Optimal Scheduling for Multi-Product Batch Processes Under Consideration of Non-zero Transfer Times and Set-up Times

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Abstract

Simple recurrence relations for calculating completion times of various storage polices (unlimited intermediate storages(UIS), finite intermediate storages(FIS), no intermediate storage(NIS), zero wait(ZW)) for serial multi-product multi-unit processes are suggested. Not only processing times but also transfer times, set-up (clean-up) times of units and set-up times of storages are considered. Optimal scheduling strategies with zero transfer times and zero set-up times had been developed as a mixed integer linear programming(MILP) formulation for several intermediate storage policies. In this paper those with non-zero transfer times, non-zero set-up times of units and set-up times of storages are newly proposed as a mixed integer nonlinear programming(MINLP) formulation for various storage polices (UIS, NIS, FIS, and ZW). Several examples are tested to evaluate the robustness of this strategy and reasonable computation times.

INTRODUCTION

According as the life times of products are shorter and the processing of small quantity, high value-added products is increased, batch or semi-continuous processes are highly required in chemical process industry(CPI). In the batch processes operations, there are many interesting research fields to enhance the productivity and operability. The scheduling of multi-product batch process turns out to be an important field. It had been said the flowshop scheduling problem. This problem has N products to be processed through unit1, unit2, ..., unit$M$. The sequence of processing unit is fixed for every products. Processing times of product i ($i=1,2,...,N$) at unit j ($j=1,2,..., M$) and transfer times of product i ($i=1,2,...,N$) from unit j to unit $j+1$ ($j=1,2,...,M-1$) are fixed. Also sequence-dependent set-up times on each unit and set-up times of storages between every pair of products are fixed. At these conditions, scheduling problem is to find out the best sequence of production and the shortest total operation time of all products, so called minimization of makespan.

Optimal scheduling for serial multi-product batch processes are known as NP-complete combinatorial optimization problems. For this reason, they are restricted by problem size. But optimal solutions are obtainable for relatively small size problems. The methods for optimal solution i.e., MILP, MINLP formulation and solve it by mathematical optimization methods, are important because they give a target(optimal solution) for developing another methods which obtain suboptimal solutions for large size problems. Optimal scheduling and calculation of completion times for multi-product batch scheduling have been studied for a decade. Additionally they have been researched for different operating types of intermediate storage policies. The different types of interstage storage policies which frequently have been studied are UIS, FIS, NIS, ZW, and MIS(mixed intermediate storage). Completion times algorithm under consideration of non-zero transfer and set-up times had been developed by Rajagopalan and Karimi(5). They considered transfer and set-up times splendidly but did not consider sequence dependent storage clean-up times. Optimal scheduling of UIS and NIS policy have been developed via MILP formulations with consideration of only processing times. Optimal scheduling of FIS policy had also been developed with consideration of only processing times by Ku and Karimi(4). However optimal scheduling of various storage policies under consideration of non-zero transfer and set-up times has rarely been studied.

In this study, completion times algorithm for various storage policies which considers not only processing times, transfer times, set-up times of units but also clean-up times of storage are suggested. To obtain the optimal solutions of scheduling with these various storage policies, we developed MINLP formulations. Several examples are solved for the efficiency of these methods.

COMPLETION TIMES DETERMINATION FOR VARIOUS STORAGE POLICIES

Completion times for various storage policies with non-zero transfer and set-up times are well developed by Rajagopalan and Karimi. They assumed that set-up times for storage units are negligible. But set-up times for storage is as important as set-up times for units when intermediate storages are introduced. Set-up times for storages should be also determined sequence-dependent non-zero parameters. To develop recurrence relations for the formulations of completion times following variables should be first defined.

Differently from the method of Rajagopalan and Karimi(5), we obeyed the conventional definitions of each variables.

- N = number of products to be produced.
- M = number of batch units in the plant.
- t_{ij} = processing time of product i on batch unit j
- a_{ij} = transfer time of product i out of batch unit j to batch unit $j+1$.
- S_{ijk} = set-up time required for product j after product i on batch unit k.
- C_{ij} = completion time of ith product in the sequence on batch unit j where the product is finished to transfer out of the unit j and to fill in the unit $j+1$.
- SC_{ij} = set-up time required for product i after product j
- z_j = number of storage units between unit j and unit $j+1$
- $F(z_j)$ = a function defined as $F=0$ if $z_j=0$ and $F=1$ if $z_j>0$

UIS policy

Batch process plant with UIS policy has N-1 storages between every pair of consecutive units. It means that every product i which is finished at unit j needs not holding(waiting) until ready to process product i on next unit. If next unit (unit $j+1$) is busy, product i can transfer out of unit j to available storage any time. At Fig. 1. general schematic feature of UIS flowshop batch process is shown and at Fig. 2. general Gannt chart of UIS flowshop is shown for developing completion times.

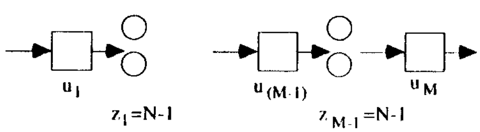

Fig. 1. UIS flowshop batch process

In the case of that only processing times are considered, completion times of UIS flowshop is

$$C_{ij} = \max\,[C_{(i-1)j}, C_{i(j-1)}] + t_{ij} \quad \text{------------------- (1)}$$

and when transfer times and set-up times are considered equation (1) is changed as follows:

$$C_{ij} = \max\,[C_{i(j-1)}, C_{(i-1)j} + S_{(i-1)ij} + a_{i(j-1)}] + t_{ij} + a_{ij} \quad \text{------------------- (2)}$$

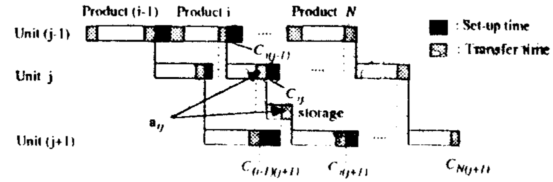

Fig. 2. General Gannt chart of UIS policy

Set-up times for storages are not introduced at equation (2) because the storages are fully available in UIS flowshop process. We assumed that transfer times of product i from unit j to unit $j+1$ are same as transfer times of product i from unit j to an available storage and from the used storage to unit $j+1$ as shown in Fig. 2.

NIS policy

NIS policy batch process plant has no intermediate storages and product i should be holding on unit j when next unit (unit $j+1$) is busy. General Gannt chart of NIS policy for developing completion times is shown in Fig. 3.

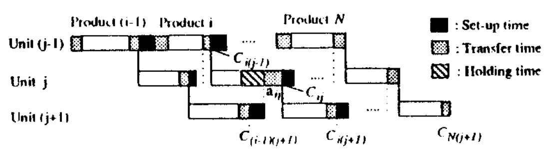

Fig. 3. General Gannt chart of NIS policy

When only production times are considered, C_{ij} for NIS policy is as follows.

$$C_{ij} = \max\,[C_{(i-1)j}, C_{i(j-1)}, C_{(i-1)(j+1)} - t_{ij}] + t_{ij} \quad \text{-------- (3)}$$

where C_{oj}, C_{io}, t_{io}, t_{oj} and $C_{i(M+1)}$ are zero. Equation (3) can be reduced from the condition of $C_{i(j-1)} \geq C_{(i-1)j}$ for $j>1$ as follows:

$$C_{ij} = \max\,[C_{(i-1)j}, C_{(i-1)(j+1)} - t_{ij}] + t_{ij} \quad \text{for } j=1$$

$$C_{ij} = \max\,[C_{i(j-1)}, C_{(i-1)(j+1)} - t_{ij}] + t_{ij} \quad \text{for } j>1 \quad \text{------ (4)}$$

In case of that sequence-dependent set-up times and transfer times are considered, completion time for NIS flowshop is:

$$C_{ij}' = [C_{i(j-1)}, C_{(i-1)j} + S_{(i-1)ij} + a_{i(j-1)}, C_{(i-1)(j+1)} + S_{(i-1)i(j+1)} - t_{ij}] + t_{ij} + a_{ij} \quad \text{------------------------- (5)}$$

Equation (5) can also be reduced from $C_{i(j-1)} \geq C_{(i-1)j} + S_{(i-1)ij} + a_{i(j-1)}$ for $j>1$.

for $j = 1$:

$$C_{ij} = \max [\, C_{(i-1)j} + S_{(i-1)ij} + a_{i(j-1)},\ C_{(i-1)(j+1)} + S_{(i-1)i(j+1)} - t_j \,] + t_{ij} + a_{ij}$$

for $j = 2, \ldots, M$:

$$C_{ij} = \max [\, C_{i(j-1)},\ C_{(i-1)(j+1)} + S_{(i-1)i(j+1)} - t_j \,] + t_{ij} + a_{ij} \qquad \text{------------------------------} \ (6)$$

FIS policy

In the FIS policy, finite number of storage unit is available between stages. The numbers of storage between unit j and unit $j+1$, z_j ($j=1, \ldots, M-1$) are fixed. Completion times algorithm for FIS flowshop under consideration of only processing time is developed by Ku and Karimi(4) as follows:

$$C_{ij} = \max [\, C_{(i-1)j},\ C_{i(j-1)},\ C_{(i-z_j-1)(j+1)} - t_{ij} \,] + t_{ij} \quad \text{------} \ (7)$$

where $C_{ij} = 0$, if $i \leq 0$ or $j \leq 0$ or $j > M$. In case of considering not only processing times but transfer times, sequence-dependent set-up times for units and storages, equation (7) should be changed as follows:

$$C_{ij} = \max [\, C_{i(j-1)} + t_{ij} + q_j,\ C_{(i-1)j} + S_{(i-1)ij} + a_{i(j-1)} + t_{ij} + a_{ij},\ C_{(i-z_j-1)(j+1)} + S_{(i-z_j-1)(i-z_j)(j+1)} + F(z_j)(SC_{(i-z_j)i} + a_{(i-z_j)j}) + q_j \,] \quad \text{----------------------------} \ (8)$$

The Gannt chart, as shown in Fig. 4 may give an explanation that why equation (8) represents the recurrence relations of completion times for FIS policy.

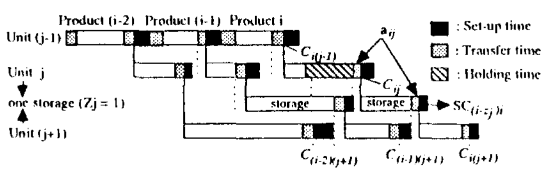

Fig. 4. General Gannt chart of FIS policy

If all storages and next unit (unit $j+1$) are busy, product i which have finished on unit j must be holded by unit j. To complete processing product i on unit j, one of the storages (z_j) should be available. And for this, unit $j+1$ must be ready to process product (i-z_j) which have been stored in one of the storages. So set-up time of storage is dependent on product (i-z_j) and i ie., using product i after product (i-z_j). We assumed that the conditions of all storages are equal.

ZW policy

Products must be transferred immediately from unit j to next unit (j+1) in ZW policy. It does not allow any intermediate storages. Jung et al (7) studied new completion time algorithms and optimal scheduling with MINLP under consideration of non-zero transfer times and set-up times of units. For only the processing time, completion times algorithm is

$$C_{ij} = C_{i(j-1)} + t_{ij}$$

$$C_{iM} = \max [\, C_{(i-1)1} + \sum_{k=1}^{M} t_{ik},\ C_{(i-1)2} + \sum_{k=2}^{M} t_{ik}, \cdots,\ C_{(i-1)(M-1)} + \sum_{k=(M-1)}^{M} t_{ik},\ C_{(i-1)M} + \sum_{k=M}^{M} t_{ik} \,] \quad \text{------} \ (9)$$

$$C_{ij} = C_{iM} - \sum_{k=(j+1)}^{M} t_{ik} \quad \text{--------------------------} \ (10)$$

Equation (9) and (10) should be changed for additional consideration of transfer and sequence dependent set-up times to the equation (11) and (12).

$$C_{i1} = C_{i0} + t_{i1} + a_{i1}$$

$$C_{ij} = C_{i(j-1)} + t_{ij} + a_{ij}$$

$$C_{iM} = \max [\, (C_{(i-1)1} + S_{(i-1)i1} + a_{i0} + \sum_{k=1}^{M} t_{ik} + \sum_{k=1}^{M} a_{ik}),\ (C_{(i-1)2} + S_{(i-1)i2} + \sum_{k=2}^{M} t_{ik} + \sum_{k=2}^{M} a_{ik}), \ldots\ldots,\ (C_{(i-1)M} + S_{(i-1)iM} + \sum_{k=M}^{M} t_{ik} + \sum_{k=M}^{M} a_{ik}) \,] \quad \text{----} \ (11)$$

$$C_{ij} = C_{iM} - \sum_{k=(j+1)}^{M} t_{ik} - \sum_{k=(j+1)}^{M} a_{ik} \quad \text{----------------} \ (12)$$

To show how the equation(11) and (12) are developed, the general Gannt chart for ZW policy is shown in Fig. 5.

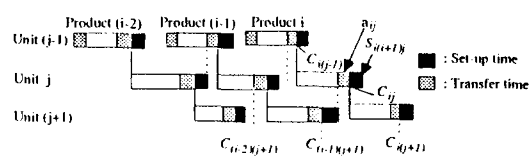

Fig. 5. General Gannt chart for ZW policy

SCHEDULING VIA MINLP FORMULATION FOR VARIOUS STORAGE POLICIES

There has been studied optimal scheduling of multi-product serial batch units which assumed zero transfer and set-up times, by the way of mathmatical formation i.e., MILP formation. But for real operations, non-zero tranfer times and set-up times for units are considered. Recently completion time algorithm with transfer and set-up times for various storage policies are studied. As the number of products and units are larger, the importance of transfer and set-up times is increased. Optimal scheduling method thus must consider the transfer and set-up times. However optimal scheduling under consideration of non-zero transfer and set-up times has been rarely studied.

Sequence dependent set-up times make the problem more complex. Because of the variable which depends on production sequence i.e., set-up times for units, the problem becomes non-linear form, and optimal schedulings are formulated as a MINLP form. The objective function of optimal scheduling is to minimize C_{NM}. For the formation of optimal scheduling, new binary variables are defined as follow:

$$X_{ij} = \begin{cases} 1 & \text{if product } i \text{ is in position } j \text{ in the sequence} \\ 0 & \text{otherwise} \end{cases}$$

And then, the constraints which means a position in the sequence must be assigned to only one products are as follow:

$$\sum_{j=1}^{N} X_{ij} = 1 \qquad (i = 1,2,\ldots,N) \qquad \text{------------------ (13)}$$

An ordered product must be processed only once. So the constraints are constructed as follow:

$$\sum_{i=1}^{N} X_{ij} = 1 \qquad (j = 1,2,\ldots,N) \qquad \text{-------------------- (14)}$$

Equation (13) and (14) are the constraints that commonly used in various intermediate storage policies. To solve the optimal scheduling of various storage policies, additional constraints which shows the characteristic of each storage policy are needed. They are formulated from the completion times algorithm which is shown in above section for each storage policy respectively.

MINLP formulation of UIS policy

The remaining constraints for UIS policy under consideration of non-zero transfer and set-up times which are formulateded by equation (2) are as follows:

for unit 1 (j =1) :

$$C_{i1} \geq C_{(i-1)1} + \sum_{l=1}^{N}\sum_{k=1}^{N} S_{lk1} X_{l(i-1)} X_{ki} + \sum_{k=1}^{N} (a_{k0}+a_{k1}+t_{k1}) X_{ki} \qquad \text{-------------------- (15)}$$

for unit 2,...,M (j = 2,...,M) :

$$C_{ij} \geq C_{i(j-1)} + \sum_{k=1}^{N} (a_{kj}+t_{kj}) X_{ki} \qquad \text{------------------- (16)}$$

$$C_{ij} \geq C_{(i-1)j} + \sum_{k=1}^{N} (a_{k(j-1)}+a_{kj}+t_{kj}) X_{ki} + \sum_{l=1}^{N}\sum_{k=1}^{N} S_{lkj} X_{l(i-1)} X_{ki} \qquad \text{------------------ (17)}$$

The quadratic term $\sum_{l=1}^{N}\sum_{k=1}^{N} S_{lkj} X_{l(i-1)} X_{ki}$ in equation (17) is the expression of sequence dependent set-up times in the optimal scheduling problem. Due to this term, the problem is formulated as an MINLP form.

C_{ij}, X_{ij} = 0 for i or j equal 0. With the objective function of min. C_{NM}, a formulation of UIS policy for optimal scheduling is constructed with constraints by equations (13 -17).

MINLP formulation of NIS policy

The constraints from equation (6) are as follows:

for unit 1 (j = 1)

$$C_{i1} \geq C_{(i-1)1} + \sum_{l=1}^{N}\sum_{k=1}^{N} S_{lk1} X_{l(i-1)} X_{ki} + \sum_{k=1}^{N} (a_{k0}+a_{k1}+t_{k1}) X_{ki} \qquad \text{------------------ (18)}$$

$$C_{i1} \geq C_{(i-1)2} + \sum_{l=1}^{N}\sum_{k=1}^{N} S_{lk2} X_{l(i-1)} X_{ki} + \sum_{k=1}^{N} a_{k1} X_{ki} \qquad \text{--------------------- (19)}$$

for unit 2,...,M-1 (j=2,...,M-1)

$$C_{ij} \geq C_{i(j-1)} + \sum_{k=1}^{N} (a_{kj}+t_{kj}) X_{ki} \qquad \text{-------------------- (20)}$$

$$C_{ij} \geq C_{(i-1)(j+1)} + \sum_{l=1}^{N}\sum_{k=1}^{N} S_{lk(j+1)} X_{l(i-1)} X_{ki} + \sum_{k=1}^{N} a_{kj} X_{ki} \qquad \text{---------------------- (21)}$$

for unit M (j = M)

$$C_{iM} \geq C_{i(M-1)} + \sum_{k=1}^{N} (a_{kM}+t_{kM}) X_{ki} \qquad \text{---------------- (22)}$$

C_{ij}, X_{ij} = 0 for i or j equal 0. With the objective function of min. C_{NM}, a formulation of NIS policy for optimal scheduling is constructed with constraints by equations (13-14) and (18-22).

MINLP formulation of FIS policy

As above mentioned, set-up times of storages(SC_{ij}) shoule be considered for FIS policy additionally. It is shown in equation (8) in detail. Additional constraints for FIS policy are as follows:

for unit 1 (j = 1) :

$$C_{i1} \geq C_{(i-1)1} + \sum_{l=1}^{N}\sum_{k=1}^{N} S_{lk1} X_{l(i-1)} X_{ki} + \sum_{k=1}^{N} (a_{k0}+a_{k1}+t_{k1}) X_{ki} \qquad \text{-------------- (23)}$$

$$C_{i1} \geq C_{(i-z_j-1)2} + \sum_{l=1}^{N}\sum_{k=1}^{N} S_{lk2} X_{l(i-z_j-1)} X_{k(i-z_j)} + \sum_{k=1}^{N} a_{k1} X_{ki} + \sum_{k=1}^{N} F(z_j)(SC_{kj} X_{k(i-z_j)} + a_{k1} X_{k(i-z_j)}) \qquad \text{--------- (24)}$$

for unit 2,...,M-1 (j=2,...,M-1) :

$$C_{ij} \geq C_{i(j-1)} + \sum_{k=1}^{N} (a_{kj}+t_{kj}) X_{ki} \qquad \text{---------------- (25)}$$

$$C_{ij} \geq C_{(i-Zj-1)(j-1)} + \sum_{l=1}^{N} \sum_{k=1}^{N} S_{lk(j+1)} X_{l(i-zj-1)} X_{k(i-zj)} + \sum_{k=1}^{N} a_{kj} X_{ki} + \sum_{k=1}^{N} F(z_j)(SC_{ki} X_{k(i-z_j)} + a_{kj} X_{k(i-z_j)}) \quad \text{-------} \quad (26)$$

$$C_{ij} \geq C_{(i-1)j} + \sum_{k=1}^{N} (a_{k(j-1)} + a_{kj} + t_{kj}) X_{ki} + \sum_{l=1}^{N} \sum_{k=1}^{N} S_{lkj} X_{l(i-1)} X_{ki} \quad \text{-------} \quad (27)$$

for unit M (j = M) :

$$C_{iM} \geq C_{i(M-1)} + \sum_{k=1}^{N} (a_{kM} + t_{kM}) X_{ki} \quad \text{-------} \quad (28)$$

$$C_{iM} \geq C_{(i-1)M} + \sum_{k=1}^{N} (a_{k(M-1)} + a_{kM} + t_{kM}) X_{ki} + \sum_{l=1}^{N} \sum_{k=1}^{N} S_{lkM} X_{l(i-1)} X_{ki} \quad \text{-------} \quad (29)$$

C_{ij}, X_{ij} = 0 for i or j equal 0. With the objective function of min. C_{NM}, a formulation of FIS policy for optimal scheduling is formulated with constraints by equations (13-14) and (23-29). UIS, NIS and FIS policy have several similarities, and one of them is the completion times of the first product on each units and the constraints from the first product. To reduce the number of constraints, equality constraints which are constructed by the condition of the first product may be useful. Equality constraints from the condition of the first product can be applied to UIS, NIS and FIS policy and they are as follows:

for unit 1 (j = 1)

$$C_{11} = \sum_{k=1}^{N} (a_{k0} + a_{k1} + t_{k1}) X_{k1} \quad \text{-------} \quad (30)$$

for unit 2,...,M (j = 2,...,M) :

$$C_{1j} = C_{1(j-1)} + \sum_{k=1}^{N} (a_{kj} + t_{kj}) X_{k1} \quad \text{-------} \quad (31)$$

MINLP formulation of ZW policy

Optimal formulation of ZW policy under consideration of transfer and set-up times had been studied by Jung et al (7). In this paper, we develop an MINLP formulation for ZW policy based on Jung et al (7). From the second to the Nth(final) product, the followings are formulated.

for unit 1:

$$C_{iM} \geq C_{(i-1)M} - \sum_{l=1}^{N} \sum_{k=2}^{M} (t_{lk} + a_{lk}) X_{l(i-1)} + \sum_{l=1}^{N} \sum_{k=1}^{N} S_{lk1} X_{l(i-1)} X_{ki} + \sum_{l=1}^{N} \sum_{k=0}^{M} a_{lk} X_{li} + \sum_{l=1}^{N} \sum_{k=1}^{M} t_{lk} X_{li} \quad \text{-------} \quad (32)$$

for unit 2:

$$C_{iM} \geq C_{(i-1)M} - \sum_{l=1}^{N} \sum_{k=3}^{M} (t_{lk} + a_{lk}) X_{l(i-1)} + \sum_{l=1}^{N} \sum_{k=1}^{N} S_{lk2} X_{l(i-1)} X_{ki} + \sum_{l=1}^{N} \sum_{k=1}^{M} a_{lk} X_{li} + \sum_{l=1}^{N} \sum_{k=2}^{M} t_{lk} X_{li} \quad \text{-------} \quad (33)$$

$\vdots \qquad \vdots \qquad \vdots$

for unit (M-1) :

$$C_{iM} \geq C_{(i-1)M} - \sum_{l=1}^{N} \sum_{k=M}^{M} (t_{lk} + a_{lk}) X_{l(i-1)} + \sum_{l=1}^{N} \sum_{k=1}^{N} S_{lk(M-1)} X_{l(i-1)} X_{ki} + \sum_{l=1}^{N} \sum_{k=(M-2)}^{M} a_{lk} X_{li} + \sum_{l=1}^{N} \sum_{k=(M-1)}^{M} t_{lk} X_{li} \quad \text{-------} \quad (34)$$

for unit M :

$$C_{iM} \geq C_{(i-1)M} + \sum_{l=1}^{N} \sum_{k=1}^{N} S_{lkM} X_{l(i-1)} X_{ki} + \sum_{l=1}^{N} \sum_{k=(M-1)}^{M} a_{lk} X_{li} + \sum_{l=1}^{N} \sum_{k=M}^{M} t_{lk} X_{li} \quad \text{-------} \quad (35)$$

and in the case of 2nd product (i=2), $C_{(i-1)M}$'s in equations (32),(33),(34), and (35) are became C_{1M}, so they should be substituted by

$$\sum_{l=1}^{N} \sum_{k=0}^{M} a_{lk} X_{l1} + \sum_{l=1}^{N} \sum_{k=1}^{M} t_{lk} X_{l1}.$$

NUMERICAL EVALUATIONS

We solve an example (example 1) which is 4 unit, 4 product serial batch process problem and the process data is as shown in Table 1. Example 1 is solved in UIS, NIS, FIS, and ZW policies. In the case of FIS system, it is assumed that one storage tank is introduced between unit 3 and unit 4 (z_1=0, z_2=0, and z_3=1). Optimal sequences of 1-4-3-2 (makespan 120), 1-2-4-3 (makespan 121), 1-4-2-3 (makespan 126), and 1-4-2-3 (makespan 130) were obtained for UIS, FIS, NIS and ZW system respectively within reasonable computational time.

TABLE 1. Process data for examlpe 1.

Products (N)	Processing times Units (M)				Transfer times Units (M)				
	U1	U2	U3	U4	U0	U1	U2	U3	U4
P1	10	20	5	30	2	2	2	2	3
P2	15	8	12	10	3	3	3	3	1
P3	20	7	9	5	2	4	2	2	1
P4	13	7	17	10	2	2	1	4	2

Prod. seq. S (N-N)	Set-up times(unit)				Set-up times(storages) (for FIS policy)
	U1	U2	U3	U4	
S(1-2)	3	1	2	4	2
S(1-3)	2	2	1	3	2
S(1-4)	1	4	2	2	2
S(2-1)	4	1	2	3	1
S(2-3)	1	1	4	3	2
S(2-4)	3	2	3	2	1
S(3-1)	2	1	4	3	2
S(3-2)	1	2	3	2	3
S(3-4)	2	2	2	2	3
S(4-1)	4	3	4	3	1
S(4-2)	1	4	3	3	1
S(4-3)	3	2	2	1	3

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