'93 KACC (1993.10.20~22)

On the Generalized Truncated Least Squares Adaptive Algorithm and Two-Stage Design Method with Application to Adaptive Control

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ABSTRACT

This paper presents a generalized truncated least squares adaptive algorithm and a two-stage design method. The proposed algorithm is directly derived from the normal equation of the generalized truncated least squares method (GTLSM). The special case of the GTLSM, the truncated least squares (TLS) adaptive algorithm, has a distinct features which includes the case of minimum steps estimator. This algorithm seemed to be best in the deterministic case. For real applications in the presence of disturbances, the GTLS adaptive algorithm is more effective. The two-stage design method proposed here combines the adaptive control system design with a conventional control design method and each can be treated independently. Using this method, the validity of the presented algorithms are examined by the simulation studies of an indirect adaptive control.

1. INTRODUCTION

The least squares method (LSM) for on-line parameter estimation in linear regression models depends on the use of historic data, which are a collection of all previous obserbations. This method does not necessarily work well, however, with parameter convergence and many other methods have been developped to improve its convergence properties.[1-6]

In this paper a generalized truncated least squares method (GTLSM) is proposed. A special case of GTLSM is a truncated least squares method (TLSM). The TLSM depends on the truncated data which are a collection of the last M observations. Here, M is any number greater than N which is the number of unknown parameters in a regression model. In the ideal circumstance, where there is no uncertainty, the algorithm converges to its true value in a finite steps M. So, if M equals to N, the TLSM algorithm gives a minimum steps estimator. But, in the presence of disturbances, the algorithm is very sensitive to a variation of the unknown parameter, and so is to disturbances. To moderate the estimated parameter responses, a performance of GTLSM consists of a linear combination of the one of TLSM and a term of variation of estimates. A weighting parameter introduced presents an adaptive ability in the closed loop system when the algorithm is used in adaptive control, special cases of which are adaptive control with TLSM algorithm and the fixed parameter control.

In the second part of this paper, a two-stage design method is proposed. The method includes double feedback loops , inner loop of which is to design a input-output characteristics by a model matching method and is able to use as an adaptive loop and the outer loop is designed to compensate unmodeled dynamics and disturbance without affecting input-output characteristic which is independent with an inner loop design.

The effectiveness of the algorithm and the two-stage design method are examined by means of the simulation studies of indirect adaptive control systems synthesis method.

2. PROBLEM STATEMENT and LSM

Consider a linear regression model

$$y_k = v_k^T \theta \quad k=1,2,\ldots \tag{2-1}$$

where y_k is a scalar output at the k^{th} instant, v_k is a N-dimensional known vector function and θ is a N-dimensional unknown parameter vector. The problem of concern is to derive an algorithm which estimates θ in a recurrent form. Often this is done by minimizing the performance criterion

$$J_k = \frac{1}{2}\sum_{i=1}^{k} \lambda^{k-i}(y_i - v_i^T \theta)^2 \tag{2-2}$$

where λ is a weighting parameter. This is the LSM with an forgetting factor. Where $\lambda=1$ it is the standard LSM. The normal form of (2-2) is

$$\left(\sum_{i=1}^{k} \lambda^{k-i} v_i v_i^T\right)\theta = \sum_{i=1}^{k} \lambda^{k-i} v_i y_i \tag{2-3}$$

and the recurrent form of the LSM is

$$\Delta\theta_k = \theta_k - \theta_{k-1} = g_k e_k, \tag{2-4-1}$$

$$e_k = y_k - v_k^T \theta_{k-1}, \tag{2-4-2}$$

$$g_k = P_k v_k = \frac{P_{k-1} v_k}{\lambda + v_k^T P_{k-1} v_k}, \tag{2-4-3}$$

$$P_k = \lambda^{-1}(I - g_k v_k^T) P_{k-1}. \tag{2-4-4}$$

3. GENERALIZED TRUNCATED LEAST SQUARES METHOD (GTLSM)

3.1 Derivation of a GTLS Adaptive Algorithm

The GTLSM is a method of finding a θ_k which minimizes

$$J_k(\beta, M, \theta_k) = \tfrac{1}{2}(1-\beta)\sum_{i=0}^{M-1}(y_{k-i} - \theta_k^T v_{k-i})^2 + \tfrac{1}{2}\beta(\theta_k - \theta_{k-1})^T \Gamma (\theta_k - \theta_{k-1}), \quad 0 \leqq \beta \leqq 1 \tag{3-1}$$

where θ_{k-1} is a priori estimate at instant k, Γ is a positive definite symmetric matrix and the performance is measured over the last M observations. It is also possible to introduce a forgetting factor λ in (3-1) as in (2-2), but it is omitted here for simplicity. The case β=0 is called the Truncated Least Squares Method (TLSM) because the data are truncated. The second term on the right side of equation (3-1) is introduced to make the parameter estimates smooth. When β=1 the case is trivial such that $\theta_k = \theta_{k-1}$ which reduces to an initial estimate. Therefore, the parameter β can be thought as measure of adaptive ability in a feedback control system.

The normal form of equation (3-1) is

$$(\beta\Gamma + (1-\beta)\sum_{i=0}^{M-1} v_{k-i} v_{k-i}^T)\theta_k = \beta\Gamma\theta_{k-1} + (1-\beta)\sum_{i=0}^{M-1} v_{k-i} y_{k-i}. \tag{3-2}$$

Define

$$Q_k^{-1} = \beta\Gamma + (1-\beta)\sum_{i=0}^{M-1} v_{k-i} v_{k-i}^T. \tag{3-3}$$

Then, it follows from the definition that

$$\theta_k = \beta Q_k \Gamma \theta_{k-1} + (1-\beta) Q_k \sum_{i=0}^{M-1} v_{k-i} y_{k-i}, \tag{3-4}$$

$$Q_k^{-1} = Q_{k-1}^{-1} + (1-\beta)(v_k v_k^T - v_{k-M} v_{k-M}^T) \tag{3-5}$$

where Q_k is positive definite for $0 < \beta \leqq 1$. Another excellent feature is an initial setting of Q_k. By using (3-3) recursively, it follows that

$$Q_k^{-1} = Q_0^{-1} + (1-\beta)\sum_{i=0}^{M-1} V_{k-i} V_{k-i}^T \tag{3-6}$$

and, from (3-3) and (3-6),

$$Q_0^{-1} = \beta\Gamma, \qquad Q_0 = \frac{1}{\beta}\Gamma^{-1}. \tag{3-7}$$

If β tends to zero, $1/\beta$ in (3-7) should be replaced with a sufficiently large constant number as is done in the standard LSM. Equations (3-4) and (3-5) give an GTLS adaptive algorithm. To obtain a computationally more effective algorithm, consider the following.

[Fact 1] If the matrices A_0 and A satisfy the equation

$$A_0^{-1} = A^{-1} + b c^T + d f^T \tag{3-8}$$

for vectors b, c, d and f with an appropriate size, then it follows that

$$A_0 = A - g_{11} A b c^T A + g_{12} A d c^T A + g_{21} A b f^T A - g_{22} A d f^T A \tag{3-9}$$

where

$$g_{11} = \frac{1 + f^T A d}{g}, \qquad g_{12} = \frac{f^T A b}{g}, \tag{3-10}$$

$$g_{21} = \frac{c^T A d}{g}, \qquad g_{22} = \frac{1 + c^T A b}{g}, \tag{3-11}$$

$$g = (1 + f^T A d)(1 + c^T A b) - (f^T A b)(c^T A d). \tag{3-12}$$

(Proof) This fact is proved by the well known matrix inversion lemma.

Equation (3-3) then becomes

$$Q_k = Q_{k-1} - (1-\beta) g_{1k} w_{1k}^T + (1-\beta) g_{2k} w_{2k}^T \tag{3-13}$$

where the following variables are defined not only for notational convenience, but also for ease of computer programming.

$$w_{1k} = Q_{k-1} v_k, \qquad w_{2k} = Q_{k-1} v_{k-M}, \tag{3-14-1}$$

$$s_{1k} = v_k^T w_{1k}, \qquad s_{2k} = v_{k-M}^T w_{2k}, \qquad s_{0k} = v_{k-M}^T w_{1k} = v_k^T w_{2k}, \tag{3-14-2}$$

$$d_k = (1 - (1-\beta) s_{2k})(1 + (1-\beta) s_{1k}) + (1-\beta)^2 s_{0k}^2, \tag{3-14-3}$$

$$g_{1k} = ((1 - (1-\beta) s_{2k}) w_{1k} + (1-\beta) s_{0k} w_{2k}) / d_k, \tag{3-14-4}$$

$$g_{2k} = ((1 + (1-\beta) s_{1k}) w_{2k} - (1-\beta) s_{0k} w_{1k}) / d_k, \tag{3-14-5}$$

$$e_{Mk} = y_{k-M} - \theta_{k-1}^T v_{k-M}, \tag{3-14-6}$$

$$e_k = y_k - \theta_{k-1}^T v_k, \tag{3-14-7}$$

$$\Delta\theta_k = \theta_k - \theta_{k-1}. \tag{3-14-8}$$

The following relationships are also proved by direct substitutions.

$$Q_k v_k = g_{1k}, \qquad Q_k v_{k-M} = g_{2k}. \tag{3-15}$$

From (3-2),

$$\begin{aligned} Q_k^{-1}\theta_k &= \beta\Gamma\theta_{k-1} + (1-\beta)\sum_{i=0}^{M-1} v_{k-i} y_{k-i} \\ &= \beta\Gamma\theta_{k-1} + (1-\beta)\{v_k y_k - v_{k-M} y_{k-M} + \sum_{i=0}^{M-1} v_{k-i} y_{k-i}\} \\ &= \beta\Gamma\theta_{k-1} + (1-\beta)\{v_k y_k - v_{k-M} y_{k-M}\} + Q_{k-1}^{-1}\theta_{k-1} - \beta\Gamma\theta_{k-2} \\ &= \beta\Gamma(\theta_{k-1} - \theta_{k-2}) + (1-\beta)\{v_k y_k - v_{k-M} y_{k-M}\} + \{Q_k^{-1} - (1-\beta)(v_k v_k^T - v_{k-M} v_{k-M}^T)\}\theta_{k-1}. \end{aligned} \tag{3-16}$$

As a result, the following algorithm is obtained.

[Theorem 1: GTLS Adaptive Algorithm 1]

$$\Delta\theta_k = \beta Q_k \Gamma \Delta\theta_{k-1} + (1-\beta)\{g_{1k} e_k - g_{2k} e_{Mk}\}. \tag{3-17}$$

Equation (3-17) and (3-13) with (3-14) give an adaptive algorithm for the performance (3-1).

Substituting (3-13) into the right hand side of

(3-17) gives an another version of the algorithm.

[Theorem 2: GTLS Adaptive Algorithm 2]

$$\Delta\theta_k=\beta Q_{k-1}\Gamma\Delta\theta_{k-1}+(1-\beta)g_{1k}(e_k-\beta w_{1k}^T\Gamma\Delta\theta_{k-1})-(1-\beta)g_{2k}(e_{Mk}-\beta w_{2k}^T\Gamma\Delta\theta_{k-1}). \quad (3\text{-}18)$$

Equations (3-18) and (3-13) with (3-14) give an adaptive algorithm for the performance (3-1).

The algorithm performs two roles simultaneously. One is to add a new datum point and the other is to discard an old datum point. These two activities can be carried separately as follows

[Corollary 1] Let

$$w_{2k}=0,\quad s_{0k}=0 \quad (3\text{-}19)$$

in the [Theorem 1, 2]. It follows then that

$$s_{2k}=0,\quad d_k=1+(1-\beta)s_{1k},$$
$$g_{1k}=w_{1k}/d_k,\quad g_{2k}=0. \quad (3\text{-}20)$$

and

$$Q_k=Q_{k-1}-(1-\beta)g_{1k}w_{1k}^T, \quad (3\text{-}21)$$
$$\Delta\theta_k=\beta Q_{k-1}\Gamma\Delta\theta_{k-1}+(1-\beta)g_{1k}(e_k-\beta w_{1k}^T\Gamma\Delta\theta_{k-1}). \quad (3\text{-}22)$$

This results in the addition of a new datum point at each step in time and, when $\beta=0$, it coincides with eq.(2-4) where $\lambda=1$.

[Corollary 2] Let

$$w_{1k}=0,\quad s_{0k}=0 \quad (3\text{-}23)$$

in the [Theorem 1, 2]. It follows then that

$$s_{1k}=0,\quad d_k=1-(1-\beta)s_{2k},$$
$$g_{1k}=0,\quad g_{2k}=\lambda w_{2k}/d_k \quad (3\text{-}24)$$

and

$$Q_k=Q_{k-1}+(1-\beta)g_{2k}w_{2k}^T, \quad (3\text{-}25)$$
$$\Delta\theta_k=\beta Q_{k-1}\Gamma\Delta\theta_{k-1}-(1-\beta)g_{2k}(e_{Mk}-\beta w_{2k}^T\Gamma\Delta\theta_{k-1}). \quad (3\text{-}26)$$

In this case, only an old datum point is discarded and time step is not proceeded.

3.2 Stability of the algorithm

To prove the stabitity of the algorithm, it is assumed that the unknown parameter vector θ is constant. From (3-4), (3-5) and (2-1), it follows that

$$Q_k^{-1}(\theta-\theta_k)=\beta\Gamma(\theta-\theta_{k-1}). \quad (3\text{-}27)$$

For this system, the following function

$$L_k=(\theta-\theta_k)^T\Gamma(\theta-\theta_k) \quad (3\text{-}28)$$

becomes a Lyapunov function. The proof is as follows. From (3-27) and (3-28),

$$\Delta L_k=L_k-L_{k-1}$$
$$=-(\theta-\theta_{k-1})^T\Gamma Q_k D Q_k\Gamma(\theta-\theta_{k-1}) \quad (3\text{-}29)$$

where

$$D=Q_k^{-1}\Gamma^{-1}Q_k^{-1}-\beta^2\Gamma. \quad (3\text{-}30)$$

Define

$$\Lambda_k=\sum_{i=0}^{M-1}v_{k-i}v_{k-i}^T\geqq 0 \quad (3\text{-}31)$$

and assume that Λ_k is positive definite. Then from (3-3), (3-30) and (3-31)

$$D=(1-\beta)(2\beta\Lambda_k+(1-\beta)\Lambda_k\Gamma^{-1}\Lambda_k) \quad (3\text{-}32)$$

and

$$\Delta L_k<0$$

follows for $0\leqq\beta<1$. This means that the system defined by (3-27) is asymptotically stable. It is also clear that for $\beta=0$, $\theta_k=\theta$ when Λ_k becomes positive definite for constant θ.

Next, consider the following function which is a difference of the squares sums of a posteriori output estimation error and a priori one.

$$\Delta\varepsilon_k^2=\sum_{i=0}^{M-1}(y_{k-i}^T-v_{k-i}^T\theta_k)^2-\sum_{i=0}^{M-1}(y_{k-i}-v_{k-i}^T\theta_{k-1})^2. \quad (3\text{-}33)$$

Then this is reformed to

$$\Delta\varepsilon_k^2=(\theta-\theta_k)^T\Lambda_k(\theta-\theta_k)-(\theta-\theta_{k-1})^T\Lambda_k(\theta-\theta_{k-1})$$
$$=-(\theta-\theta_{k-1})^T\Gamma Q_k C Q_k\Gamma(\theta-\theta_{k-1}) \quad (3\text{-}34)$$

where

$$C=Q_k^{-1}\Gamma^{-1}\Lambda_k\Gamma^{-1}Q_k^{-1}-\beta^2\Lambda_k$$
$$=(1-\beta)\Lambda_k(2\beta\Gamma^{-1}+(1-\beta)\Gamma^{-1}\Lambda_k\Gamma^{-1})\Lambda_k. \quad (3\text{-}35)$$

Here Λ_k is positive semidefinite and so is C and

$$\Delta\varepsilon_k^2\leqq 0 \quad (3\text{-}36)$$

holds for any M and $0\leqq\beta<1$. Equality in (3-36) holds when

$$\beta\Lambda_k Q_k\Gamma(\theta-\theta_{k-1})=\Lambda_k(\theta-\theta_k)=0. \quad (3\text{-}37)$$

Therefore, if Λ_k is positive definite

$$\Delta\varepsilon_k^2<0. \quad (3\text{-}38)$$

Furthermore, it is shown that the introduction of β has a role to decrease the expectation of the varince of the estimated parameters errors under the condition that the system is corrupted with a white gaussian output disturbance.

3.3 TLS Adaptive Algorithm

As is seen in the proof of stability, the case where $\beta=0$ has a special feature in the GTLS algorithm. This case is called the TLS algorithm because this is the same with a standard LS algorithm (2-4) except the data are truncated. In this case, the nonsingularity of Q_k and the initial setting (3-7) are not valid. For the initial value of Q_0, two methods can be considered.

[Setting 1] Given the initial estimate θ_0, it may be assumed that the system parameter vector had a value θ_0 for $k\leqq 0$ and then changed to the real value θ for $k>0$. These past fictitious values v_j $(1-M\leqq j\leqq 0)$ can be constructed in such a way that the system equation (2-1) is satisfied for the parameter θ_0 and initial value y_0, and Q_0 determined by (3-3) where

$\beta=0$ is nonsingular.

[Setting 2] As with the standard LSM,

$$Q_0=\gamma I, \quad v_j=0, \quad 1-M \leqq j \leqq 0 \qquad (3\text{-}39)$$

where I is a unit matrix and γ a sufficiently large number.

[Setting 1] corresponds not only to the initial time step but also to any instant where the system parameter vector is changed. This setting is generally not easy to construct and is of only theoretical interest. In contrast to this case, [Setting 2] is more easy to perform in practice although it shows only approximated version. Some properties of TLS algorithm, without proof, are listed below.

[Proposition 1] Adaptive algorithm 1 or 2 where $\beta=0$ with a [Setting 1] and the assumption that Q_k is nonsingular satisfies the normal equation (3-2).

[Proposition 2] For Q_k to be nonsingular, it is necessary and sufficient that $d_k \neq 0$.

[Proposition 3] k=M is the minimum number of steps for θ_k to converge to its true value θ.

[Proposition 4] If M=N and Q_k^{-1} is nonsingular, then

$$y_{k-i}=v_{k-i}^T \theta_k, \quad i=0,1,\ldots,N-1 \qquad (3\text{-}40)$$

holds.

[Proposition 5] If Q_j, j<k-1, are nonsingular and Q_k^{-1} becomes singular, then

$$y_{k-M}-v_{k-M}^T \theta_{k-1}=0. \qquad (3\text{-}41)$$

To treat the case where Q_k is singular, two methods can be considered.

1) Utilize [Theorem 1 or 2] and [Corollary 1 and 2] where $\beta=0$. When Q_k becomes singular, which is recognized by the value of d_k by the [Proposition 2], [Corollary 1] is used to keep Q_k nonsingular. In this case, M is augumented. When Q_k is nonsingular, then [Corollary 2] is used to reduce the augumented value M to the prescribed value, if possible. In this case, the algorithm always satisfies the normal equation (3-2) with $\beta=0$, but may not be practical since the upper bound of M is not known in advance.

2) A more practical method is as follows. If Q_k becomes singular, then $\theta_k=\theta_{k-1}$ is used instead of (3-17) or (3-18) and the data at this time step k are discarded. The nonsingularity of Q_k can then be maintained. This method is easy to implement, but provides for only a theoretical approximation.

It is more efficient, however, to introduce a threshold such that

$$\text{If } |e_k|<\varepsilon, \text{ then } \theta_k=\theta_{k-1}, \qquad (3\text{-}42)$$

since the algorithm is very sensitive not only to the parameter variations, but also to any inaccuracies that may exist. It becomes clear from the simulation studies that the introduction of (3-42) should be used with e_k instead of d_k.

4. TWO-STAGE DESIGN METHOD

4.1 Design Method

Let the process to be controlled be described by

$$Py=R(u+w) \qquad (4\text{-}1)$$

where u is the control variable, y is the measured output and w is an input disturbance. The symbols P and R denote relatively prime polynomials in the forward shift operator z with degrees

$$\deg P=n, \quad \deg R=m, \qquad (4\text{-}2)$$

respectively, and P is a monic polynomial. The desired model be described by

$$P_d y_d=R_d u_d, \qquad (4\text{-}3)$$

where u_d is the reference input and y_d is the desired output. P_d is monic and P_d and R_d are stable polynomials with degrees

$$\deg P_d=n_d=2n-m-1, \quad \deg R_d=m_d=n-1, \qquad (4\text{-}4)$$

respectively. In this formulation, control u is designed as follows.

$$K_1 R_d u=R_d v+A_1 u+B_1 y \qquad (4\text{-}5)$$

where

$$P_d=Q_1 P+S_1, \quad A_1=K_1 R_d-Q_1 R,$$
$$B_1=-S_1, \quad \deg A_1=m_d=n-1, \qquad (4\text{-}6)$$

and v is an intermediate variables designed below. This control constitute an inner feedback loop and it is easily seen that the feedback system coincides with the desired model (4-3) when $v=u_d$. If the plant parameters are unknown, then (4-5,6) can be used as an adaptive loop to adjust the parameters of the unknown matrices A_1 and B_1 using the GTLS adaptive algorithm as an adaptive law. In this setting, the model matching design is also performed by setting $\beta=1$ when a tolerable initial estimate of θ is given and the plant parameter variation are not expected.

The second feedback loop v can be designed as

$$QR_d v=TR_d u_d-SP_d y \qquad (4\text{-}7)$$

for a stable monic polynomial T, and

$$T=Q+S, \quad \deg T=\rho \geqq n-m,$$
$$\deg Q=\rho, \quad \deg S=s \leqq \rho-n+m. \qquad (4\text{-}8)$$

The design of the outer loop (4-7) and (4-8) is independent of the inner loop. This is considered as one of the presentations of design method with two derees of freedom. By a direct substitution, second loop does not affect the input output characteristics and is devoted to compensate system uncertainties like unmodeled dynamics and disturbances.

4.2 Design Method of Outer Loop

The sensitivity function Se and complementary sensitivity Te of the outer loop to the inner loop can be derived as follows.

$$Se=\frac{Q}{T}, \quad Te=\frac{S}{T}. \tag{4-9}$$

In order to minimize Se and/or Te, define T, Q and S as

$$T=\sum_{i=0}^{\rho} t_i z^{\rho-i}, \quad Q=\sum_{i=0}^{\rho} q_i z^{\rho-i}, \quad S=\sum_{i=0}^{m} s_i z^{m-i}, \quad t_0=q_0=1, \tag{4-10}$$

and consider the performance indices

$$J_T=\tfrac{1}{2}\Sigma(t_i)^2, \quad J_Q=\tfrac{1}{2}\Sigma(q_i)^2, \quad J_S=\tfrac{1}{2}\Sigma(s_i)^2, \tag{4-11}$$

It is impossible to minimize the above separately, so the following performance index is introduced.

$$\begin{aligned} J &= \alpha(J_T+J_Q)+(1-\alpha)(J_T+J_S) \\ &= J_T+\alpha J_Q+(1-\alpha)J_S \end{aligned} \tag{4-12}$$

where α is a weighting factor of the sensitivity and complementary sensitivity parameters. In order to have a zero steady state error for a step input or disturbance, polynomial Q has to include a factor $z-1$. Two methods are available for this purpose. One is to replace J_Q with

$$J_{Q1}=\tfrac{1}{2}\sum_{i=0}^{\rho}\Big(\sum_{k=0}^{i} q_{i-k}\Big)^2, \quad J_{Q2}=\tfrac{1}{2}\sum_{i=0}^{\rho}\Big(\sum_{k=0}^{i} (k+1)\, q_{i-k}\Big)^2 \tag{4-13}$$

and, in general,

$$J_{Q\eta}=\tfrac{1}{2}\sum_{i=0}^{\rho}\Big(\sum_{k=0}^{i} {}_{k+\eta-1}C_{\eta-1}\, q_{i-k}\Big)^2. \tag{4-14}$$

It is shown that (4-14) can be minimized using η integrators in Q. The other is to force the term $z-1$ in Q such that

$$Q=(z-1)^{\gamma}Q' . \tag{4-15}$$

As a result, the J has four parameters,

$$J=J(\alpha\,;\,\rho,\ \eta,\ \gamma) \tag{4-16}$$

where η shows the number of integrators desired and γ is the total number of compulsory integrators included. The roles of η and γ are the same when $\alpha=1$. When the parameters of J are determined, the coefficients of the polynomials T, Q and S are derived by differentiating J.

5. SIMULATION STUDIES

Consider as an example a second order system described by

$$(z^2+p_1 z+p_2)y=(r_0 z+r_1)u \tag{5-1}$$

and a reference model by

$$(z^2+p_{d1} z+p_{d2})y_d=(r_{d0} z+r_{d1})u_d, \tag{5-2}$$

where

$$\begin{aligned} &p_1=-1.112, \quad p_2=0.243, \quad r_0=0.133, \quad r_1=0.0835, \\ &p_{d1}=-0.906, \quad p_{d2}=0.156, \quad r_{d0}=0.292, \quad r_{d1}=0.108. \end{aligned} \tag{5-3}$$

For the parameter variation, consider the case:

$$p_1=1.2p_1=-1.3344, \quad k\geqq 80. \tag{5-4}$$

The poles of the system are 0.813, 0.299 and, after variation, they become $-1.117$and -0.218 which show that the system becomes unstable. The control input is also restricted to the interval $-2\leqq u\leqq 2$. In some cases which follow, a disturbace is added to the input signal. The disturbance used in the simulations is a pseudo-random binary sequence generated by means of shift register with feedback[5] and the magnitude of PRBS is chosen as 0.1.

The inner loop is determined by (4-5) and (4-6) as

$$u=\frac{1}{K_1}\Big(v+\frac{a_0}{r_{d0}z+r_{d1}}u+\frac{b_0 z+b_1}{r_{d0}z+r_{d1}}y\Big), \tag{5-5}$$

where

$$\begin{aligned} &K_1=r_0/r_{d0}, \quad A_1=a_0=K_1 r_{d1}-r_1, \\ &B_1=b_0 z+b_1=(p_1-p_{d1})z+(p_2-p_{d2}). \end{aligned} \tag{5-6}$$

Let $\rho=1$, then the outer loop is

$$v=\frac{z+t_1}{z+q_1}u_d+\frac{s_0(z^2+p_{d1}z+p_{d2})}{(z+q_1)(r_{d0}z+r_{d1})}y, \tag{5-7}$$

where

$$T=z+t_1, \quad Q=z+q_1, \quad S=s_0. \tag{5-8}$$

Obviously, these loops (5-5) and (5-7) can be combined into one and system equation (5-1) is transformed to the form of (2-1) and

$$\theta^T=(p_1, p_2, r_0, r_1). \tag{5-9}$$

To show the evaluation of the initial estimate of θ, parameter δ is introduced as

$$\theta_0=\delta\,\theta. \tag{5-10}$$

If $\delta\fallingdotseq 1$, then (5-5) acts as a model matching control, and (5-7) has a conventional integral action if $\eta=1$ and $\alpha=1$ or $\gamma=1$. The case where $\eta=1$ and $\alpha=0$ is the same with the case where $\eta=0$ which mean the control without integrators. When $\eta=1$, the steady state error decreases, in general, to zero as α tends to 1. The intermediate value of α, $0<\alpha<1$, is effective if the integral action is so strong that the closed loop system becomes oscillatory or unstable. These situations will be happened in the cases that the δ is far from 1, system is under noisy circumstances and a system model has indisregarded unmodeled dynamics. In these cases, adaptive control becomes more useful. Fig.1 and Fig.2 demonstrate the simulations for adaptive control using GTLSM without and with disturbance, respectively, where $\delta=0.2$, $\eta=1$, $\alpha=1$, $\beta=0.1$ and $M=10$. Fig.1 is almost the same with the case using TLSM where the exact coincidence with the real values can be attained in the parameter estimation. In the presence of disturbance, the output response of the adaptive control using GTLSM is satisfactory and the parameter responses are fairy tolerable compared to the cases of TLSM and LSM. The numerical values listed just below the Figs.b indicate the real system parameter values and their estimates at 80 and 160 steps. As figures show, the GTLS adaptive algorithm is also effective for systems whose parameter change drastically. It is clear by the formulation that, since the GTLSM and the TLSM treat truncated data, the algorithms are not affected by any parameter variations before M steps past.

5. CONCLUSIONS

In this paper, the generalized truncated least squares adaptive algorithm is presented. The proposed algorithm is directly derived from the normal equation of the generalized truncated least squares criterion. The special case of the GTLSM, the TLS adaptive algorithm, seemed to be best in the deterministic case. For real applications, the GTLS adaptive algorithm is more effective. The two-stage design method is also presented here. Using this method, the validity of the presented algorithms are examined by the simulation studies. In this studies, the two-stage design method is used only as an integrator, therefore more thorough researches are expected.

The problem is how to determine the parameters ρ, η, γ and α, and how to choose the values β and M (in the case of adaptive control). These are highly depending upon a system itself, its environments and the objects of control. But, in authors experinces, it is recommended that $\rho = \eta = \alpha = 1$, $\gamma = 0$, $\beta = 0.01 \sim 0.1$ in most cases. The value M in adaptive algorithm is also important. This depends upon a computer ability and a sampling time. For a slow system parameter variation, M can be a large number, but for a rapid variation, small M gives a quick convergence. This becomes also a trade off.

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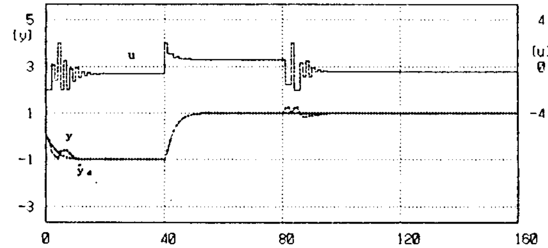
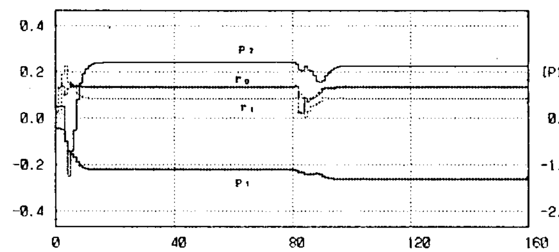
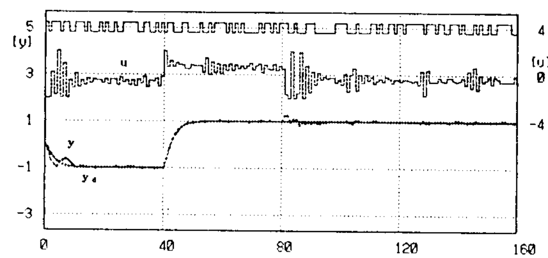


[Fig.1a]: Adaptive Control using GTLSM, $\delta = 0.2$, $\eta = 1$, $\alpha = 1$, $\beta = 0.1$, M=10, Input and Output Responses.

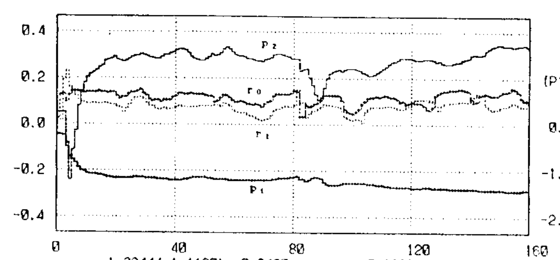


	-1.3344(-1.1120)	0.2430	0.1330	0.0835
80	-1.1118	0.2429	0.1330	0.0835
160	-1.3189	0.2269	0.1337	0.0842

[Fig.1b]: Parameters Responses.



[Fig.2a]: Adaptive Control with disturbance using GTLSM, $\delta = 0.2$, $\eta = 1$, $\alpha = 1$, $\beta = 0.1$, M=10, Input and output Responses.



	-1.3344(-1.1120)	0.2430	0.1330	0.0835
80	-1.1451	0.2835	0.1449	0.0814
160	-1.4169	0.3390	0.0957	0.0790

[Fig.2b]: Parameters Responses.