불확실하고 상태변수의 지연이 있는 시스템에 대한 H∞제어기

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Robust H∞ Control for State Delayed Linear Systems with Uncertainties

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Abstract

In this paper, we present a robust H^∞ controller for a state delayed system with uncertainties. The unstructured and norm bounded uncertainties enter into both the state and the input matrix, where the matching condition of the uncertainties is not assumed. A robust stabilization condition and also a robust H^∞ stabilization condition are suggested. The robust H^∞ controller is obtained by solving a Riccati equation which is derived from the suggested robust H^∞ stabilization condition.

1 INTRODUCTION

In recent years, there have been several publications [1]-[4], about the robust H^∞ control problems which combine the robust controller with the H^∞ controller. And there are also a few publications about the H^∞ control for delayed linear systems [5]. In such publication, the objective is to find a feedback control law to stabilize the uncertain system or the delayed system, and guarantee the prescribed level of the disturbance attenuation in the H^∞ sense. In [1] and [2], Xie *et.al* suggested the robust H^∞ controller when the structured uncertainty enters either purely into the input matrix or purely into the state matrix. Also, Xie *et.al* [3] suggested the robust H^∞ controller for uncertain systems with norm bounded time-varying uncertainties in both the state and input matrix, which must satisfy the matching condition. Lee *et.al* suggested the memoryless H^∞ controller for state delayed system.

In this paper, we will propose a robust H^∞ controller for a state delayed linear time-invariant system with norm bounded and unstructured uncertainties in both the input and state matrix, which need not necessarily satisfy the matching condition. In particular, we will extend the robust H^∞ controller in [6] and the memoryless H^∞ controller in [5], to a robust H^∞ controller for a state delayed system with uncertainties. First, we suggest a robust stability condition for the state delayed uncertain system, and then derive a robust H^∞ controller using this suggested stability condition. The derived controller is obtained by solving an algebraic Riccati equation.

2 MAIN RESULTS

We consider the state delayed linear system with uncertainties given by

$$\begin{aligned} \frac{dx(t)}{dt} &= [A + \Delta A(\sigma)]x(t) + A_h x(t-h) \\ &\quad + [B + \Delta B(\sigma)]u(t) + Dw(t) \\ z(t) &= Ex(t) \\ x(t) &= \Phi(t),\ t \in [-h, 0] \end{aligned} \tag{1}$$

where $x \in R^n$ is the state, $u \in R^m$ the control, $w \in R^p$ the disturbance, and $z \in R^q$ the controlled output. $A, A_h \in R^{n\times n}, B \in R^{n\times m}, D \in R^{n\times p}, E \in R^{q\times n}$, and $\Phi(t) \in C[-h, 0]$ is a continuous vector valued initial function. $\Delta A(\sigma) \in R^{n\times n}$ and $\Delta B(\sigma) \in R^{n\times m}$ are the time-invariant uncertainties in the state matrix and the input matrix, respectively, where σ is a vector of uncertain parameters which is restricted to a prescribed bounding compact set $\sum$. The uncertainties are bounded as follows:

$$\begin{aligned} \Delta A^T(\sigma)\Delta A(\sigma) &\le Q_S \\ \Delta B^T(\sigma)\Delta B(\sigma) &\le R_I, \quad \text{for all } \sigma \in \textstyle\sum, \end{aligned} \tag{2}$$

where Q_S and R_I are positive definite matrices with appropriate dimensions. In this paper, ΔA and ΔB will be used to denote $\Delta A(\sigma)$ and $\Delta B(\sigma)$, respectively.

We are interested in designing a linear state feedback control

$$u(t) = Fx(t), \tag{3}$$

where $F \in R^{m \times n}$ is a constant matrix. The closed loop transfer function T_{zw} from w to z is given by

$$T_{zw}(s) = E(sI - \hat{A} - \hat{\Delta} + A_h e^{-sh})^{-1} D, \qquad (4)$$

where $\hat{A} = A + BF$ and $\hat{\Delta} = \Delta A + \Delta B F$.

The purpose of this paper is to construct a controller $u(t)$, which stabilizes the closed loop system and simultaneously guarantees the H^∞ norm bound, γ, of the transfer function T_{zw}, namely $\| T_{zw} \|_\infty \leq \gamma$, where γ is a positive real value. In order to find the robust H^∞ controller, we will derive a robust stabilization condition for stabilizing the closed loop system and also a robust H^∞ stabilization condition for guaranteeing the H^∞ norm bound. The following lemma suggests the stability condition for a state delayed time-invariant linear system, in which uncertainties enter into both the input and state matrix without the matching condition.

LEMMA 1 *The closed loop system of the state delayed linear system (1) with norm bounded uncertainties (2) is quadratically stable with the state feedback control (3) ,if there exist positive definite matrices P, Q, R, and Π, which satisfies the following inequality:*

$$(A + BF)^T P + P(A + BF) + Q + \Pi + F^T R F$$
$$+ P(2I + A_h \Pi^{-1} A_h^T) P < 0, \qquad (5)$$

where $Q \geq Q_S$ and $R \geq R_I$.

Proof: The proof follows immediately from Lemma 1 in [5] and Lemma 1 in [6].

<Q.E.D.>

The above lemma provides a robust stabilization condition which is dependent only on the upper bounds of the uncertainties and is independent of the delay h. Lemma 1 will be used to obtain a robust H^∞ controller.

Now, we will present a sufficient condition under which the controller (3) guarantees the H^∞ norm bound of the transfer function T_{zw}, i.e. $\| T_{zw} \|_\infty \leq \gamma$, and simultaneously stabilizes the closed loop system.

THEOREM 1 *If there exist positive definite matrices P, Q, R, and Π which satisfy the following inequality:*

$$(A + BF)^T P + P(A + BF) + Q + \Pi + F^T R F$$
$$+ P(2I + A_h \Pi^{-1} A_h^T) P + \frac{1}{\gamma} E^T E + \frac{1}{\gamma} P D D^T P < 0, \qquad (6)$$

where $Q \geq Q_S$ and $R \geq R_I$, then the uncertain and state delayed linear system(1) with the control (3) is quadratically stable and the H^∞ norm of the transfer function T_{zw} is less than or equal to γ, i.e. $\| T_{zw} \|_\infty \leq \gamma$ for all ΔA and ΔB satisfying (2), and for all $h \geq 0$.

Proof: From Lemma 1, the controller with the state feedback gain F satisfying the inequality (6) stabilizes the uncertain system (1) for all ΔA and ΔB satisfying (2). Let's define a positive definite matrix S as follows:

$$S := -(\hat{A}^T P + P\hat{A} + Q + \Pi + F^T R F$$
$$+ P(2I + A_h \Pi^{-1} A_h^T) P \frac{1}{\gamma} E^T E + \frac{1}{\gamma} P D D^T P), \quad (7)$$

Then we have

$$\hat{A}^T P + P\hat{A} + Q + \Pi + F^T R F$$
$$+ P(2I + A_h \Pi^{-1} A_h^T) P + \frac{1}{\gamma} E^T E + \frac{1}{\gamma} P D D^T P + S = 0 \quad (8)$$

and

$$(-jwI - \hat{A}^T - \hat{\Delta}^T - e^{jwh} A_h^T) P$$
$$+ P(jwI - \hat{A} - \hat{\Delta} - e^{-jwh} A_h)$$
$$-F^T R F - 2P^2 - Q - \Pi - S - \frac{1}{\gamma} E^T E - \frac{1}{\gamma} P D D^T P$$
$$= -\Delta^T P - P\Delta - e^{jwh} A_h^T P - e^{-jwh} P A_h \quad (9)$$

for all $w \in R$. Let's define W and $X(jw)$ as follows:

$$W(jw) := Q + F^T R F + 2P^2 - \hat{\Delta}^T P - P\hat{\Delta} + \Pi$$
$$+ P A_h \Pi^{-1} A_h^T P - e^{jwh} A_h^T P - e^{-jwh} P A_h \quad (10)$$

$$X(jw) := (jwI - \hat{A} - \hat{\Delta} - A_h e^{-jwh})^{-1} \quad (11)$$

for all $w \in R$. Since

$$W(jw) = Q + F^T R F + 2P^2 - \Delta A^T P - P \Delta A$$
$$- F^T \Delta B^T P - P \Delta B F$$
$$+ [P A_h e^{-jwh} - \Pi] \Pi^{-1} [A_h^T P e^{jwh} - \Pi] \quad (12)$$
$$\geq Q + F^T R F - \Delta A^T \Delta A - F^T \Delta B^T \Delta B F$$
$$\geq Q - Q_S + F^T (R - R_I) F, \quad (13)$$

$W(jw)$ is nonnegative definite. Using the matrices $W(jw)$ and $X(jw)$, the equation (9) may be rewritten as

$$(X^T(-jw))^{-1} P + P X^{-1}(jw) - W(jw)$$
$$- S - \frac{1}{\gamma} E^T E - \frac{1}{\gamma} P D D^T P = 0 \quad (14)$$

also

$$P X(jw) + X^T(-jw) P - \frac{1}{\gamma} X^T(-jw) P D D^T P X(jw) =$$
$$X^T(-jw) \{ W(jw) + S + \frac{1}{\gamma} E^T E \} X(jw). (15)$$

This implies that

$$D^TPX(jw)D + D^TX^T(-jw)PD$$
$$-\frac{1}{\gamma}D^TX^T(-jw)PDD^TPX(jw)D - \gamma I =$$
$$-\gamma I + D^TX^T(-jw)\{W(jw) + S + \frac{1}{\gamma}E^TE\}X(jw)D. \quad (16)$$

It follows that

$$-\frac{1}{\gamma}(\gamma I - D^TPX(-jw)D)^T(\gamma I - D^TPX(jw)D) =$$
$$-\gamma I + D^TX^T(-jw)\{W+S\}X(jw)D$$
$$+\frac{1}{\gamma}D^TX^T(-jw)E^TEX(jw)D \quad (17)$$

for all $w \in R$. In the equation (17), the left hand side is nonpositive definite and $EX(jw)D = T_{zw}(jw)$. Hence, we have

$$-\gamma I + D^TX^T(-jw)\{W(jw) + S\}X(jw)D$$
$$+\frac{1}{\gamma}T_{zw}^T(-jw)T_{zw}(jw) \leq 0 \quad (18)$$

and

$$T_{zw}^T(-jw)T_{zw}(jw) \leq \gamma^2 I - \gamma D^TX^T(-jw)\{W+S\}X(jw)D$$
$$\leq \gamma^2 I \quad (19)$$

for all $w \in R$, i.e. $\| T_{zw} \|_\infty \leq \gamma$.

<Q.E.D.>

If there exists a matrix F which satisfies the inequality (6), then $u(t) = Fx(t)$ is a robust H^∞ controller, which stabilizes the state delayed uncertain system and guarantees the H^∞ norm bound γ.

Consider the controller

$$u(t) = -R^{-1}B^TPx(t), \quad (20)$$

where R is a positive definite matrix. Substituting this feedback gain matrix $-R^{-1}B^TP$ of the control (20) into the inequality (6), we have a matrix inequality

$$A^TP + PA - PBR^{-1}B^TP + \frac{1}{\gamma}PDD^TP + Q$$
$$+\Pi + P(2I + A_h\Pi^{-1}A_h^T)P + \frac{1}{\gamma}E^TE < 0. \quad (21)$$

Hence, if there exist positive definite matrices P, $Q \geq Q_S$, $R \geq R_I, \Pi$, and a positive value ϵ which satisfy the following equation:

$$A^TP + PA - PBR^{-1}B^TP + \frac{1}{\gamma}PDD^TP + Q$$
$$+\Pi + P(2I + A_h\Pi^{-1}A_h^T)P + \frac{1}{\gamma}E^TE + \epsilon I = 0, \quad (22)$$

then the state feedback control (20) robustly stabilizes the linear system (1) and guarantees the H^∞ norm bound of the transfer function T_{zw}. The derived Riccati equation (22) is similar to Riccati equations in [2], [5], and [6], though the problems are different. Under the special case that $A_h = 0, R = I$ and $\epsilon = 1$, the Riccati equation (22) coincides with the Riccati equation proposed in [2], when the unstructured uncertainty purely enters into the state matrix.

3 Conclusion

In this paper, we proposed a robust H^∞ controller for a state delayed linear time-invariant system with unstructured and norm bounded uncertainties. The proposed controller is an extension of the H^∞ controller proposed in [5] and [6]. The controller robustly stabilizes the state delayed uncertain system (1) and simultaneously guarantees the H^∞ norm bound of the closed loop transfer function $T_{zw}(s)$. A robust stabilization condition and also a robust H^∞ stabilization condition are suggested. We can obtain the robust H^∞ controller via the Riccati equation which is derived from the suggested robust H^∞ stabilization condition. By choosing appropriate Q, R, Π and ϵ, we can obtain the positive definite solution of (22).

References

[1] L. Xie and C.E. de Souza, "Robust H^∞ control for linear time-invariant systems with norm bounded uncertainty in the input matrix," *Systems Control Lett.*, vol. **14**, pp.389-396, 1990.

[2] L. Xie and C.E. de Souza, "Robust H^∞ control for a class of uncertain linear time-invariant systems," *IEE Proc.*, vol. **138**, pp.479-483, 1991.

[3] L. Xie and C.E. de Souza, "Robust H^∞ control for linear systems with norm-bounded time-varying uncertainty," *IEEE Trans. Automat. Contr.*, vol. **AC-37**, pp.1188-1191, 1992.

[4] L. Xie, M. Fu, and C.E. de Souza, "H^∞ control and quadratic stabilization of systems with parameter uncertainty via output feedback," *IEEE Trans. Automat. Contr.*, vol. **AC-37**, pp.1253-1256, 1992.

[5] J.H. Lee, S.W. Kim, and W.H. Kwon, "Memoryless H^∞ controllers for state delayed systems," *IEEE Trans. Automat. Contr.*, to be published.

[6] J.W. Lee, J.H. Lee, and W.H. Kwon, "Robust H^∞ control for Uncertain Linear Systems," *Proc. 1993 SICE Conf.*, to be published.