squares of the three error components along the measuring lines.

Printing out

The volumetric error information can be printed out in tabular form via any commercial printer. Nominal X,Y,Z coordinates of the nominal grids are listed in first columns, then the volumetric errors $\Delta X, \Delta Y, \Delta Z$ are calculated and printed in next columns. The printed tabular format of the volumetric error can be very much useful for variety of applications such as volumetric error listing up for error compensation, and error budget in design stages. As the printed format of volumetric error is written in readable ASCII codes, it is possible to transfer to and from commercial interfaces such as RS232C, IEEE, and CENTRONICS.

6.Practical assessment

Practical assessments proved the efficiency of the developed module, giving quick and thorough analysis on volumetric errors. Two practical examples are shown in the paper.

6.1 A CMM at GEC, Preston

A CMM at GEC, Preston was chosen for the assessment, as the parametric error calibration data were available([8]). The CMM was the moving horizontal type CMM, and classified as MH2 model by coordinate axes arrangements. A working volume of 1800mm × 1200mm × 1200 (13 steps × 9 steps × 9 steps) was chosen in the machine, and 15 parametric errors were measured, using laser interferometer, electronic level,and mechanical artefacts. The 15 parametric errs were 3 positional, 5 angular, 4 straightness, and 3 squareness errors. All the available parametric error data were inputted to the developed module, then processed. For example, fig.2a shows X positional error along 1800 mm measurement span, giving 65.59 μm bandwidth.;fig.2b shows X pitch angular error, giving 74.95 μm/m bandwidth. The volumetric error map was then constructed, and 3 dimensional display was shown in fig.2c, nominal grid as solid line and error map as dotted line. The length uncertainty values were 151.0μm,-22.2μm,8.5μm,226.1μm along space diagonal 1 to space diagonal 4, respectively, where diagonal 1 is from (0,0,0) to (X_{max}, Y_{max}, Z_{max}); diagonal 2 from (X_{max}, 0, 0) to (0, Y_{max}, Z_{max}); diagonal 3 from (0, Y_{max}, 0) to (X_{max}, 0, Z_{max}); diagonal 4 from (0, 0, Z_{max}) to (X_{max}, Y_{max}, 0).

Fig.2d shows one dimensional error display along a measuring line which is parallel to X axis,passing through a central point(0,600,600); where Δx,Δy,Δz error components are plotted, and the resultant error is calculated, giving μm bandwidth and μm maximum error.

6.2 A CMM at NPL, Teddington

As a more complete assessment, a CMM were chosen, which used to be at the NPL(National Physical Laboratory,Teddington). The CMM was the moving bridge type CMM and the working volume was chosen as 900 mm × 1150 mm × 400 mm (19 steps × 23 steps × 16 steps). 17 parametric errors data were available ([9]), they were 3 positional, 4 straightness, 7 angular, and 3 squareness errors. Fig.3a shows X positional error, giving 6.15 mm bandwidth along 1800 mm measurement span. The random errors were also available from repeat measurement and taken as ±2σ band at each nominal target position, giving 6.15 μm repeatability (maximum 2σ). Fig 3b shows Y straightness error along X axis, giving 3.40 μm bandwidth and 15.6 μm repeatability. Fig.3c shows X pitch angular error, giving 11.18 μm/m bandwidth and 12.3 μm/m repeatability. All the parametric errors were inputted and kinematically combined to give volumetric error. Random errors were also calculated, as random portion of each parametric error were available. As results, fig.3d shows the constructed volumetric error map within the working volume, giving -24,21.3,25.9, and -35.9 μm in length uncertainty along the 4 space diagonals, respectively. Fig.3e shows error display along one dimensional measuring line which is parallel to X axis, passing through a central position,(0,550,200). The resultant error was calculated and shown, 7.17 μm bandwidth and 24.72 μm maximum error.

7. Discussions and conclusions

The developed module of volumetric error generation for most types of CMMs was implemented around micro computer environment. The main features of the developed module are as follows.

(1) Volumetric error generation for most types of CMMs with efficiency

10 different CMM models have been considered for generation of volumetric errors, and 10 corresponding sets of volumetric error equations are derived and implemented, so as to accept most CMMs currently used in industry and laboratories. It has to be mentioned that random volumetric errors as well as systematic volumetric errors can be calculated. The efficiency and validity of the developed module have been proved by two cases of practical assessments.

(2) Derivation of general rules for transformation of parametric errors

General transformation rules are derived for transforming the measured parametric errors with respect to a reference point (i.e. machine origin for working volume). As the transformation rules vary depending on the CMM models, 10 sets of rules are derived from the concept of error propagation, so that parametric errors measured at convenient locations can be transformed, then used for calculation of volumetric errors in the working volume.

(3) Data file management

A convenient file name structure was designed based on the machine specification, and it was possible to manage clusters of data files of parametric errors, even when several machine models of specification were mixed in the working directory.

(4) Variety of input/output methods

The developed module offers variety of input/output methods. In case of inputting parametric errors, key board input, best fitted curve input, and data file input have been implemented. The results and outputs from the developed system can be displayed on the screen with the aid of implemented colour graphics software. Plotter driving programs (HPGL plotter language) have been also implemented, so that the results can be plotted via any commercial plotters, or can be saved into specific plotter file files. It should be mentioned that the above input/output options can be easily selected by pressing function keys or numeric keys.

(5) Possible applications

The developed module, as a general purpose volumetric error generator, can have variety of possible applications for CMM manufacturers as well as users. In view of CMM manufacturers, the developed module can be applied to the CMM controllers as generator or compensator of volumetric errors, since it has been implemented in such handy environment. Also, in the stage of design/manufacturing, the developed module can provide error information associated with error budget. In view of CMM users, the developed system will be useful for maintenance, verification, periodic re-verification, and compensation. Since all the volumetric error informations are known at each nominal position in the working volume, the developed system can also play a role of numerical CMM (or numerical simulator), which is in preparation for publications.

Acknowledgement

The authors are grateful to the DTI (Departments of Trade and Industry,U.K.) for the research fund, enabling this contract project to be carried out.

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APPENDIX

A.1 VOLUMETRIC ERROR EQUATIONS

The notations and symbols are defined as follows:
Positional error along X,Y,Z axes[μm] : $\delta x(x), \delta y(y), \delta z(z)$
Straightness error along the axes[μm] :
$\delta y(x), \delta z(x); \delta x(y), \delta z(y); \delta y(z), \delta x(z)$

Angular error along X axis[μm/m] : $E_x(x), E_y(x), E_z(x)$
Angular error along Y axis[μm/m] : $E_x(y), E_y(y), E_z(y)$
Angular error along Z axis[μm/m] : $E_x(z), E_y(z), E_z(z)$
Squareness error[μm/m] : α (XY plane), β_1(YZ plane), β_2(XZ plane)
Thermal expansion coefficient[μm/m/C] : α_x(X axis), α_y(Y axis), α_z(Z axis)
Temperature deviation[C] : T_x(X axis), T_y(Y axis), T_z(Z axis)
Offset coordinate of probe[mm] : X_p(X axis), Y_p(Y axis), Z_p(Z axis)

(1) MB1 model

$$\Delta X=\delta x(x)+\delta x(y)+\delta x(z)+z(-\beta_1+E_y(x)+E_y(y)) -Y_p(E_z(x)+E_z(y)+E_z(z))+Z_p(E_y(x)+E_y(y)+E_y(z))+\alpha_x T_x$$
$$\Delta Y=\delta y(y)+\delta y(x)+\delta y(z)+x(-\alpha+E_z(y))-z(\beta_2+E_x(y)+E_x(x)) +X_p(E_z(x)+E_z(y)+E_z(z))-Z_p(E_x(x)+E_x(y)+E_x(z))+\alpha_y T_y$$
$$\Delta Z=\delta z(z)+\delta z(y)+\delta z(x)-xE_y(y) -X_p(E_y(x)+E_y(y)+E_y(z))+Y_p(E_x(x)+E_x(y)+E_x(z))+\alpha_z T_z$$

(2) MB2 model

$$\Delta X=\delta x(x)+\delta x(y)+\delta x(z)+z(-\beta_1+E_y(x)+E_y(y))-yE_z(x) -Y_p(E_z(x)+E_z(y)+E_z(z))+Z_p(E_y(x)+E_y(y)+E_y(z))+\alpha_x T_x$$
$$\Delta Y=\delta y(y)+\delta y(x)+\delta y(z)-x\alpha+z(-\beta_2-E_x(y)-E_x(x)) +X_p(E_z(x)+E_z(y)+E_z(z))-Z_p(E_x(x)+E_x(y)+E_x(z))+\alpha_y T_y$$
$$\Delta Z=\delta z(z)+\delta z(x)+\delta z(y)+yE_x(x) -X_p(E_y(x)+E_y(y)+E_y(z))+Y_p(E_x(x)+E_x(y)+E_x(z))+\alpha_z T_z$$

(3) FB1 model

$$\Delta X=\delta x(x)+\delta x(y)+\delta x(z)+yE_z(y)+z(-\beta_1+E_y(x)-E_y(y)) +Y_p(-E_z(x)+E_z(y)-E_z(z))+Z_p(E_y(x)-E_y(y)+E_y(z))+\alpha_x T_x$$
$$\Delta Y=\delta y(y)+\delta y(x)+\delta y(z)+x(-\alpha-E_z(y))+z(-\beta_2-E_x(x)+E_x(y)) +X_p(E_z(x)-E_z(y)+E_z(z)+Z_p(-E_x(x)+E_x(y)-E_x(z))+\alpha_y T_y$$
$$\Delta Z=\delta z(z)+\delta z(x)+\delta z(y)+xE_y(y)-yE_x(y) +X_p(-E_y(x)+E_y(y)-E_y(z))+Y_p(E_x(x)-E_x(y)+E_x(z))+\alpha_z T_z$$

(4) FB2 model

$$\Delta X=\delta x(x)+\delta x(y)+\delta x(z)+z(-\beta_1-E_y(x)+E_y(y))+yE_z(x) +Y_p(E_z(x)-E_z(y)-E_z(z))+Z_p(-E_y(x)+E_y(y)+E_y(z))+\alpha_x T_x$$
$$\Delta Y=\delta y(y)+\delta y(x)+\delta y(z)+x(-\alpha-E_z(X))+z(-\beta_2+E_x(x)-E_x(y)) +X_p(-E_z(x)+E_z(y)+E_z(z))+Z_p(E_x(x)-E_x(y)-E_x(z))+\alpha_y T_y$$
$$\Delta Z=\delta z(z)+\delta z(x)+\delta z(y)+xE_y(x)-yE_x(x) +X_p(E_y(x)-E_y(y)-E_y(z))+Y_p(-E_x(x)+E_x(y)+E_x(z))+\alpha_z T_z$$

(5) MH1 model

$$\Delta X=\delta x(x)+\delta x(y)+\delta x(z)+z(-\beta_1+E_y(y))+\phi_1 xz +Y_p(-E_z(x)-E_z(y)-E_z(z))+Z_p(E_y(x)+E_y(y)+E_y(z))+\alpha_x T_x$$
$$\Delta Y=\delta y(y)+\delta y(x)+\delta y(z)+x(-\alpha+E_z(y)+E_z(z))+z(-\beta_2-E_x(y)) +X_p(E_z(x)+E_z(y)+E_z(z))-Z_p(E_x(x)+E_x(y)+E_x(z))+\alpha_y T_y$$
$$\Delta Z=\delta z(z)+\delta z(x)+\delta z(y)-x(E_y(y)+E_y(z))-\phi_2 x\circ x/2 -X_p(E_y(x)+E_y(y)+E_y(z))+Y_p(E_x(x)+E_x(y)+E_x(z))+\alpha_z T_z$$

(6) MH2 model

$$\Delta X=\delta x(x)+\delta x(y)+\delta x(z)-y(Ez(z)+Ez(x))+z(-\beta_1+Ey(x))$$
$$+Y_p(-Ez(x)-Ez(y)-Ez(z))+Z_p(Ey(x)+Ey(y)+Ey(z))+\alpha_x T_x$$
$$\Delta Y=\delta y(y)+\delta y(x)+\delta y(z)-x\alpha+z(-\beta_2-Ex(x))+\phi_1 yz$$
$$+X_p(Ez(x)+Ez(y)+Ez(z))+Z_p(-Ex(x)-Ex(y)-Ex(z))+\alpha_y T_y$$
$$\Delta Z=\delta z(z)+\delta z(x)+\delta z(y)+y(Ex(x)+Ex(z))+\phi_2 y\circ y/2$$
$$+X_p(-Ey(x)-Ey(y)-Ey(z))+Y_p(Ex(x)+Ex(y)+Ex(z))+\alpha_z T_z$$

(7) FH1 model

$$\Delta X=\delta x(x)+\delta x(y)+\delta x(z)+z(Ey(y)-Ey(x)-\beta_1)$$
$$+Y_p(Ez(x)-Ez(y)-Ez(z))+Z_p(-Ey(x)+Ey(y)+Ey(z))+\alpha_x T_x$$
$$\Delta Y=\delta y(y)+\delta y(x)+\delta y(z)+x(-\alpha-Ez(x))+z(-Ex(y)+Ex(x)-\beta_2)$$
$$+X_p(-Ez(x)+Ez(y)+Ez(z))+Z_p(Ex(x)-Ex(y)-Ex(z))+\alpha_y T_y$$
$$\Delta Z=\delta z(z)+\delta z(x)+\delta z(y)+xEy(x)-yEx(x)$$
$$+X_p(Ey(x)-Ey(y)-Ey(z))+Y_p(-Ex(x)+Ex(y)+Ex(z))+\alpha_z T_z$$

(8) FH2 model

$$\Delta X=\delta x(x)+\delta x(y)+\delta x(z)+z(-\beta_1+Ey(x)-Ey(y))+yEz(y)$$
$$+Y_p(-Ez(x)+Ez(y)-Ez(z))+Z_p(Ey(x)-Ey(y)+Ey(z))+\alpha_x T_x$$
$$\Delta Y=\delta y(y)+\delta y(x)+\delta y(z)+x(-\alpha-Ez(y))+z(-\beta_2-Ex(x)+Ex(y))$$
$$+X_p(Ez(x)-Ez(y)+Ez(z))+Z_p(-Ex(x)+Ex(y)-Ex(z))+\alpha_y T_y$$
$$\Delta Z=\delta z(z)+\delta z(x)+\delta z(y)+xEy(y)-yEx(y)$$
$$+X_p(-Ey(x)+Ey(y)-Ey(z))+Y_p(Ex(x)-Ex(y)+Ex(z))+\alpha_z T_z$$

(9) CX1 model

$$\Delta X=\delta x(x)+\delta x(y)+\delta x(z)+z(-\beta_1-Ey(x)-Ey(y))+yEz(y)$$
$$+Y_p(Ez(x)+Ez(y)-Ez(z))+Z_p(-Ey(x)-Ey(y)+Ey(z))+\alpha_x T_x$$
$$\Delta Y=\delta y(y)+\delta y(x)+\delta y(z)+z(-\beta_2+Ex(x)+Ex(y))+x(-\alpha-Ez(x)-Ez(y))$$
$$+X_p(-Ez(x)-Ez(y)+Ez(z))+Z_p(Ex(x)+Ex(y)-Ex(z))+\alpha_y T_y$$
$$\Delta Z=\delta z(z)+\delta z(x)+\delta z(y)+x(Ey(x)+Ey(y))-yEx(y)$$
$$+X_p(Ey(x)+Ey(y)-Ey(z))+Y_p(-Ex(x)-Ex(y)+Ex(z))+\alpha_z T_z$$

(10) CY1 model

$$\Delta X=\delta x(x)+\delta x(y)+\delta x(z)+y(Ez(x)+Ez(y))+z(-\beta_1-Ey(x)-Ey(y))$$
$$+Y_p(Ez(x)+Ez(y)-Ez(z))+Z_p(-Ey(x)-Ey(y)+Ey(z))+\alpha_x T_x$$
$$\Delta Y=\delta y(y)+\delta y(x)+\delta y(z)+z(-\beta_2+Ex(y)+Ex(x))+x(-\alpha-Ez(x))$$
$$+X_p(-Ez(x)-Ez(y)+Ez(z))+Z_p(Ex(x)+Ex(y)-Ex(z))+\alpha_y T_y$$
$$\Delta Z=\delta z(z)+\delta z(x)+\delta z(y)+xEy(x)+y(-Ex(x)-Ex(y))$$
$$+X_p(Ey(x)+Ey(y)-Ey(z))+Y_p(-Ex(x)-Ex(y)+Ex(z))+\alpha_z T_z$$

A.2 Transformation rules for parametric errors with measurement offsets

Let (xo,yo,zo) be the reference point; (x,y,z) be the practical measurement locations as mentioned in section 4. Symbol* at the end of every positional, straightness, squareness errors indicate the transformed errors with respect to (xo,yo,zo) position in the working volume.

In case of straightness errors, the right hand terms have to be processed by the straightness criterion, either least squares, end points fit, or minimum separation which are currently implemented in the developed system, after arithmetic calculations have been performed. ⇔ indicates the process to be performed.

(1) MB1 model
$\delta{*}x(x)=\delta x(x)-(y-yo)Ey(x)$
$\delta{*}y(y)=\delta y(y)+(z-zo)Ex(y)-(x-xo)Ez(y)$
$\delta{*}z(z)=\delta z(z)$
$\delta{*}x(y)\Leftrightarrow\delta x(y)-(z-zo)Ey(y)$
$d{*}x(z)=\delta x(z)$
$\delta{*}y(x)\Leftrightarrow\delta y(x)+(z-zo)Ex(x)$
$\delta{*}y(z)=\delta y(z)$
$\delta{*}z(y)\Leftrightarrow\delta z(y)+(x-xo)Ey(y)$
$\delta{*}z(x)=\delta z(x)$
$\alpha{*}=\alpha+Ez(y)-(z-zo)Ex(x)/(x-xo)$
$\beta_1{*}=\beta_1+Ey(y)+Ey(x)$
$\beta_2{*}=\beta_2-Ex(x)-Ex(y)-(x-xo)Ey(y)/(y-yo)$

(2) MB2 model
$\delta{*}x(x)=\delta x(x)+(y-yo)Ez(x)-(z-zo)Ey(x)$
$\delta{*}y(y)=\delta y(y)+(z-zo)Ex(y)$
$\delta{*}z(z)=\delta z(z)$
$\delta{*}x(y)\Leftrightarrow\delta x(y)-(z-zo)Ey(y)$
$\delta{*}x(z)=\delta x(z)$
$\delta{*}y(x)\Leftrightarrow\delta y(x)+(z-zo)Ex(x)$
$\delta{*}y(z)=\delta y(z)$
$\delta{*}z(y)=\delta z(y)$
$\delta{*}z(x)\Leftrightarrow\delta z(x)-(y-yo)Ex(x)$
$\alpha{*}=\alpha-(z-zo)Ex(x)/(x-xo)$
$\beta_1{*}=\beta_1+Ey(y)+Ey(x)+(y-yo)Ex(x)/(x-xo)$
$\beta_2{*}=\beta_2-Ex(y)-Ex(x)$

(3) FB1 model
$\delta{*}x(x)=\delta x(x)-(z-zo)Ey(x)$
$\delta{*}y(y)=\delta y(y)-(z-zo)Ex(y)+(x-xo)Ez(y)$
$\delta{*}z(z)=\delta z(z)$
$\delta{*}x(y)\Leftrightarrow\delta x(y)+(z-zo)Ey(y)$
$\delta{*}x(z)=\delta x(z)$
$\delta{*}y(x)\Leftrightarrow\delta y(x)+(z-zo)Ex(x)$
$\delta{*}y(z)=\delta y(z)$
$\delta{*}z(y)\Leftrightarrow\delta z(y)-(x-xo)Ey(y)$
$\delta{*}z(x)=\delta z(x)$
$\alpha{*}=\alpha-Ez(y)-(z-zo)Ex(x)/(x-xo)$
$\beta_1{*}=\beta_1+Ey(x)-Ey(y)$
$\beta_2{*}=\beta_2-Ex(x)+Ex(y)+(x-xo)Ey(y)/(y-yo)$

(4) FB2 model
$\delta{*}x(x)=\delta x(x)-(y-yo)Ez(x)+(z-zo)Ey(x)$
$\delta{*}y(y)=\delta y(y)+(z-zo)Ex(y)$

$\delta{*}z(z)=\delta z(z)$
$\delta{*}x(y)\leftrightarrow\delta x(y)-(z-zo)Ey(y)$
$\delta{*}x(z)=\delta x(z)$
$\delta{*}y(x)\leftrightarrow\delta y(x)-(z-zo)Ex(x)$
$\delta{*}y(z)=\delta y(z)$
$\delta{*}z(y)=\delta z(y)$
$\delta{*}z(x)\leftrightarrow\delta z(x)+(y-yo)Ex(x)$
$\alpha{*}=\alpha-Ez(x)+(z-zo)Ex(x)/(x-xo)$
$\beta_1{*}=\beta_1+Ey(y)-Ey(x)+(y-yo)Ex(x)/(x-xo)$
$\beta_2{*}=\beta_2-Ex(y)+Ex(x)$

(5) MH1 model
$\delta{*}x(x)=\delta x(x)$
$\delta{*}y(y)=\delta y(y)+(z-zo)Ex(y)-(x-xo)Ez(y)$
$\delta{*}z(z)=\delta z(z)+(x-xo)Ey(z)$
$\delta{*}x(y)\leftrightarrow\delta x(y)-(z-zo)Ey(y)$
$\delta{*}x(z)=\delta x(z)$
$\delta{*}y(x)\leftrightarrow\delta y(x)-(z-zo)Ex(x)$
$\delta{*}y(z)\leftrightarrow\delta y(z)-(x-xo)Ez(z)$
$\delta{*}z(y)\leftrightarrow\delta z(y)+(x-xo)Ey(y)$
$\delta{*}z(x)=\delta z(x)$
$\alpha{*}=\alpha+Ez(y)+Ez(z)$
$\beta_1{*}=\beta_1+Ey(y)$
$\beta_2{*}=\beta_2-Ex(y)+(x-xo)Ez(z)/(z-zo)-(x-xo)Ey(y)/(y-yo)$

(6) MH2 model
$\delta{*}x(x)=\delta x(x)+(y-yo)Ez(x)-(z-zo)Ey(x)$
$\delta{*}y(y)=\delta y(y)$
$\delta{*}z(z)=\delta z(z)-(y-yo)Ex(z)$
$\delta{*}x(y)=\delta x(y)$
$\delta{*}x(z)\leftrightarrow\delta x(z)+(y-yo)Ez(z)$
$\delta{*}y(x)\leftrightarrow\delta y(x)+(z-zo)Ex(x)$
$\delta{*}y(z)=\delta y(z)$
$\delta{*}z(y)=\delta z(y)$
$\delta{*}z(x)\leftrightarrow\delta z(x)-(y-yo)Ex(x)$
$\alpha{*}=\alpha-(z-zo)Ex(x)/(x-xo)$
$\beta_1{*}=\beta_1+Ey(x)-(y-yo)Ez(z)/(z-zo)+(y-yo)Ex(x)/(x-xo)$
$\beta_2{*}=\beta_2-Ex(x)$

(7) FH1 model
$\delta{*}x(x)=\delta x(x)+(z-zo)Ey(x)$
$\delta{*}y(y)=\delta y(y)+(z-zo)Ex(y)$
$\delta{*}z(z)=\delta z(z)$
$\delta{*}x(y)\leftrightarrow\delta x(y)-(z-zo)Ey(y)$
$\delta{*}x(z)=\delta x(z)$
$\delta{*}y(x)\leftrightarrow\delta y(x)-(z-zo)Ex(x)$
$\delta{*}y(z)=\delta y(z)$

$\delta*z(y)=\delta z(y)$

$\delta*z(x)\leftrightarrow\delta z(x)+(y-y_o)Ex(x)$

$\alpha*=\alpha-Ez(x)+(z-z_o)Ex(x)/(x-x_o)$

$\beta_1*=\beta_1-Ey(x)+Ey(y)-(y-y_o)Ey(y)/(x-x_o)$

$\beta_2*=\beta_2+Ex(y)-Ex(x)-(x-x_o)Ey(y)/(y-y_o)$

(8) FH2 model

$\delta*x(x)=\delta x(x)-(z-z_o)Ey(x)$

$\delta*y(y)=\delta y(y)-(z-z_o)Ex(y)+(x-x_o)Ez(y)$

$\delta*z(z)=\delta z(z)$

$\delta*x(y)\leftrightarrow\delta x(y)+(z-z_o)Ey(y)$

$\delta*x(z)=\delta x(z)$

$\delta*y(x)\leftrightarrow\delta y(x)+(z-z_o)Ex(x)$

$\delta*y(z)=\delta y(z)$

$\delta*z(y)\leftrightarrow\delta z(y)-(x-x_o)Ey(y)$

$\delta*z(x)=\delta z(x)$

$\alpha*=\alpha-Ez(y)-(z-z_o)Ex(x)/(x-x_o)$

$\beta_1*=\beta_1+Ey(x)-Ey(y)+(y-y_o)Ex(x)/(x-x_o)$

$\beta_2*=\beta_2-Ex(x)+Ex(y)+(x-x_o)Ey(y)/(y-y_o)$

(9) CX1 model

$\delta*x(x)=\delta x(x)+(z-z_o)Ey(x)$

$\delta*y(y)=\delta y(y)-(z-z_o)Ex(y)+(x-x_o)Ez(y)$

$\delta*z(z)=\delta z(z)$

$\delta*x(y)\leftrightarrow\delta x(y)+(z-z_o)Ey(y)$

$\delta*x(z)=\delta x(z)$

$\delta*y(x)\leftrightarrow\delta y(x)-(z-z_o)Ex(x)$

$\delta*y(z)=\delta y(z)$

$\delta*z(y)\leftrightarrow\delta z(y)-(x-x_o)Ey(y)$

$\delta*z(x)=\delta z(x)$

$\alpha*=\alpha-Ez(x)-Ez(y)+(z-z_o)Ex(x)/(x-x_o)$

$\beta_1*=\beta_1-Ey(y)-Ey(x)-(y-y_o)Ex(x)/(x-x_o)$

$\beta_2*=\beta_2+Ex(x)+Ex(y)+(x-x_o)Ey(y)/(y-y_o)$

(10) CY1 model

$\delta*x(x)=\delta x(x)-(y-y_o)Ez(x)+(z-z_o)Ey(x)$

$\delta*y(y)=\delta y(y)-(z-z_o)Ex(y)$

$\delta*z(z)=\delta z(z)$

$\delta*x(y)\leftrightarrow\delta x(y)+(z-z_o)Ey(y)$

$\delta*x(z)=\delta x(z)$

$\delta*y(x)\leftrightarrow\delta y(x)-(z-z_o)Ex(x)$

$\delta*y(z)=\delta y(z)$

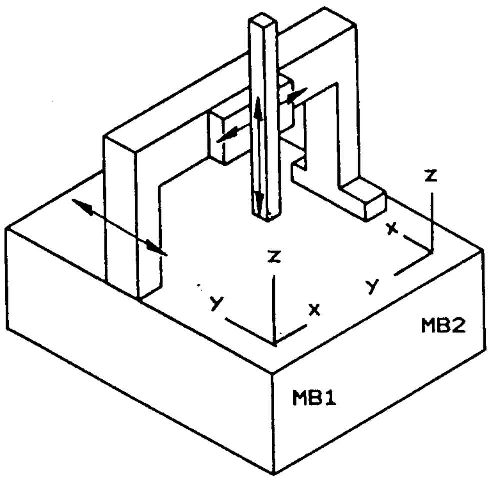
$\delta*z(y)=\delta z(y)$

$\delta*z(x)\leftrightarrow\delta z(x)+(y-y_o)Ex(x)$

$\alpha*=\alpha-Ez(x)+(z-z_o)Ex(x)/(x-x_o)$

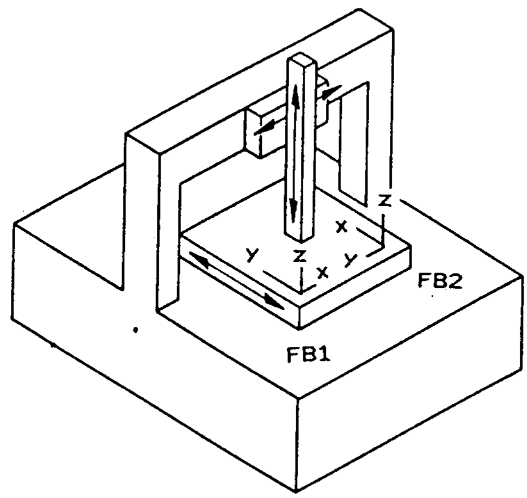
$\beta_1*=\beta_1-Ey(x)-Ey(y)+(x-x_o)Ex(x)/(y-y_o)$

$\beta_2*=\beta_2+Ex(x)+Ex(y)+(x-x_o)Ey(y)/(y-y_o)$



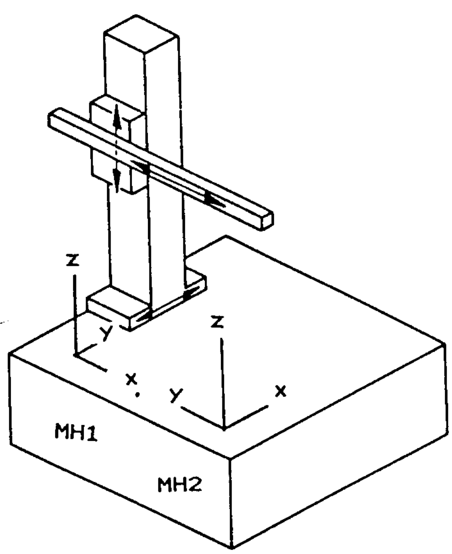
MOVING BRIDGE CMM

Fig. 1a



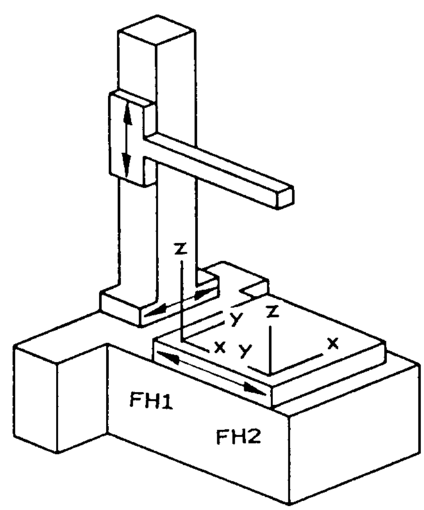
FIXED BRIDGE CMM

Fig. 1b



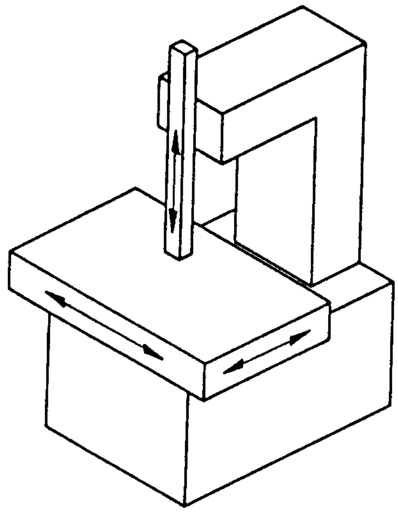
MOVING HORIZONTAL ARM CMM

Fig. 1c

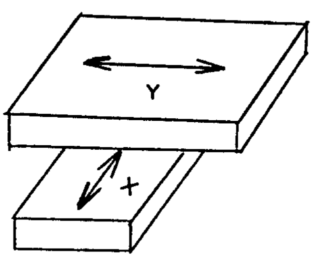


FIXED HORIZONTAL ARM CMM

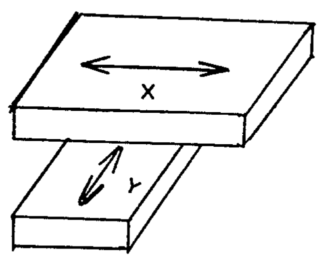
Fig. 1d



COLUMN CMM



CXI model



CYI model

Fig. 1e

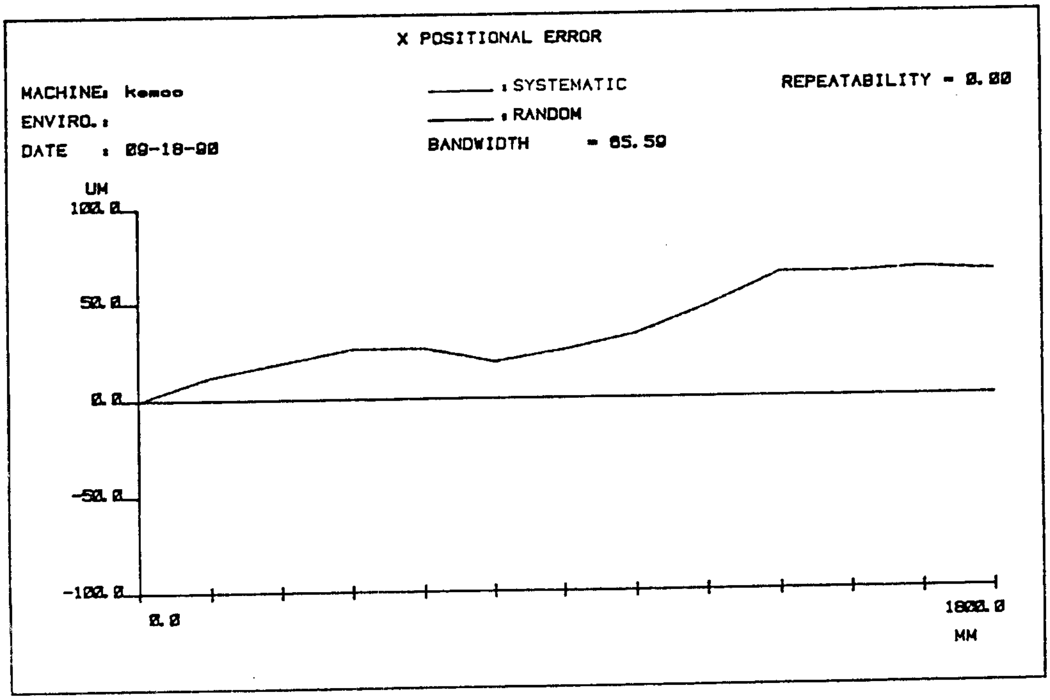

X POSITIONAL ERROR
MACHINE: kemoo
ENVIRO.:
DATE : 09-18-90
: SYSTEMATIC
: RANDOM
BANDWIDTH = 65.50
REPEATABILITY = 0.00
UM
100.0
50.0
0.0
-50.0
-100.0
0.0
1800.0
MM

Fig. 2a

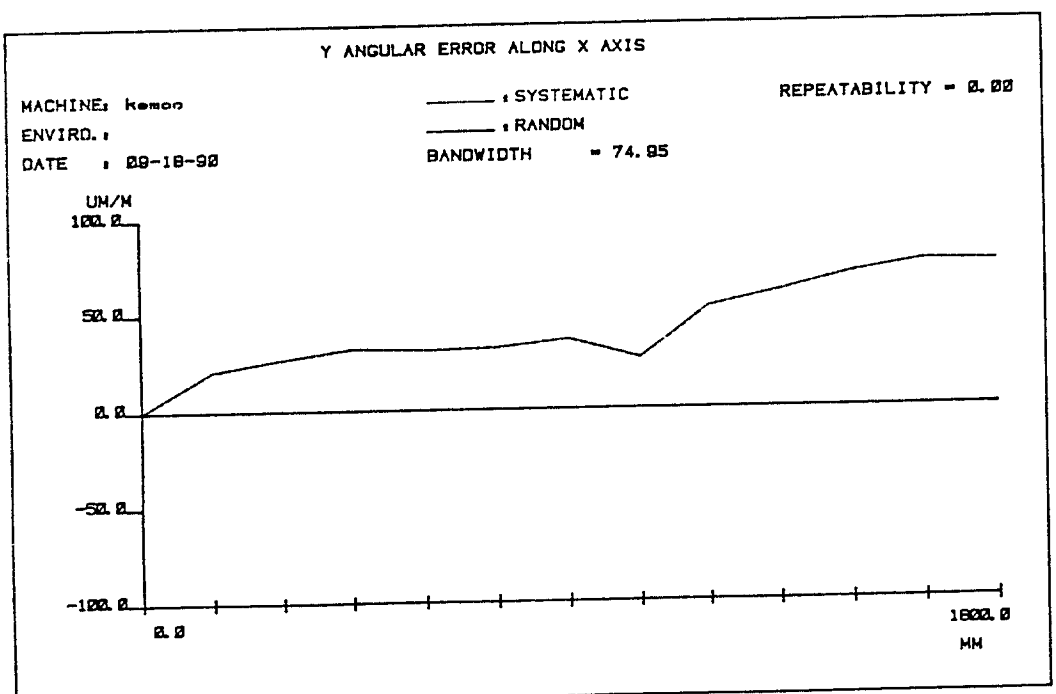

Y ANGULAR ERROR ALONG X AXIS
MACHINE: kemoo
ENVIRO.:
DATE : 09-18-90
: SYSTEMATIC
: RANDOM
BANDWIDTH = 74.85
REPEATABILITY = 0.00
UM/M
100.0
50.0
0.0
-50.0
-100.0
0.0
1800.0
MM

Fig. 2b

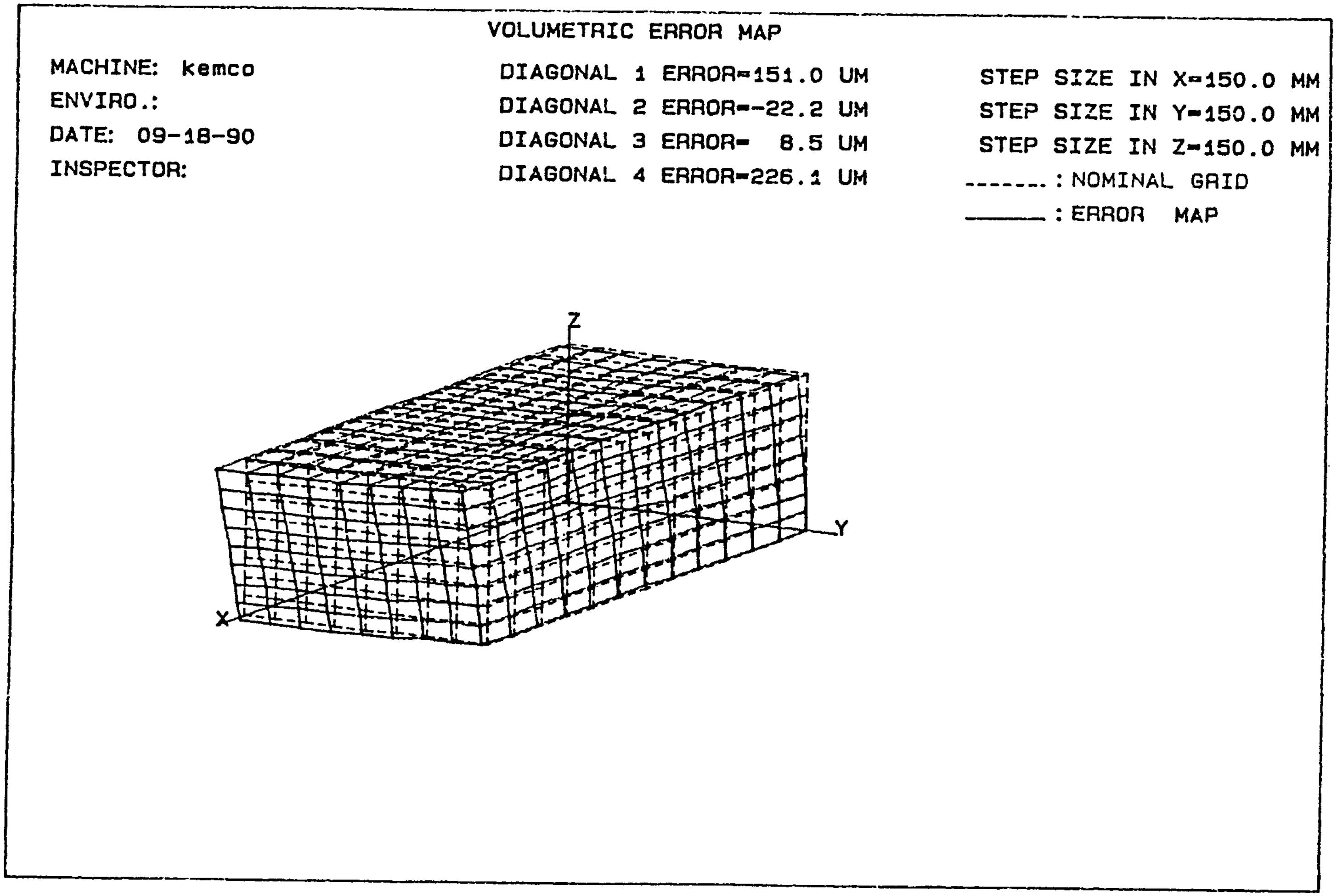

VOLUMETRIC ERROR MAP
MACHINE: kemco
ENVIRO.:
DATE: 09-18-90
INSPECTOR:
DIAGONAL 1 ERROR=151.0 UM
DIAGONAL 2 ERROR=-22.2 UM
DIAGONAL 3 ERROR= 8.5 UM
DIAGONAL 4 ERROR=226.1 UM
STEP SIZE IN X=150.0 MM
STEP SIZE IN Y=150.0 MM
STEP SIZE IN Z=150.0 MM
-------: NOMINAL GRID
———: ERROR MAP
Z
Y
X

Fig. 2c

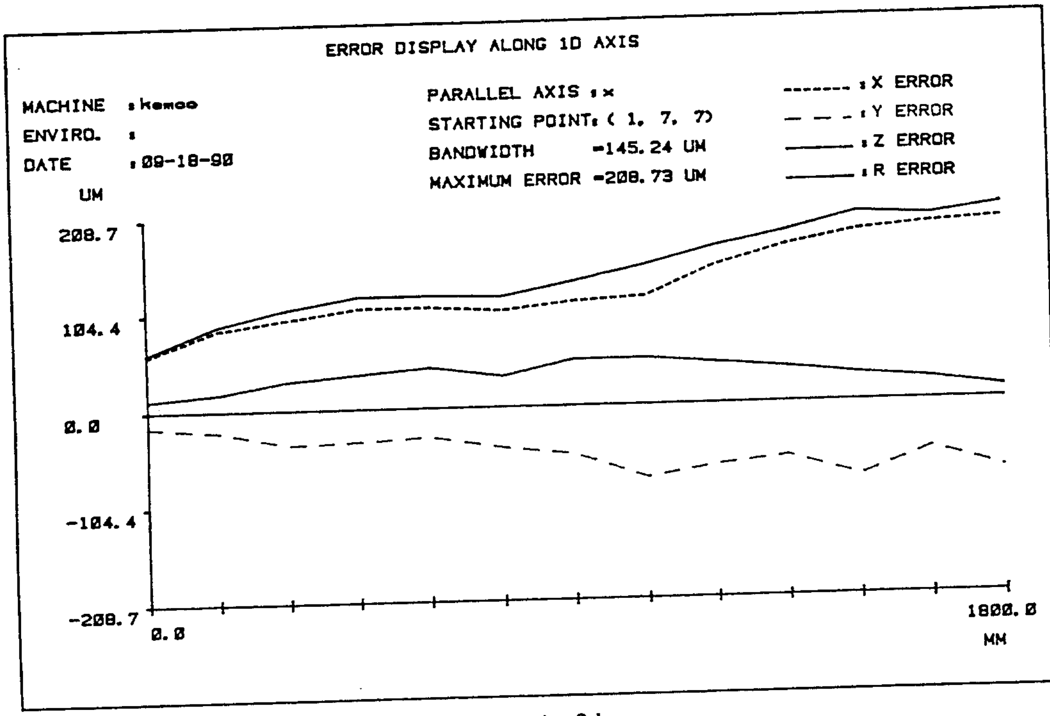

ERROR DISPLAY ALONG 1D AXIS
MACHINE : kemoo
ENVIRO. :
DATE : 09-18-90
PARALLEL AXIS : x
STARTING POINT: (1, 7, 7)
BANDWIDTH =145.24 UM
MAXIMUM ERROR =208.73 UM
: X ERROR
: Y ERROR
: Z ERROR
: R ERROR
UM
208.7
104.4
0.0
-104.4
-208.7
0.0
1800.0
MM

Fig. 2d

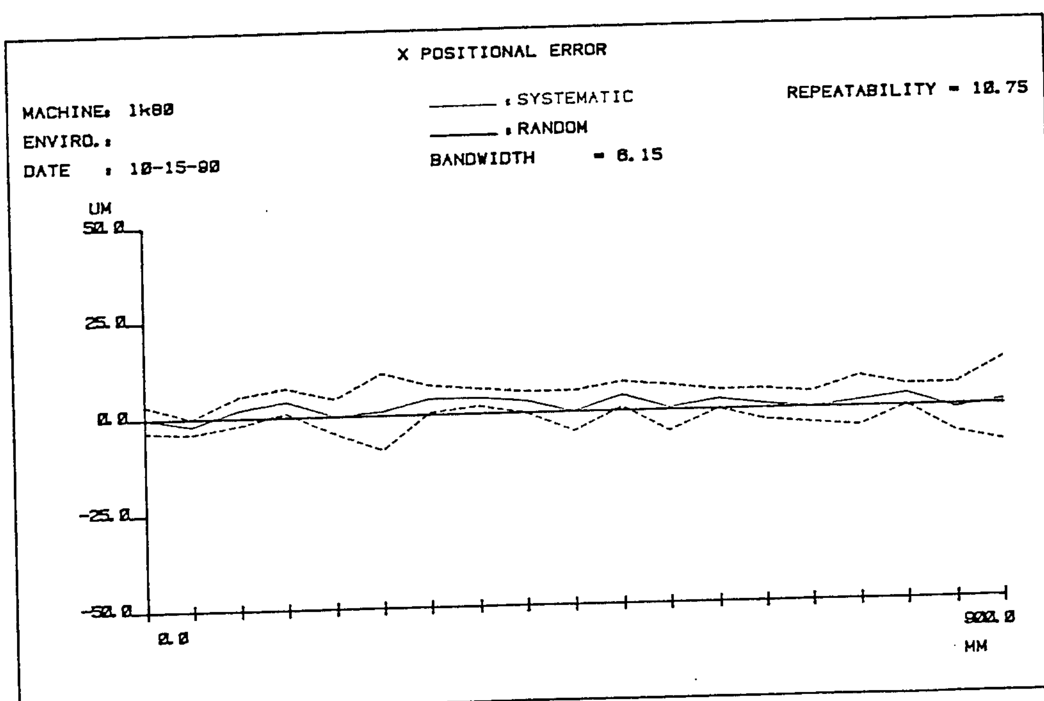

X POSITIONAL ERROR
MACHINE: 1k80
ENVIRO. :
DATE : 10-15-90
: SYSTEMATIC
: RANDOM
BANDWIDTH = 6.15
REPEATABILITY = 10.75
UM
50.0
25.0
0.0
-25.0
-50.0
0.0
900.0
MM

Fig. 3a

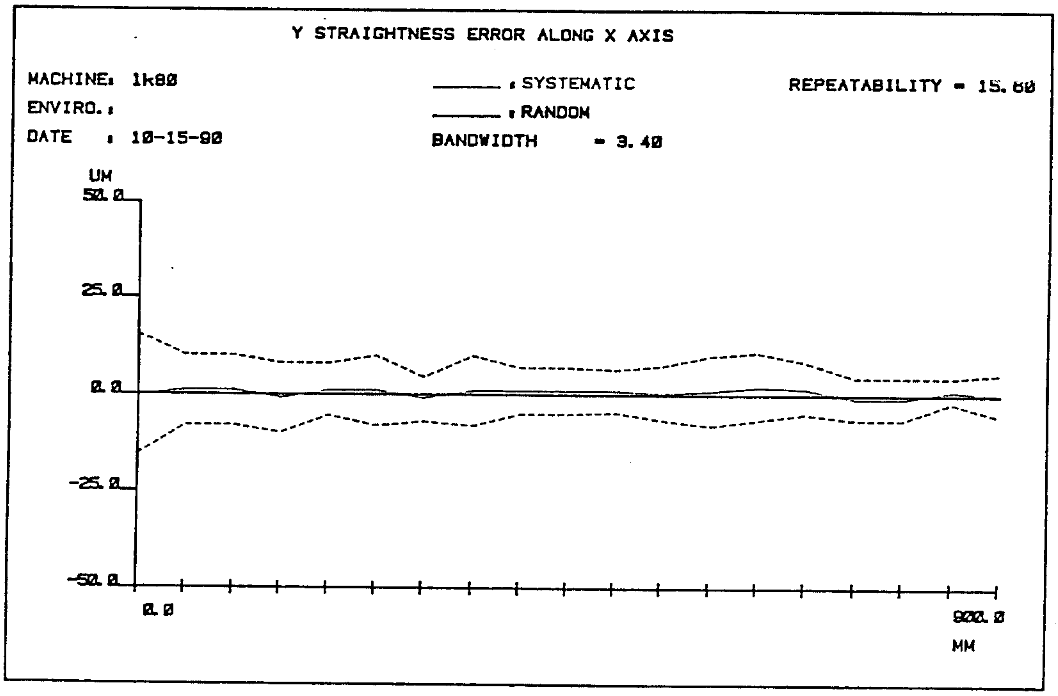

Y STRAIGHTNESS ERROR ALONG X AXIS
MACHINE: 1k80
ENVIRO.:
DATE : 10-15-90
: SYSTEMATIC
: RANDOM
BANDWIDTH = 3.40
REPEATABILITY = 15.60
UM
50.0
25.0
0.0
-25.0
-50.0
0.0
900.0
MM

Fig. 3b

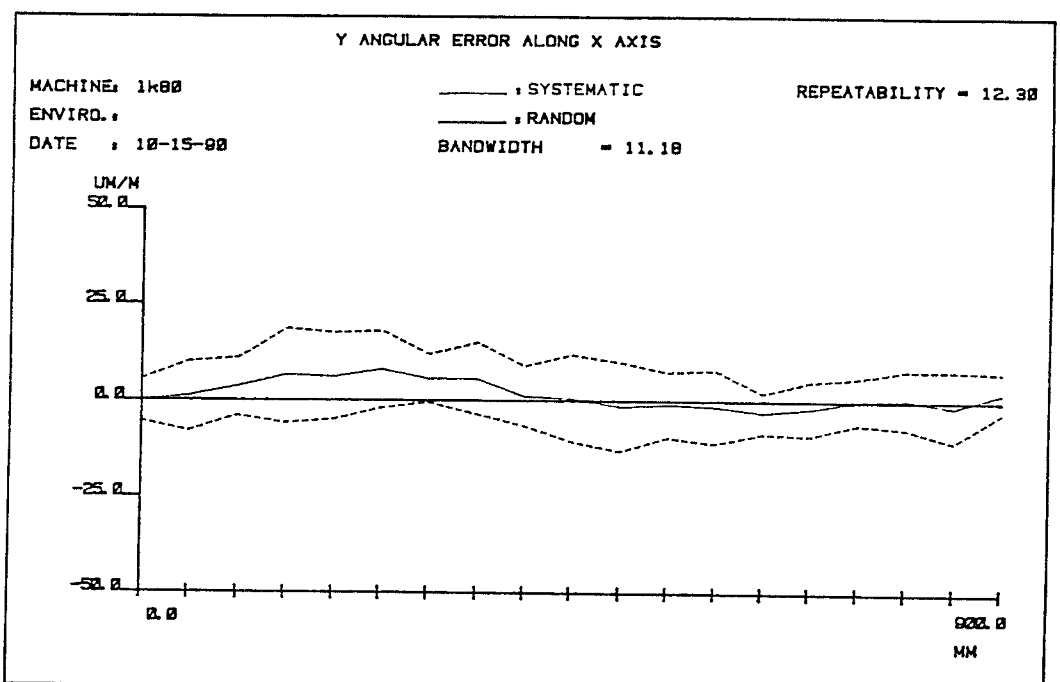

Y ANGULAR ERROR ALONG X AXIS
MACHINE: 1k80
ENVIRO.:
DATE : 10-15-90
: SYSTEMATIC
: RANDOM
BANDWIDTH = 11.10
REPEATABILITY = 12.30
UM/M
50.0
25.0
0.0
-25.0
-50.0
0.0
900.0
MM

Fig. 3c

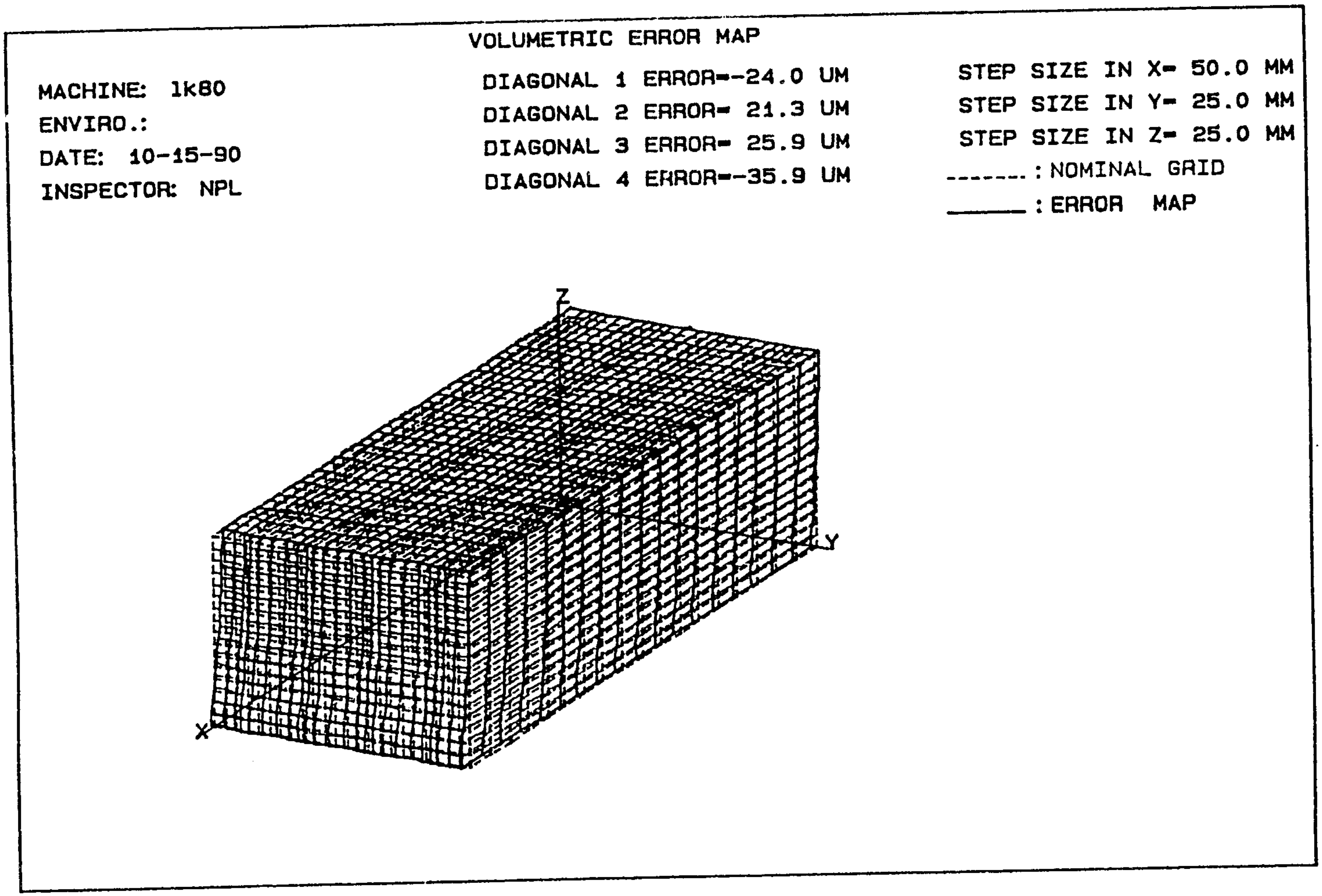

VOLUMETRIC ERROR MAP
MACHINE: 1k80
ENVIRO.:
DATE: 10-15-90
INSPECTOR: NPL
DIAGONAL 1 ERROR=-24.0 UM
DIAGONAL 2 ERROR= 21.3 UM
DIAGONAL 3 ERROR= 25.9 UM
DIAGONAL 4 ERROR=-35.9 UM
STEP SIZE IN X= 50.0 MM
STEP SIZE IN Y= 25.0 MM
STEP SIZE IN Z= 25.0 MM
------- : NOMINAL GRID
______ : ERROR MAP
Z
Y
X

Fig. 3d

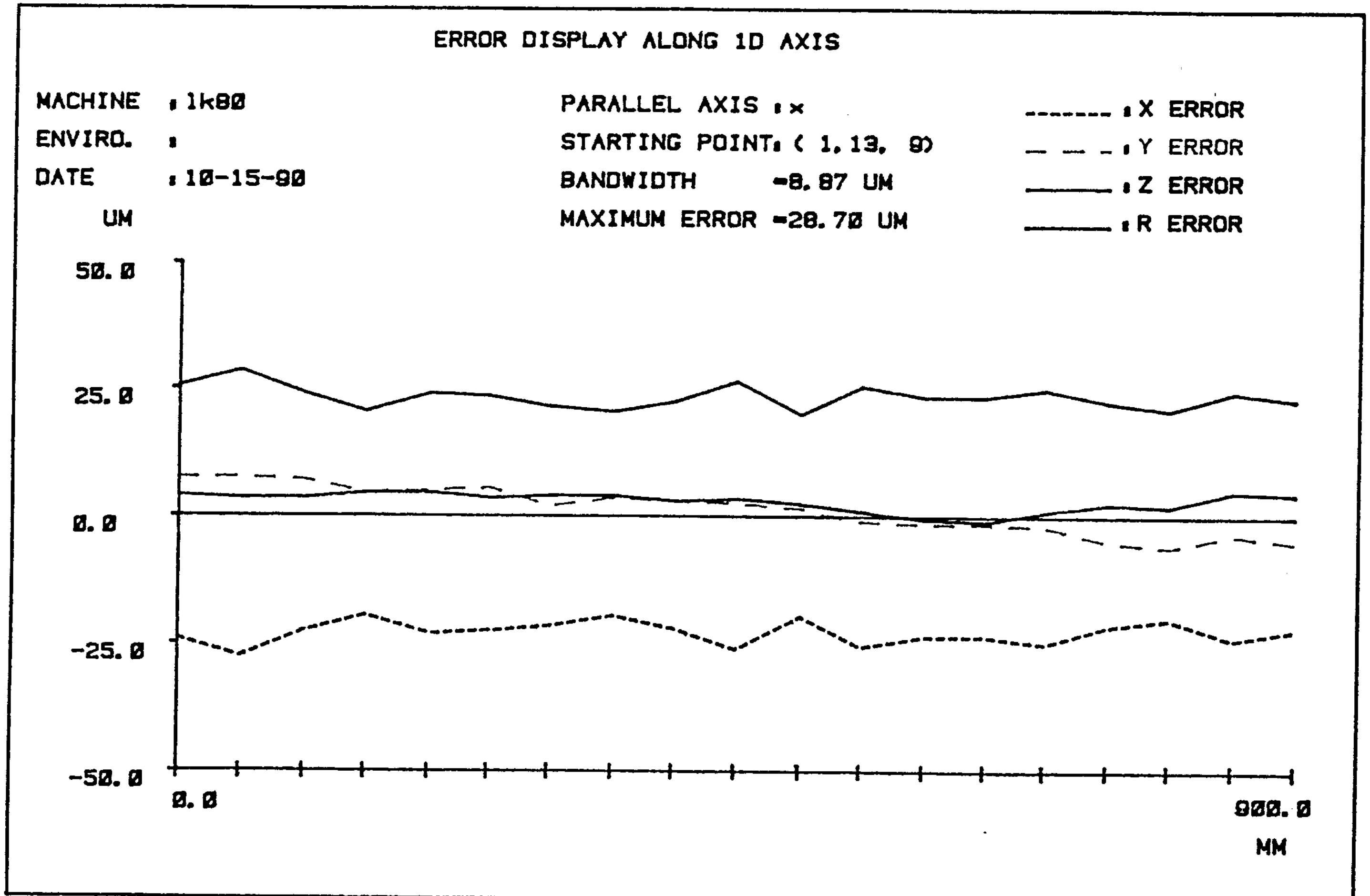
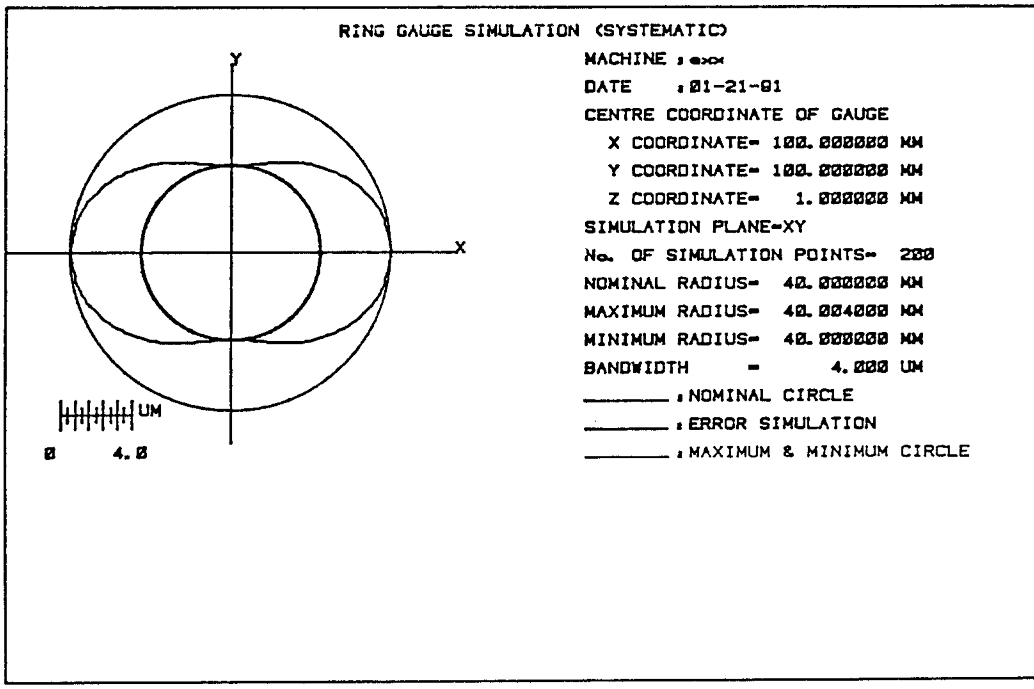
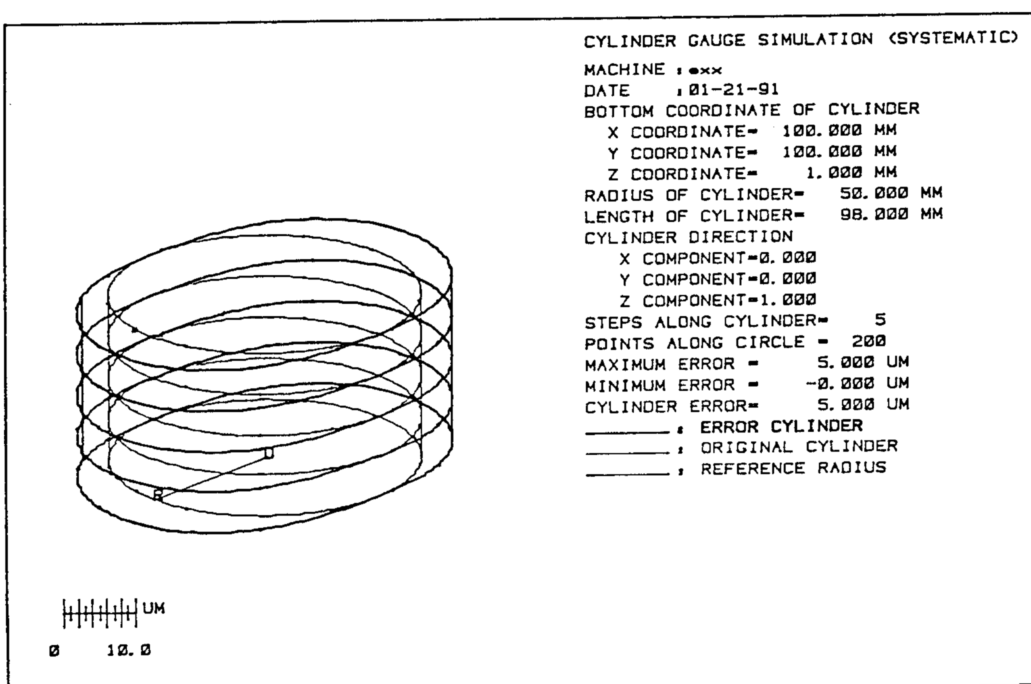

ERROR DISPLAY ALONG 1D AXIS
MACHINE : 1k80
ENVIRO. :
DATE : 10-15-90
UM
PARALLEL AXIS : x
STARTING POINT: (1, 13, 9)
BANDWIDTH =8.87 UM
MAXIMUM ERROR =28.70 UM
: X ERROR
: Y ERROR
: Z ERROR
: R ERROR
50.0
25.0
0.0
-25.0
-50.0
0.0
900.0
MM

Fig. 3e


RING GAUGE SIMULATION (SYSTEMATIC)
Y
X
MACHINE : •xx
DATE : 01-21-91
CENTRE COORDINATE OF GAUGE
X COORDINATE= 100.000000 MM
Y COORDINATE= 100.000000 MM
Z COORDINATE= 1.000000 MM
SIMULATION PLANE=XY
No. OF SIMULATION POINTS= 200
NOMINAL RADIUS= 40.000000 MM
MAXIMUM RADIUS= 40.004000 MM
MINIMUM RADIUS= 40.000000 MM
BANDWIDTH = 4.000 UM
: NOMINAL CIRCLE
: ERROR SIMULATION
: MAXIMUM & MINIMUM CIRCLE
UM
0 4.0


CYLINDER GAUGE SIMULATION (SYSTEMATIC)
MACHINE : •xx
DATE : 01-21-91
BOTTOM COORDINATE OF CYLINDER
X COORDINATE= 100.000 MM
Y COORDINATE= 100.000 MM
Z COORDINATE= 1.000 MM
RADIUS OF CYLINDER= 50.000 MM
LENGTH OF CYLINDER= 98.000 MM
CYLINDER DIRECTION
X COMPONENT=0.000
Y COMPONENT=0.000
Z COMPONENT=1.000
STEPS ALONG CYLINDER= 5
POINTS ALONG CIRCLE = 200
MAXIMUM ERROR = 5.000 UM
MINIMUM ERROR = -0.000 UM
CYLINDER ERROR= 5.000 UM
: ERROR CYLINDER
: ORIGINAL CYLINDER
: REFERENCE RADIUS
UM
0 10.0