An Impulse Noise-Robust Wiener Filter

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ABSTRACT

In this paper we propose the impulse noise-robust Wiener filter based on a combination of Wiener and modified trimmed mean(MTM) filters. The robust Wiener filter uses the trimming operation of the MTM filter to replace the outliers with the median within the window and the new set of samples which can be considered as the random process with same mean are inputted into the following Wiener filter.

We show that the robust Wiener filter is effective in frequency selective filtering of nonstationary signals while preserving signal edges with the rejection of impulse noise.

I. Introduction

Linear FIR filters, producing outputs which are linear combinations of input sequences, have been widely used in order to retain desired signal frequencies while reducing additive Gaussian noise components. In particular, if the first and second order statistics of the signal and noise are available a priori under the stationary assumption, then the minimum mean square error (MMSE) estimatior (Wiener filter) can be designed to do optimal restoration of the signal corrupted by Gaussian noise. For many applications in which this FIR filter is employed, however, it is the signal with nonstationary mean levels providing the abrupt changes (i.e., edges) in data and the noise has a probability density function (PDF) with fast decaying tails. In these situations, the conventional Wiener filter introduces blurring of edges and cannot suppress impulse noise components.

To alleviate the problems associated with linear FIR filters in filtering nonstationary signals containing impulse noise components, many nonlinear filters producing outputs based on the order statistics of the input sequence have been proposed. Probably the most widely used ones among them are filters based on the median operation. The median filter generally preserves edges and monotonic trends while suppressing impulse noise components quite well [1].

Despite of the simple implementation and good edge preservation of median filtering, median filter often fails to provide sufficient smoothing of nonimpulse noise components. To overcome this limitation, the L-filters, based on linear combination of ordered input sequenc are introduced [2,3,4]. L-filters which are also called Order Statistic Filter (OSF), and the median filter which is the special case of the OSF use only the rank-order information of the input sequence and do not depend on temporal correlation among the data. As a result in L-filtering, the signal to be filtered is assumed to be piecewise constant corrupted by additive Gaussian noise and spectral characteristics of the input signal cannot be retained. It is therefore desirable in filtering correlated signal with sharp edge and corrupted by Gaussian and impuse noise to combine the characteristics of linear FIR and nonlinear L-filters which process both temporal and rank order information of the data [5].

In this paper we propose the impulse noise-robust Wiener filter based on a combination of Wiener and Modified trimmed mean (MTM) filters. Here the MTM filter first determines the sample median m_k inside its window and then choose an interval $[m_k-q, m_k+q]$ using some preselected constant q. Within the window, data samples outside this range (i.e., outliers) are discarded and the average value of the rest of the data is used as the output [3].

The proposed robust Wiener filter uses the trimming operation of the MTM filter to replace the outliers with the median within the window and the new set of samples is inputted to the Wiener filter to result the output. In this way, frequency selective filtering with the rejection of impulse noise is achieved.

II. The Robust Wiener Filter

Let us consider the problem of estimating the signal $\{S_i\}$ from the observation $\{X_i\}$ described by $X_i = S_i + N_i$, where the signal $\{S_i\}$ is assumed to be a random process with nonstationary mean but stationary covariance, and $\{N_i\}$ is assumed to be a zero mean independently and identically distributed (i.i.d.) random noise which is uncorrelated with $\{S_i\}$.

In addition, we assume that the covariance of $\{S_i\}$ and the variance of $\{N_i\}$ are known. The output Y_k of a conventional Wiener filter of window size 2N+1 at index k is given by

$$Y_k = \sum_{j=1}^{2N+1} A_j X_j \qquad (1)$$

where $\{A_j\}$ is weighting coefficients satisfying the orthogonality principle [6], which is

$$E\{ [S_k - (\sum_{j=1}^{2N+1} A_j X_j)] X_i \} = 0 \qquad (2)$$

for $i=1, \cdots, 2N+1$

This solution is not robust to impluse noise and exhibit blurring due to nonstationary mean of the signal.

In an L-filter, the output Y_k is given by

$$Y_k = \sum_{j=1}^{2N+1} B_j X^k_{(j)} \qquad (3)$$

where $X^k_{(j)}$ is the jth smallest sample among the 2N+1 samples inside the window centered at k, and where $\{B_j\}$ is a set of weighting coefficients with $\sum_{j=1}^{2N+1} B_j = 1$.

The trimming operation of the MTM filter which is the special case of an L-filter has been used in a MTM-MMSE filter where several weights of the Wiener filter are computed and stored in a table for each canonical set of the trimmed input sequence [6]. For the practical use, the MTM-MMSE filter is modified into the robust Wiener filter where one set of weights is only computed under the covariance model with the same mean.

Combining the definition (1) and trimming

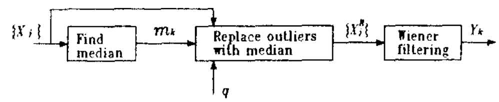

Fig. 1. Block diagram of a robust Wiener filter.

operation of the MTM filter, the output Y_k of a robust Wiener filter is defined as

$$Y_k = \sum_{j=1}^{N} A_j X_j^R \qquad (4)$$

where weighs $\{A_j\}$ are the solution of (2) and $\{X_j^R\}$ is the input sequence whose outliers are replaced by the median m_k within the window.

As shown in Fig.1, the robust Wiener filter first finds the sample median m_k inside its window and choose an interval $[m_k - q, m_k + q]$ using some preselected constant q. The outliers which are located outside the interval are replaced with the median m_k and the new set $\{X^R_j\}$ of an input sequence is inputted to the Wiener filter. Here, the outliers appear in the window due to the existence of an edge or impluse noise. Under the assumption that the edge height is sufficiently large and q is properly chosen, the new set $\{X^R_j\}$ can be considered as the random process with the same mean. Therefore the Wiener filtering for the new set of the input sequence can suppress impulse noise while retaining the spectral content of the signal.

By selecting the parameter q sufficiently large, the robust Wiener filter reduces to a conventional Wiener filter and a very small value for q makes the robust Wiener filter behave as a median filter.

To get good performance in frequency selective filtering with impulse noise suppression, the value of q can be picked in the range $[2\sigma, H-2\sigma]$, where H is the minimum hight of edges to be preserved and σ is the noise standard deviation, for H approximately equal to a large than 4σ. The edge retention and impulse noise suppression of the robust Wiener filter is illustrated in Fig. 2.

III. Simulated Performance

In this section we examine the performance characteristics of the robust Wiener filter and make comparisons with the conventional Wiener filter and the median filter. To generate the correlated signal, i.i.d. random variables with a

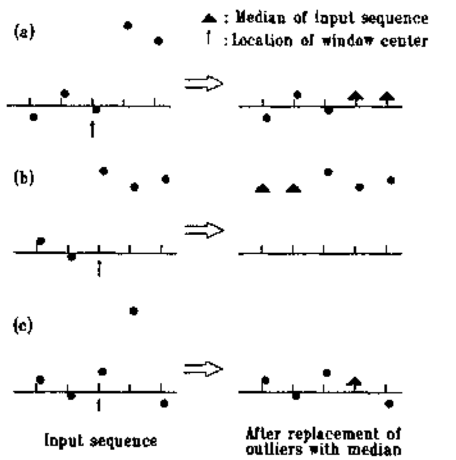

Fig. 2. Results of trimming and replacement with median with 2N+1=5. (a) when median is selected in the lower side of the edge, (b) when median is selected in the higher side of the edge, (c) when impulse noise component exists within the window.

Gaussian density function of zero mean and standard deviation 4 were passed through the ideal low-pass filter with cutoff frequency $\pi/2$. The resulted signal is a realization of a zero mean wide-sense stationary random process and the noise free input signal was obtained by adding piecewise constant values to the wide-sense stationary random process as shown in Fig. 3(a). Fig. 3(b) shows the noisy signal corrupted by additive zero mean white Gaussian noise of variance 4 and two impulses.

The results of filtering the noisy signal by the median, the conventional Wiener, and the robust Wiener filter, respectively, are shown in Fig. 3(c)-(e). A window size 2N+1=11 was used for all filters and q=10 was used for the robust Wiener filter since the sum of the standard deviation of the signal and noise is approximately five and the minimum edge height is twenty. It is seen that the conventional Wiener filter smears the edges and fails to suppress the impulse noise sufficiently while the spectral contents of the signal are retained. On the other hand, the median filtering preserves the edges and suppresses impulse noise while frequency selective filtering capability is lost. The robust Wiener filter shows the best performance in preserving the edges and the desired frequencies while suppressing the white Gaussian and impulse noise.

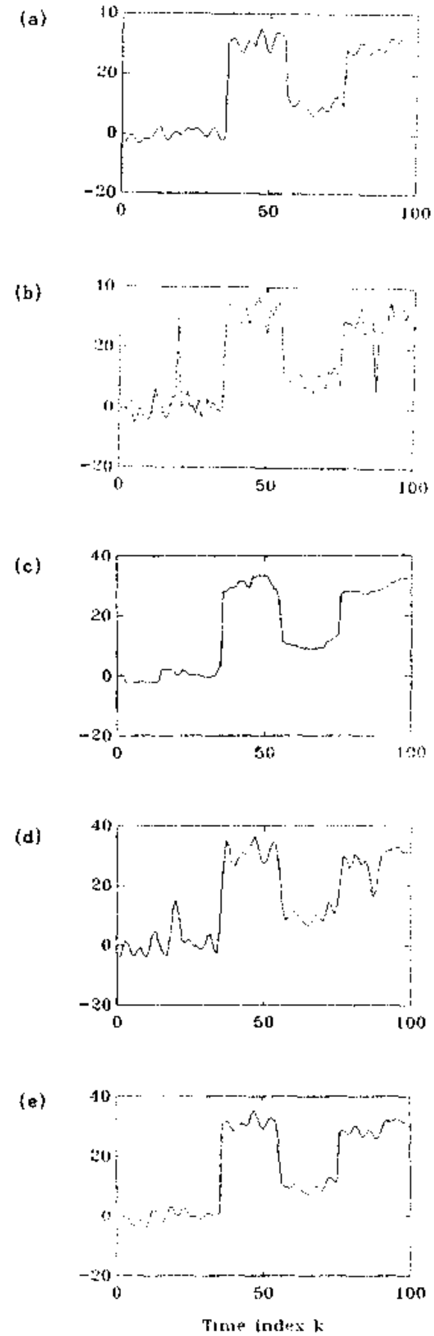

Fig. 3. Simulation results with 2N+1=11.
(a) Noise free signal, (b) Noisy signal,
(c) Output of the median filter,
(d) Output of the Wiener filter,
(e) Output of the robust Wiener filter.

IV. Conclusions

We have considered the robust Wiener filter which combines the characteristics of linear FIR and nonlinear L-filters in order to process both temporal and rank order information of the data. The trimming operation of the MTM filter was employed as the preprocessor to convert the nonstationary mean of the input sequence into the stationary one.

It was observed that the robust Wiener filter is effective in frequency selective filtering of nonstationary signals with the rejection of impulse noise.

References

[1] N. C. Gallagher and G. L. Wise,"A theoretical analysis of the properties of median filters", IEEE Trans. Acoust.,Speech, Signal Processing, vol. ASSP-29, pp.1136-1141,Dec.1982.

[2] A. C. Bovik, T. S. Huang,and D.C.Munson,Jr.,"A generalization of median filtering using linear combinations of order statistics", IEEE Trans. Acoust., Speech, Signal Processing,vol.ASSP-31,pp.1342-1349,Dec.1983.

[3] Y. H. Lee and S. A. Kassam,"Generalized median filtering and related nonlinear filtering techniques", IEEE Trans. Acoust., Speech, Signal Processing,vol.ASSP-33,pp.672-683,June 1985.

[4] A. Restrepo and A. C. Bovik," Adaptive trimmed mean filters for image restoration", IEEE Trans.Acoust., Speech,Signal Processing, vol. ASSP-36,pp.1326-1337,Aug.1988.

[5] P.P.Gandhi and S.A.Kassam,"Design and performance of combination filters for signal restoration",IEEE Trans.Acoust.,Speech,Signal Processing,vol.ASSP-39,pp.1524-1540,July 1991.

[6] A. Papoulis,"Probability,Random variables and stochastic Processes",2nd Ed.,MaGraw-Hill,1984.

[7] Y.H.Lee and S.Park,"A frequency selective nonlinear filter based on a generalized median filter",in Proc.Conf.Inform.Sci.Syst.(Baltimore, MD),Mar.1985,pp.384-389.