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Rank Reduction for Wideband Signals incident on a Uniform Linear Array

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Abstract

A new class of data transformation matrix is introduced for estimation of angles of arrivals by the rank reduction of multiple wideband sources. The proposed unitary focusing matrix minimizes the average of the squared norm of focusing error over the angles of interest without *a priori* knowledge of source locations. The merit that result as a consequence is a lower resolution threshold. These matrices can be applied to the case of the multigroup sources. Simulations and the comparison of statistical performance are compared with the algorithms (especially, spatial resampling method) which does not require the pre-estimation.

I. Introduction

Array signal processing is concerned with the detection and the estimation of signals and their parameters from data collected by spatially distributed sensors or antennas. Interest in array signal processing comes from the numerous applications-sonar, radar, radio astronomy, tomography and seismic exploration, etc. One of the important issues in array signal processing is that of finding the location of a number of sources. Many methods has been developed such as the Maximum Likelihood method[1] and eigenspace based methods[2, 3, 4, 5].

The eigenspace based methods exploit the eigenstructure of the spatial correlation matrix. The literature about eigenspace based methods include MUSIC[5], MIN-NORM[6], ROOT-MUSIC[7], ESPRIT[8] and their variations. They have been shown to perform well and capable of resolving closely spaced sources which are not completely correlated[6,7,8]. The high resolution property of the eigen based spatial estimation methods for noncoherent narrow-band sources made it a possible candidate for use in wideband signal DOA estimation problems.

In this paper, we propose the new unitary focusing method[9] by which wideband estimation problem can be changed to narrowband problem. This method does not require the pre-estimation and minimizes the weighted average of the squared norm of focusing error over the angles of interest. The paper is organized as follows. Section 2 presents the model formulation. In Section 3, a new focusing matrix is proposed. Simulations and the comparison of statistical performance are given in Section 5, followed by concluding remarks.

2. MODEL FORMULATION

The data is observed from the outputs of an array of *M* sensors with a known arbitrary geometry. The data consists of multiple *d* (< M) plane waves with additive white gaussian noise. The source signals are wide-band signals with bandwidth B centered at frequency f_c, characterized as zero mean, stationary stochastic processes over the observation intervals T_0. The source signal vectors s(t) can be written as

$$s(t)=[s_1(t), \ldots , s_d(t)]^T, \qquad (2.1)$$

where ' T ' denotes transpose of a vector or a matrix. The signal $x_m(t)$, received at the m th sensor, can be expressed as

$$x_m(t) = \sum_{i=1}^{d} a_{im}(f)\, s_i(t - \tau_{im}) + n_m(t). \qquad (2.2)$$

$a_{im}(f)$ is the frequency dependent complex gain of the m the sensor to the i th source, τ_{im} is the propagation time delay between the m th sensor and the phase reference sensor and is given by $\tau_{im} = m\, d \sin\theta_i / c$. $n_m(t)$ is the wideband noise process which consists of the receiver noise and background noise in the medium. The observation interval $\mathbf{T}$ is divided into K non-overlapping snapshot intervals T_0 and for each of these intervals the array output signals $x_m(t)$ are decomposed into J frequency components $X_m(f_n)$, n=1, ..., J, $f_1 = f_c - B/2$ and $f_J = f_c + B/2$, via Fast Frequency Transformation(FFT). So, we sample K times each frequency component of the output signals, thus obtaining the data vector $X_k(f_n)$,n=1,...,J , k=1,..., K . Using eqn. (2.2) $X_m(f_n)$ will be given by

$$X_m(f_n) = \sum_{i=1}^{d} a_{im}(f_n)\ e^{-j2\pi(fc+fn)\,\tau im}\, S_i(f_n) + N_m(f_n), \tag{2.3}$$

where $S_i(f_n)$ and $N_m(f_n)$ are the n-th frequency components of $s_i(t)$ and $n_m(t)$ respectively. Eqn. (2.3) can be written in vector-matrix notations as

$$X(f_n) = A(\theta, f_c+f_n)\, S(f_n) + N(f_n), \tag{2.4}$$

where $A(\theta, f_c+f_n) = [\mathbf{a}(\theta_1, f_c+ f_n), \ldots, \mathbf{a}(\theta_d, f_c+f_n)]$ is the M x d *direction matrix* (*steering or location matrix*) at frequency f_n, $\mathbf{a}(\theta_i, f_c+f_n) =[a_{i1}\, e^{-j2\pi(fc+fn)\,\tau i1}, \ldots, a_{iM}\, e^{-j2\pi(fc+fn)\,\tau iM}]^T$ is the *direction vector* ($a_{im}(f_n) = a_m$ if the sensors are *omnidirectional* and frequency independent) and τ_{im}= $md\sin\theta_i / c$ is a time delay between the reference sensor and m th sensor. Based on the above notations, the spatial correlation matrix $\mathbf{R}_x(f_n)$ is given by

$$\begin{aligned}\mathbf{R}_x(f_n) &= E\,[\, X(f_n)\ X^H(f_n)\,] \\ &= A(\theta, f_c+ f_n)\, \mathbf{R}_s(\theta, f_n)\, A^H(\theta, f_c+f_n) + \mathbf{R}_n(f_n)\,,\end{aligned} \tag{2.5}$$

where $\mathbf{R}_s(f_n) = E\,[\, S(f_n)\, S^H(f_n)\,]$, $\mathbf{R}_n(f_n) = E\,[\, N(f_n)\, N^H(f_n)\,]$. E[.] and 'H' denote the expectation and conjugate transpose respectively. $\mathbf{R}_x(f_n)$ is the statistic on which the estimates of the *angle of arrival*(AOA) of wide-band plane waves are based.

It is noted that in the above model the spatial correlation matrices are functions of the temporal frequencies f_n, n=1, ... , J. Therefore this model leads to a frequency domain processing. Our aim is to estimate the angle θ_i, i=1, ... , d from the data $X_k(f_n)$, k=1, ..., K; n=1, ... , J.

3. NEW FOCUSING MATRIX

In [10], it is shown that nonunitary focusing matrices, in general, lead to poorer performance than appropriately selected unitary focusing matrices. Hence, we derive a focusing matrix within the unitary matrix.

The solution which is proposed here is based on the following constrained minimization problem for G group sources:

$$\begin{aligned}&\min_{\mathbf{T}(f_n)} \int_{\mu_{i1}}^{\mu_{f1}} \| \mathbf{T}(f_n)\, \mathbf{a}(\theta, f_c+f_n) - \mathbf{a}(\theta, f_c) \|^2 d\mu \\ +&\min_{\mathbf{T}(f_n)} \int_{\mu_{i2}}^{\mu_{f2}} \| \mathbf{T}(f_n)\, \mathbf{a}(\theta, f_c+f_n) - \mathbf{a}(\theta, f_c) \|^2 d\mu \\ &\quad\vdots \\ +&\min_{\mathbf{T}(f_n)} \int_{\mu_{iG}}^{\mu_{fG}} \| \mathbf{T}(f_n)\, \mathbf{a}(\theta, f_c+f_n) - \mathbf{a}(\theta, f_c) \|^2 d\mu, \\ &\qquad n=1, \ldots, J \\ &\text{subject to } \mathbf{T}^H(f_n)\mathbf{T}(f_n) = I,\end{aligned} \tag{3.1}$$

This transformation matrix minimizes the weighted average of the squared norm of focusing error over the angles of interest. Here μ is defined as $\mu = \sin\theta$. With the unitary constraint, if we consider single group for simplicity, this minimization problem changes to maximization problem of the following form :

$$\begin{aligned}\max_{\mathbf{T}(f_n)} \int_{\mu_{ig}}^{\mu_{fg}} [\, &\mathbf{a}^H(\theta, f_c+f_n)\, \mathbf{T}^H(f_n)\, \mathbf{a}(\theta, f_c) \\ +\, &\mathbf{a}^H(\theta, f_c)\, \mathbf{T}^H(f_n)\, \mathbf{a}(\theta, f_c+f_n)]\, d\mu, \\ &n=1, \ldots, J.\end{aligned} \tag{3.2}$$

It is easy to see that the above problem is equivalent to

$$\max_{\mathbf{T}(f_n)} \text{Real}\,[\, \mathrm{Tr}\,(\, \mathbf{R}_{gcn}\, \mathbf{T}^H(f_n)\,)], \; n=1, \ldots, J, \tag{3.3}$$

where Tr[·] is the trace of the matrix in the bracket and $\mathbf{R}_{gcn}$ is the outer product of steering vectors given by

$$\mathbf{R}_{gcn} = \int_{\mu_i}^{\mu_f} \mathbf{a}(\theta, f_c)\, \mathbf{a}^H(\theta, f_c+f_n)\ d\mu, \tag{3.4}$$

If the search interval is symmetric (i.e. $\theta_i = -\theta_f$), then the element $[\mathbf{R}_{gcn}]_{pq}$ of M x M matrix $\mathbf{R}_{gcn}$ has a simple form of $2\sin\theta_i\ \mathrm{sinc}\,\{\frac{d\sin\theta_i}{c}\,(\,(p-1)\omega_c - (q-1)\omega_n\,)\}$, $1 \le p \le M$; $1 \le q \le M$. The outer product $\mathbf{R}_{gcn}$ of steering vectors can be factored by the singular value decomposition as $\mathbf{R}_{gcn} = \mathbf{U}(f_n)\, \Sigma(f_n)\, \mathbf{V}^H(f_n)$, where $\mathbf{U}(f_n)$ and $\mathbf{V}(f_n)$ are M by M unitary matrices formed by left and right singular vectors of $\mathbf{R}_{gcn}$ and the matrix $\Sigma(f_n)$ is a diagonal matrix with singular values of $\mathbf{R}_{gcn}$.

Hence we have the following problem :

$$\max_{\mathbf{T}(f_n)} \text{Real}\,[\, \mathrm{Tr}\,(\, \mathbf{U}(f_n)\, \Sigma(f_n)\, \mathbf{V}^H(f_n)\, \mathbf{T}^H(f_n)\,)],$$

$$n=1, \ldots, J. \quad (3.5)$$

It can be easily shown using the similar method to derive the focusing matrix in [10] that $\mathbf{T}(f_n)$ is given by $\mathbf{U}(f_n)\,\mathbf{V}^H(f_n)$. For G group sources, $\mathbf{U}(f_n)$ and $\mathbf{V}(f_n)$ are M by M unitary matrices formed by left and right singular vectors of

$$\mathbf{R}_{cn} = \mathbf{R}_{1cn} + \cdots + \mathbf{R}_{Gcn}. \quad (3.6)$$

Now we can design a bank of transformation matrices which try to focus far-field wideband sources from the angles of interest onto rank-1 subspace spanned by $\mathbf{a}(\theta, f_c)$. Using this transformation matrices $\mathbf{T}(f_n)$ at each f_n, the transformed data and its correlation matrix under the assumption that noise field is spatially uncorrelated and has an unknown noise power $\sigma_n^2(f_n)$ at each frequency, can be obtained as follows:

$$\mathbf{Y}(f_n) = \sum_{n=1}^{J} \mathbf{T}(f_n)\mathbf{X}(f_n) \quad (3.7)$$

and
$$\mathbf{R_Y} = \sum_{n=1}^{J} \mathbf{T}(f_n)\,\mathbf{R_X}(f_n)\,\mathbf{T}^H(f_n)$$
$$= \mathbf{A}(\theta, f_c)\,\mathbf{R_{YS}}\,\mathbf{A}^H(\theta, f_c) + \sigma^2\,\mathbf{R_{YN}} \quad (3.8)$$

where σ^2 is the sum of noise power $\sigma_n^2(f_n)$ over the J frequency bins, $\mathbf{R_{YS}} = \sum_{n=1}^{J} \mathbf{R_S}(f_n)$ and $\mathbf{R_{YN}} = \sum_{n=1}^{J} (\sigma_n^2(f_n) / \sigma^2)\,\mathbf{T}(f_n)\,\mathbf{T}^H(f_n)$. Now we can apply any narrow-band direction finding algorithm using the matrix pencil ($\mathbf{R_Y}$, $\mathbf{R_{YN}}$), if we know array geometry whatever it is.

4. PERFORMANCE STUDIES

A linear array of 17 sensors, uniformly spaced with an inter-element spacing d equals to 0.5 wavelength of the temporal frequency f_c which is normalized to one, was used. The wide-band signals have a bandwidth of 0.4 of the center frequency f_c. The total observation time T_0 is divided into 64 segments. Each segment of this wide-band signals are decomposed into J = 33 frequency bins which gives a time-bandwidth product of BT_0=2048. The noise is assumed to be stationary, statistically independent and identical white Gaussian bandpass processes with zero mean. The signal-to-noise ratio (SNR) was defined here as the ratio of the power of each source signal to the power of the noise at a single sensor. Fifty independent trials were used to obtain the approximate performance measures. The sources were considered to be resolved if two estimates of the direction of arrivals were obtained and each was located within ±2 degrees of the true angle of arrival.

Single Group Sources

The array receives two equi-powered uncorrelated wide-band signals impinging from directions 9° and 12°, which are assumed to be temporarily stationary bandpass white Gaussian processes.

Fig.1 shows the plot of the probability of resolution. It is clear that at low SNR the proposed method outperforms the spatial resampling.

Fig.2 shows the standard deviation comparisons of angle estimate of the source at θ_1=9°. Again The proposed method show lower standard deviation than the spatial resampling at the low SNR.

Also the means are plotted in Fig.3. When the properly selected search interval (i.e. [-45,45]) is used, the proposed method is seen to have a mean close to true angle of arrival. The focusing with large search interval gives the large deviations from true angle of arrival although it shows lower resolution threshold.

Multi-Group Sources

Three group of sources, $\{[\theta_1,\theta_2],[\theta_3,\theta_4],\theta_5\} = \{[-15°,-10°], [9°,12°], 40°\}$, with {12dB, 16dB, 6dB} are used. If the search interval is not symmetric, numerical difficulty arises to obtain $\mathbf{U}_g(f_n)$ and $\mathbf{V}_g(f_n)$ using the singular value decomposition with ***MATLAB*** or ***LINPACK***. So multiple non-symmetric search intervals are obtained by the combination of multiple symmetric search intervals. So the focusing matrices come from the singular vectors of

$\mathbf{R}_{cn} = [\,\mathbf{R}_{cn}|_{\mu_1} - \mathbf{R}_{cn}|_{\mu_2}] + [\,\mathbf{R}_{cn}|_{\mu_3} - \mathbf{R}_{cn}|_{\mu_4}] + [\,\mathbf{R}_{cn}|_{\mu_5} - \mathbf{R}_{cn}|_{\mu_6}]$, where μ_1= sin(20°), μ_2= sin(5°), μ_3= sin(17°), μ_4= sin(4°), μ_5= sin(45°) and μ_6= sin(35°). The same scenario as in the single group search is used. Fig. 4 shows spatial spectral estimates obtained via the proposed method for ten independent trials. All five sources in this multigroup scenario are resolved.

5. CONCLUSION

In this paper, we have proposed a new focusing matrix which does not require preliminary estimates. Its performance has been compared with spatial resampling method. And also, it has been demonstrated that the new focusing matrix gives good estimates at low SNR where the spatial resampling method can not resolve multiple closely space sources.

REFERENCE

[1] Bohme, J. F., " Array processing" , in *Signal Processing, Vol. II,* (J. L. Lacoume et al, editors) Elsevier

Science Publishers B. V., 1987.

[2] Schmidt, R. O., " Multiple emitter location and signal parameter estimation", *Proc. of RADC Spectrum Estimation Workshop*, Griffiths AFB, NY, pp. 243-258, 1979.

[3] Kumaresan, R. and Tufts, D., " Estimating the angles of arrival of multiple plane waves", *IEEE trans. Aerospace and Electronic Systems*, Vol. AES-19, No. 1, pp. 134-139, Jan. 1983.

[4] Barabell, A. J., " Improving the resolution performance of eigenstructute-based direction-finding algorithms", *Proc. of ICASSP83*, Boston, pp. 336-338, 1983.

[5] Roy, R., and Kailath, K., " ESPRIT - Estimation of Signal Parameters via Rotational Invariance Techniques", *IEEE Trans. Acoust. Speech and Signal Proc.*, Vol. ASSP -37, No. 7, pp 984-995, 1989

[6] Kaveh, M. and Barabell, A. J., " The statistical properties of the MUSIC and the Minimum-Norm algorithms in resolving plane waves in noise", *IEEE Trans. Acoust. Speech and Signal Proc.*, Vol. ASSP-34, pp. 331-341, 1986

[7] Rao, B. D. and Hari, V. S., " Performance analysis of Root-MUSIC ", *IEEE Trans. Acoust. Speech and Signal Proc.*, Vol. ASSP, No. 12, pp. 1939-1949, Dec. 1989

[8] Stoica, P. and Nehorai, A., " MUSIC, maximum likelihood, and Cramer-Rao Bound", *IEEE Trans. Acoust. Speech and Signal Proc.*, Vol. ASSP-37, No. 5, pp. 720-741, May 1989.

[9] Hong, W. and Tewfik, A. H., " Estimation of angles of arrivals of multiple wideband sources ", *in Proc. of IEEE Antenna and Propagation Society Symposium*, Ontario, Canada, pp.368-371, June 1991

[10] Hung, H and Kaveh, M., " Focusing Matrices for Coherent Signal-Subspace Processing,"*IEEE Trans. Acoust. Speech and Signal Proc.*, Vol. ASSP-36, No. 8, pp. 1272-1281, August 1988

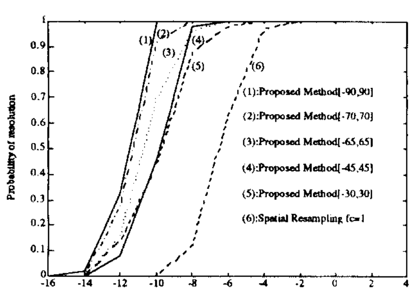

Fig.1 Resolution performance comparisons.

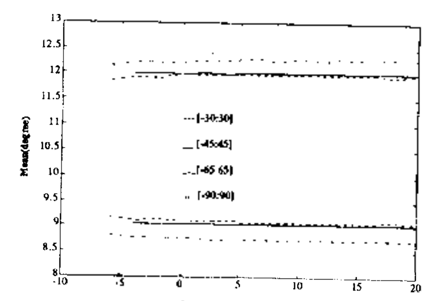

Fig.3 Mean comparisons.

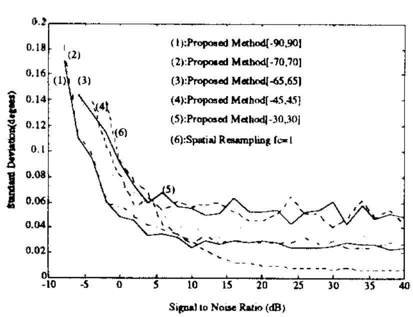

Fig.2 Standard deviation comparisons at $\theta=9^{o}$.

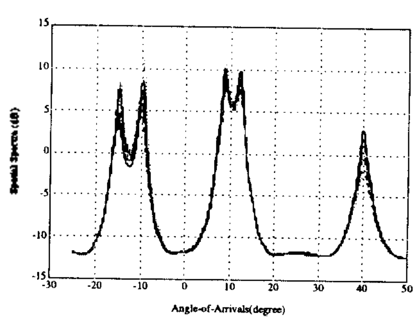

Fig.4 Spatial spectra for ten independent trials in a five uncorrelated sources, three group scenario with three group consideration.