'93 KACC (1993.10.20~22)

Compensation for Temperature-Level Control of Tanked Water System with Time Delay

Masatoshi Nakamura and Kiyoto Watanabe
Department of Electrical Engineering, Saga University, Honjomachi, Saga 840, JAPAN

Abstract

Importance of separation of a nonlinear dynamical system into nonlinear static part and linear dynamical part was insisted in designing a controller for the nonlinear system. We further proposed compensation techniques for oscillation of controlled variables caused by system time delay and compensation of steady state errors caused by modelling errors of the systems. The proposed principle of designing procedure and the compensation methods were discussed by applying them for temperature and level control of an actual tanked water system.

1. INTRODUCTION

Since the dawn of modern control theories in early '60s[1],[2],[3], there had been attempted to apply those new theories to actual fields, but those attempts had rarely been successful except for aerospace technology[4] because of lack of computational tools. The barrier for the application of control theories to actual fields was overcome in late '70s[s] by the wide spread use of computational tools of micro computers and personal computers. Some of linear control theories including estimation and identification theories were applied to restricted actual systems successfully. Meanwhile, we had encountered many gaps between modern control theories and their actual application fields. To fill those gaps, linear control theories have been developed into nonlinear theories[7], adaptive control theories and so on[8]. Some of those theories were reported to be successful in applying to actual systems. This kind of investigations in control theories and their application is one fruitful and healthy way of development. However, sophisticated theories would often cause other new problems in application fields. Authors' philosophy is that acceptable control theories and technologies to actual fields should be always clear and simple. By suitably arranging a procedure of controller design, we can apply simple and well defined conventional linear control theories ideally to design controllers for wide actual systems.

In this paper, we proposed a principle of controller designing procedure for nonlinear dynamical systems of actual fields, and proposed also simple compensation techniques for oscillation of controlled variables caused by system time delay and compensation for steady state errors caused by modelling errors of the systems. The main idea of the principle is that any nonlinear dynamical system should be separated into two parts, nonlinear static part and linear dynamical part, in designing a controller for the nonlinear system. The proposed principle of designing procedure and the compensation methods were discussed by applying them for temperature and level control of a tanked water system of a hand made pilot plant. Usefulness of the principle of the procedure and the compensation techniques were proved by control experiments of the pilot plant of temperature and level of tanked water system.

2. LEVEL AND TEMPERATURE CONTROL OF TANKED WATER SYSTEM

2.1 Equipment of Tanked Water System and its Linear Model

Pilot plant of a tanked water system adopted in this study is illustrated in Fig. 1. Control objective of this study is to maintain level and temperature of the lower tanked water at respective desired values. Inflow rates of cold water and hot water of upper tanks are regulated by the valves driven by respective DC servo motors.

The tanked water system is ideally modelled by the following equations. The first equation for the water level of the lower tank H_m is derived based on mass balance as

$$\dot{H}_m = (G_c + G_h)/S_m - G_m/S_m. \tag{1}$$

where G_c and G_h denote the inflow rates of the cold water and hot water respectively, and S_m is the cross section area of tank. The second equation for the water temperature of the lower tank T_m is derived based on energy balance as

$$\dot{T}_m = (T_c - T_m)G_c/S_m H_m + (T_h - T_m)G_h/S_m H_m. \tag{2}$$

where T_c and T_h are the temperatures of the cold water and hot water respectively. The equations are linearized around a desired level H_d and a desired temperature T_d, and assembled into a state equation of the state space representation as follows:

$$\dot{x} = Ax + Bu + w \tag{3}$$

where

$$x = \begin{bmatrix} H_m - H_d \\ T_m - T_d \end{bmatrix}, u = \begin{bmatrix} G_c - G_c^0 \\ G_h - G_h^0 \end{bmatrix}, w = \begin{bmatrix} -\dot{H}_d \\ -\dot{T}_d \end{bmatrix},$$
$$A = \begin{bmatrix} -G_d/2S_m H_d & 0 \\ 0 & -G_d/S_m H_d \end{bmatrix},$$
$$B = \begin{bmatrix} 1/S_m & 1/S_m \\ (T_c - T_d)/S_m H_d & (T_h - T_d)/S_m H_d \end{bmatrix},$$

$G_c^0 = G_d(T_h - T_d)/(T_h - T_c)$, $G_h^0 = G_d(T_d - T_c)/(T_h - T_c)$, $G_d = G_c^0 + G_h^0$.

The continuous state equation is further transformed into a transition equation of discrete time type with a sampling interval ΔT as

$$x(k+1) = \phi x(k) + Du(k) + w(k). \tag{4}$$

where

$$\phi = \begin{bmatrix} \exp(-G_d\Delta T/2S_m H_d) & 0 \\ 0 & \exp(-G_d\Delta T/S_m H_d) \end{bmatrix},$$
$$D = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix},$$

$d_{11} = d_{12} = \{1 - \exp(-G_d\Delta T/2S_m H_d)\}2H_d/G_d$, $d_{21} = (T_c - T_d)\{1 - \exp(-G_d\Delta T/S_m H_d)\}/G_d$, $d_{22} = (T_h - T_d)\{1 - \exp(-G_d\Delta T/S_m H_d)\}/G_d$. $w(k) = w\Delta T$. As the level and temperature of the lower tanked water are detected directly, the measurement equation is described as

$$y(k) = x(k). \tag{5}$$

The transition equation (4) and the measurement equation (5) are basic models for designing the controller of the tanked water system.

2.2 Nonlinear Characteristics of Detectors

The tanked water levels are detected by each level meter, consisted of a float and a potentio meter. Figure 2(a) shows the characteristic relationship between the water level H_m and output voltage of the potentio meter V_{H_m} of the lower tank. Each point was plotted for the preliminary experimental measurement data of

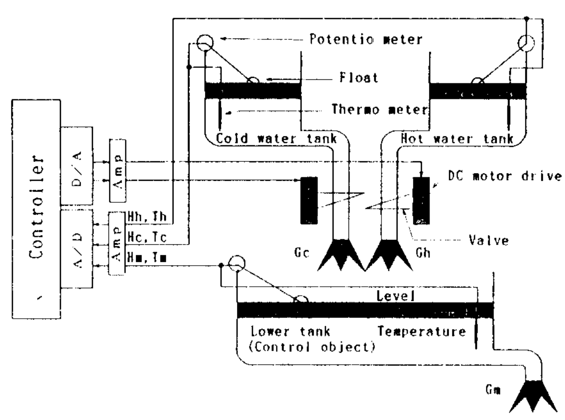

Figure 1 Pilot plant of tanked water system.

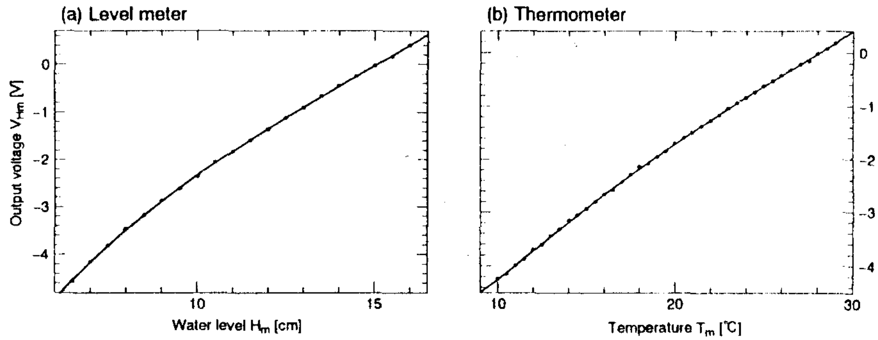

Figure 2 Characteristics of the detectors for cold tanked water: (a) level meter, (b) thermometer.

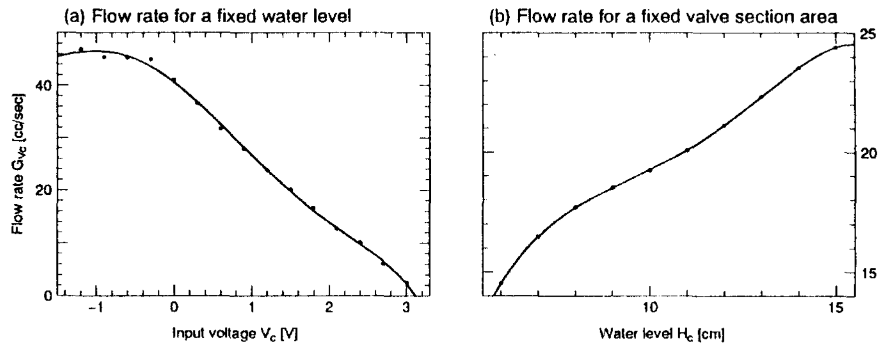

Figure 3 Characteristics of the actuator for cold tanked water: (a) input voltage applied to DC servomotor vs. flow rate of cold water for a fixed water level, (b) tanked water level vs. flow rate of cold water for a fixed valve section area.

the level meter. The characteristics of the level meter showed a slight nonlinearity. The nonlinear relationship was approximated by a polynomial equation of third order based on the least squares method as

$$V_{H_m} = 1.66\times10^{-3}(H_m)^3 - 7.22\times10^{-2}(H_m)^2 + 1.48H_m - 11.5. \quad (6)$$

Characteristics of the level meters for upper tanks were also obtained as $V_{H_c} = -1.63\times10^{-3}(H_c)^3 + 7.90\times10^{-2}(H_c)^2 - 1.67H_c + 11.6$, $V_{H_h} = 1.80\times10^{-3}(H_h)^3 - 9.14\times10^{-2}(H_h)^2 + 1.91H_h - 13.7$.

The temperatures of the tanked waters are detected by thermometers. Figure 2(b) illustrates the characteristic relationship between temperature T_m and output voltage of the thermometer V_{T_m} of the lower tanked water. A slight nonlinear relationship was approximated by third order polynomial equation as

$$V_{T_m} = 9.04\times10^{-6}(T_m)^3 - 3.01\times10^{-3}(T_m)^2 + 3.40\times10^{-1}T_m - 7.37. \quad (7)$$

Similarly, the thermometers for the upper hot water and cold water tanks were characterized by the following equations: $V_{T_c} = 5.21\times10^{-5}(T_c)^3 - 6.66\times10^{-3}(T_c)^2 + 4.21\times10^{-1}T_c - 7.33$, $V_{T_h} = 1.02\times10^{-4}(T_h)^3 - 1.94\times10^{-2}(T_h)^2 + 1.29T_h - 27.7$.

All characteristics of the detectors showed nonlinearity in the input and output relationship.

2.3 Nonlinear Characteristics of Actuators

Inflow rates of cold water and hot water are regulated by valves driven by DC servo motors. Each actuator is characterized by a section area (A) of the valve and a level (H) of the upper tanked water as $G(A, H)$. The section area of the valve is determined by an input voltage (V) of the DC servo motor connected to the valve. Figure 3 shows characteristics of the actuator for cold tanked water: (a) relationship between voltage applied to the DC servo motor V_c and the flow rate G_{V_c} for a fixed water level H_c^0, (b) relationship between tanked water level H_c and the flow rate G_{H_c} for a fixed input voltage V_c^0 of the DC servo motor. The plotted data were approximated by the following equations

$$\begin{aligned} G_{V_c}(V_c, H_c^0) &= -1.49\times10^{-1}(V_c)^5 + 3.50\times10^{-1}(V_c)^4 \\ &\quad + 1.60(V_c)^3 - 4.35(V_c)^2 - 11.4V_c + 40.7, \\ G_{H_c}(V_c^0, H_c) &= -1.68\times10^{-4}(H_c)^5 + 3.95\times10^{-3}(H_c)^4 \\ &\quad + 3.53\times10^{-2}(H_c)^3 - 1.69(H_c)^2 + 16.6H_c \\ &\quad - 35.7 \end{aligned} \quad (8)$$

Based on the curves (8), the characteristics of the actuator for arbitrary water level (H_c) and input voltage (V_c) are approximately expressed as

$$G_c(V_c, H_c) = G_{V_c}(V_c, H_c^0)G_{H_c}(V_c^0, H_c)/G_{H_c}(V_c^0, H_c^0). \quad (9)$$

The actuator characteristics showed a high nonlinearity in the input and output relation (Fig. 3). The actuator for hot tanked water was characterized as the same way and showed similar characteristics to equation (8).

The total tanked water system between the input voltage of the actuator and the output voltages of the detectors is expressed by nonlinear dynamical equations including eqns. (4)-(9).

3. CONTROLLER DESIGN FOR NONLINEAR DYNAMICAL SYSTEM

3.1 Separation of Nonlinear Dynamical System into Two Parts

The tanked water system of the pilot plant, consisted of actuators, control object and detectors, was characterized by a nonlinear dynamical system. Controller design for nonlinear dynamical

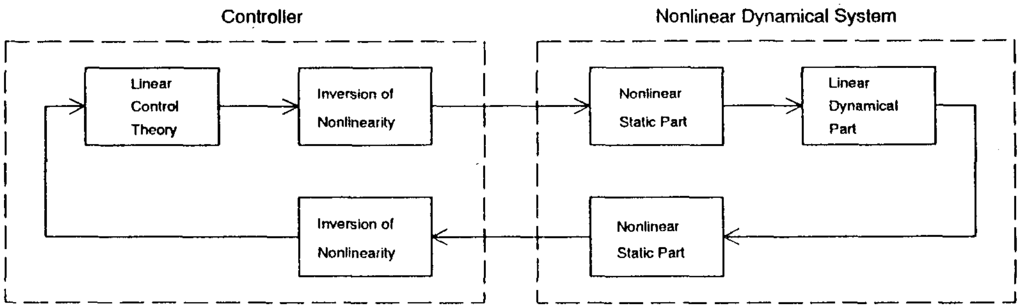

Figure 4 Separation of nonlinear dynamical system.

system, in general, is not an easy task. We insist, right now, an important idea of controller designing procedure for nonlinear dynamical system of actual fields. In designing a controller for an actual system, the nonlinear dynamical system, in almost all cases, should be separated into two parts: nonlinear static part and linear dynamical part (see right hand side of Fig. 4). In constructing a controller for the separated system, we can eliminate the nonlinearities by introducing an inversion characteristics of the nonlinear static parts as shown in Fig. 4 and then apply any conventional linear control theories ideally to the linear dynamical part of the system.

For the tanked water system treated in this study, the nonlinear static parts correspond to the detectors and the actuators, and the linear dynamical part corresponds to the control object represented by eqn (4) and (5).

3.2 Controller Design for Tanked Water System

Schematic representation of the controller design for nonlinear dynamical system is illustrated in the left hand side of Fig. 4. Restoration characteristics of nonlinearities in the detectors and the actuators were realized by the inversion functions of (6)-(8), and were obtained based on the least squares method by using the same data for eqns (6)-(8). The restoration characteristics of the detectors were obtained as follows:

$$\begin{aligned} H_m &= -8.22\times10^{-3}(V_{H_m})^3 + 7.26\times10^{-2}(V_{H_m})^2 + 2.38V_{H_m} \\ &\quad +15.1, \\ H_c &= 1.79\times10^{-2}(V_{H_c})^3 + 0.159(V_{H_c})^2 - 2.29V_{H_c} + 12.5, \\ H_h &= -1.67\times10^{-2}(V_{H_h})^3 + 0.162(V_{H_h})^2 + 2.40V_{H_h} \\ &\quad +13.9; \end{aligned} \tag{10}$$

$$\begin{aligned} T_m &= 1.45\times10^{-2}(V_{T_m})^3 + 0.284(V_{T_m})^2 + 5.20V_{T_m} + 28.2, \\ T_c &= 2.25\times10^{-2}(V_{T_c})^3 + 0.425(V_{T_c})^2 + 5.51V_{T_c} + 25.9, \\ T_h &= -4.14(V_{T_h})^3 + 25.4(V_{T_h})^2 - 34.5V_{T_h} + 64.4. \end{aligned} \tag{11}$$

The restoration characteristics of the actuators were represented by

$$\begin{aligned} V_c &= -2.54\times10^{-7}(G_{V_c})^5 + 2.72\times10^{-5}(G_{V_c})^4 \\ &\quad -1.03\times10^{-3}(G_{V_c})^3 + 1.69\times10^{-2}(G_{V_c})^2 - 0.197G_{V_c} \\ &\quad +3.44, \\ V_h &= 6.45\times10^{-8}(G_{V_h})^5 - 9.85\times10^{-6}(G_{V_h})^4 \\ &\quad +5.61\times10^{-4}(G_{V_h})^3 - 1.50\times10^{-2}(G_{V_h})^2 + 0.218G_{V_h} \\ &\quad -3.28. \end{aligned} \tag{12}$$

The variables G_{V_c} and G_{V_h} in (12) are adjusted ones depending on the water levels of the upper tanks as $G_{V_c} = G_c G_{H_c}(V_c^0, H_c^0)/G_{H_c}(V_c^0, H_c)$, $G_{V_h} = G_h G_{H_h}(V_h^0, H_h)/G_{H_h}(V_h^0, H_h^0)$, which show the inversion characteristics of the actuator (9).

These equations showed inversion characteristics of equations (6)-(8), respectively. By using (10)-(12), we can eliminate nonlinearities of the system almost exactly.

As the rest of the system is expressed by linear dynamical equations of (4) and (5), a linear control theory such as the pole assignment regulator can be ideally applicable to the linear dynamical part of the control object. The manipulated variable is calculated by the following equations as

$$u(k) = Kx(k). \tag{13}$$

The gain matrix of the pole assignment regulator K is obtained by

$$K = FM^{-1}. \tag{14}$$

The vector components of the matrix M are calculated by

$$m_i = (\lambda_i I - \phi)^{-1} D f_i. \tag{15}$$

where f_i is an independent arbitrary vector of the ith component in the matrix F, and λ_i denotes the pole of the regulator.

The controller is calculated by a series combination of the restoration characteristics of detectors (10), (11), the linear control strategy (13)-(15) and the restoration characteristics of actuators (12). An experiment of level and temperature control of the tanked water system was implemented by using the constructed controller. In the experimental results, we found some oscillations and steady state errors of the level and the temperature of the tanked water around the respective desired values. Those phenomena in the experiment were not preferable as for control performances. We shall discuss compensation techniques for these deteriorated control performance in the following chapters.

4. COMPENSATION FOR OSCILLATION CAUSED BY SYSTEM TIME DELAY

4.1 System with Detection Time Delay

A compensation technique for oscillations of controlled variables caused by system time delay of the system is discussed in this chapter. We had proposed a compensation method for oscillations caused by a drive time delay in '91 KACC[9]. In this chapter, we modified the previous compensation method into a new one for the system with detection time delay i.e. the system time delay is transformed into the detector part and compensation of time delay is done as if it appeared in the detection side.

A system with detection time delay is described by

$$x(k+1) = \phi x(k) + Du(k), \tag{16}$$

$$y(k) = \begin{pmatrix} x_1(k-L_1) \\ x_2(k-L_2) \end{pmatrix} = x(k-L) = Z^{-L}x(k) \tag{17}$$

where

$$Z^{-L} = \begin{bmatrix} Z^{-L_1} & 0 \\ 0 & Z^{-L_2} \end{bmatrix}.$$

If the system represented by (16) and (17) is controlled by using the manipulated variable without any consideration of the detection time delay as

$$u(k) = Ky(k) = KZ^{-L}x(k) \tag{18}$$

the system behaves as follows:

$$x(k+1) = (\phi + DKZ^{-L})x(k). \tag{19}$$

The system will oscillate because of the detection time delay Z^{-L}.

4.2 Compensation for Oscillation

Main idea of compensation for oscillation caused by detection time delay is to obtain a future value of the measurement based on a real time computer simulation. The real time simulation is implemented based on the transition equation (16) and measurement

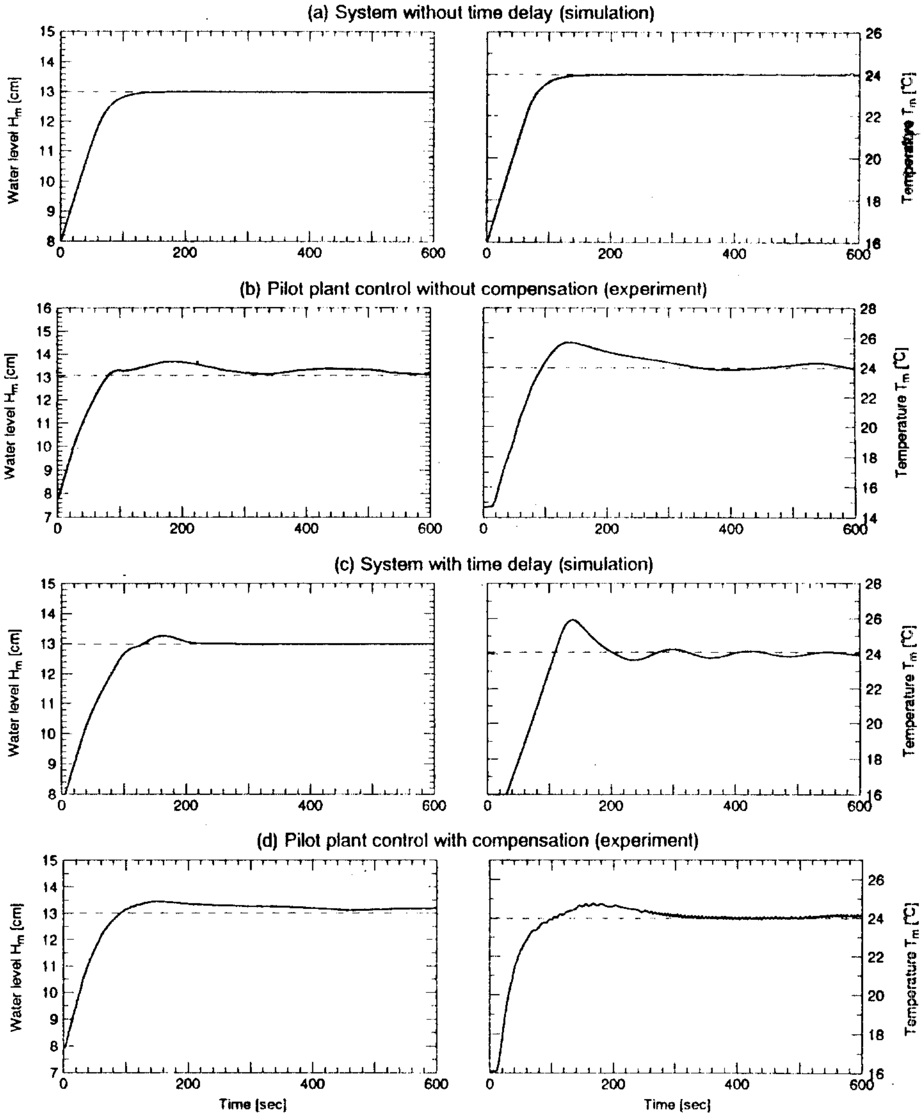

Figure 5 Control results of water level and water temperature of lower tank: (a) computer simulation of the system without any time delay controlled by the pole assignment regulator, (b) experimental result of the pilot plant controlled by the pole assignment regulator, (c) computer simulation of the system with detection time delay controlled by the pole assigument regulator, (d) experimental result of the pilot plant controlled by the proposed compensation method.

equation (17). The prediction of the state variable is obtained iteratively as

$$\hat{x}(k-l+i) = \phi\hat{x}(k-l+i-1) + D\hat{u}(k-l+i-1),$$
$$\hat{u}(k-l+i-1) = K\hat{x}(k-l+i-1), \quad i = 1, 2, \cdots, l. \tag{20}$$

where the initial state of known values is given as $\hat{x}(k-l) = x(k-l)$, and $l = \max(L_1, L_2)$.

The Lth step ahead prediction of the estimated measurement is obtained by using the prediction of the state variables of (20) as

$$\hat{y}(k+L) = \hat{x}(k) = x(k). \tag{21}$$

The second equality in (21) is fulfilled if the actual system is exactly modelled by the model (16) and (17). Compensated manipulated variable is determined by using the Lth step ahead prediction $\hat{y}(k+L)$ as

$$u(k) = KZ^L\hat{y}(k) = K\hat{x}(k) = Kx(k). \tag{22}$$

The system with detection time delay controlled by the compensated manipulated variable behaves as

$$x(k+1) = \phi x(k) + Du^0(k) = (\phi + KD)x(k) \tag{23}$$

As equation (23) is identically equal to the system without any time delay controlled by the conventional pole assignment regulator, the oscillation of the state will be eliminated completely by the control action of the compensated manipulated variable .

4.3 Simulation Studies and Experimental Result

Figure 5 illustrates four kinds of results: two simulation results and two experimental results. Left hand side of the figures illustrates behavior of the level of the tanked water and right hand side illustrates the corresponding temperature of the water. Figure 5 (a) shows the result by a computer simulation of the system without time delay (equations (4) and (5)) controlled by the pole assignment regulator (equations (13)-(15)). The level and temperature of the water in the lower tank approached to the respective

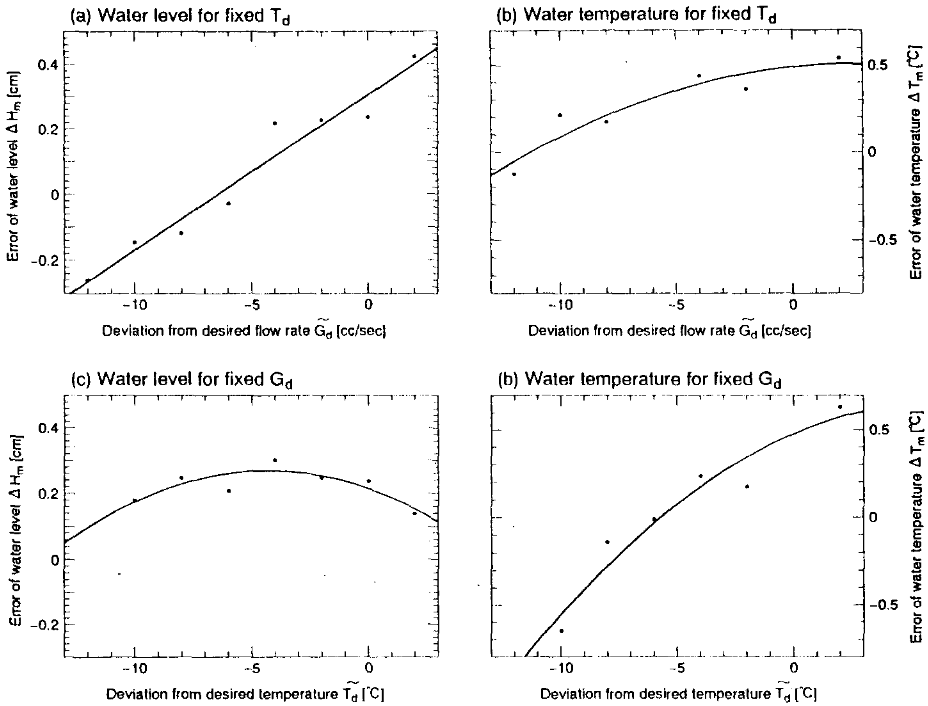

Figure 6 Relationship between the steady state errors and tuning parameters $(\tilde{G}_d, \tilde{T}_d)$: (a) deviation from desired flow rate vs. steady state error of the water level for fixed $\tilde{T}_d$, (b) deviation from desired flow rate vs. steady state error of the water temperature for fixed $\tilde{T}_d$, (c) deviation from desired temperature vs. steady state error of the water level for fixed $\tilde{G}_d$, (d) deviation from desired temperature vs. steady state error of the water temperature for fixed $\tilde{G}_d$.

desired values appropriately. The simulation conditions were as follows: temperature of cold water and hot water $T_c \doteq 13$[°C], $T_h = 60$[°C]; desired values of level and temperature $H_d = 13$[cm], $T_d = 24$[°C]; initial values of level and temperature $H_m(0) = 8$[cm], $T_m(0) = 16$[°C]; total control time =600[sec]; sampling interval 1[sec]; poles of the regulator $\lambda = 0.955, 0.956$ respectively.

Figure 5 (b) shows the experimental result of the pilot plant controlled by the conventional pole assignment regulator (equations (13)-(15)) without any compensation. There appeared oscillations around the respective desired values of the water level and temperature . The experimental condition were the same as the conditions of the simulation study.

The phenomena of the oscillation in the experiment were simulated by system with detection delay times (equations (16), (17)) controlled by the pole assignment regulator (equations (13)-(15)). Many simulation studies were performed for different values of the detection time delay. The Figure 5(c) illustrates the simulation result of the system with detection delay time $L_1 = 5$[sec] and $L_2 = 30$[sec], respectively. The period of the oscillation of the experiment was about 100 [sec] and 0.3 [°C] in the water temperature. Those oscillations created by the simulation study showed similarity to the experimental oscillations in Fig. 5(b). This implies that the actual plant is equivalent to the simulated system with detection delay time 5 [sec] and 30 [sec], respectively.

The experimental control result by using the proposed compensation method of (20)-(22) is shown in Fig. 5(d). The oscillations of the level and temperatures appeared in Fig.5(b) were eliminated almost completely in the experimental result by using the compensation method. We could prove the effectiveness of the proposed compensation technique through the control experiment of the pilot plant.

If we look at the last experimental results in more detail, a slight steady state errors can be found. Next objective is to eliminate this kind of steady state errors.

5. COMPENSATION FOR STEADY STATE ERROR CAUSED BY MODELLING ERRORS

5.1 Tuning Parameters for Steady State Error

The inflow rates of cold water and hot water were determined as $G_c = \hat{u}_1(k) + G_c^0$, $G_h = \hat{u}_2(k) + G_h^0$, by using the compensated manipulated variable of (22). Steady inflow rates G_c^0 and G_h^0 affects the steady state errors of the level and temperature of the tanked water, remarkably. By introducing modified desired flow rate $\hat{G}_d$ and modified desired water temperature $\hat{T}_d$ as

$$\begin{aligned} \hat{G}_d &= G_d + \tilde{G}_d, \\ \hat{T}_d &= T_d + \tilde{T}_d, \end{aligned} \tag{24}$$

we can regulate the steady inflow rate as follows:

$$\begin{aligned} \hat{G}_h^0 &= (G_d + \tilde{G}_d)(T_h - T_d - \tilde{T}_d)/(T_h - T_c), \\ \hat{G}_c^0 &= (G_d + \tilde{G}_d)(T_d + \tilde{T}_d - T_c)/(T_h - T_c). \end{aligned} \tag{25}$$

We implemented many control experiments of tanked water systems by using $\hat{G}_d$ and $\hat{T}_d$ instead of G_d and T_d. Fig. 6(a),(b) illustrate the steady state error for different $\hat{G}_d$ and fixed $\hat{T}_d = T_d$. The variable $\hat{G}_d$ affected the steady state error of the water level ΔH_m remarkably than that of water temperature ΔT_m. Fig. 6(c),(d) illustrate the steady state errors for different $\hat{T}_d$ and fixed $\hat{G}_d = G_d$. The variable $\hat{T}_d$ affected the steady state error of the water temperature ΔT_m more than that of water level ΔH_m.

5.2 Compensation for Steady State Error

We derive a compensation method for eliminating the steady state error of the level ΔH_m and that of the temperature ΔT_m of the tanked water system by using the modified outflow rate $\hat{G}_d$ and the modified desired temperature $\hat{T}_d$.

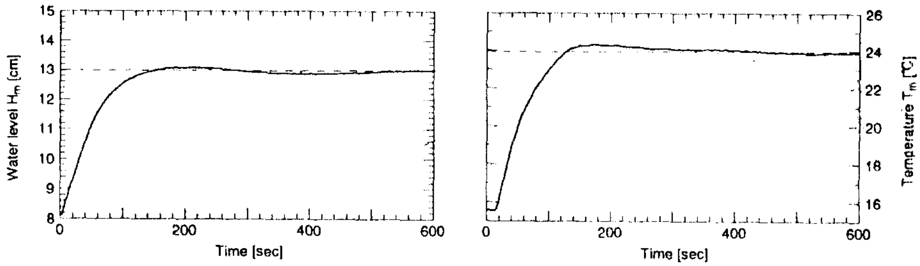

Figure 7 Experimental result of the pilot plant controlled by the compensation techniques for oscillation and steady state error.

As the ΔH_m is sensitive to $\tilde{G}_d$ and the ΔT_m is sensitive to $\tilde{T}_d$ respectively (Fig. 6), the steady state errors ΔH_m and ΔT_m are approximated by the polynomials of second order as

$$\begin{aligned} \Delta H_m &= \alpha_0 + \alpha_1 \tilde{G}_d + \alpha_2 (\tilde{G}_d)^2, \\ \Delta T_m &= \beta_0 + \beta_1 \tilde{T}_d + \beta_2 (\tilde{T}_d)^2. \end{aligned} \tag{26}$$

The coefficients of the polynomial $(\alpha_0, \alpha_1, \alpha_2)$ are determined based on the least squares method so as to minimize the following criterion

$$J_{H_m} = \sum_{i=1}^{N} \{\Delta H_m(i) - \alpha_0 - \alpha_1 \tilde{G}_d(i) - \alpha_2 (\tilde{G}_d(i))^2\}^2. \tag{27}$$

The set $\{\Delta H_m(i), \tilde{G}_d(i)\}$ are preliminary experimental data for a given fixed value $\tilde{T}_d$. The value of $\tilde{G}_d$ is obtained by solving the second order polynomial (26) whose left hand side is set at zero as follows:

$$\tilde{G}_d = \left\{-\alpha_1 + \sqrt{(\alpha_1)^2 - 4\alpha_0\alpha_2}\right\} / 2\alpha_2. \tag{28}$$

Following the similar procedure mentioned above, the coefficients of the polynomial $(\beta_0, \beta_1, \beta_2)$ are calculated so as to minimize the criterion as

$$J_{T_m} = \sum_{i=1}^{N} \{\Delta T_m(i) - \beta_0 - \beta_1 \tilde{T}_d(i) - \beta_2 (\tilde{T}_d(i))^2\}^2 \tag{29}$$

where the set $\{\Delta T_m(i), \tilde{T}_d(i)\}$ are the next experimental data for the fixed value of $\tilde{G}_d$ of (28). The value of $\tilde{T}_d$ is obtained by solving the second order polynomial whose left hand side is set at zero as follows:

$$\tilde{T}_d = \left\{-\beta_1 + \sqrt{(\beta_1)^2 - 4\beta_0\beta_2}\right\} / 2\beta_2. \tag{30}$$

The algorithm goes back to (27) to calculate $(\alpha_0, \alpha_1, \alpha_2)$ again for the fixed values of $\tilde{T}_d$ of (30). The values of $\tilde{G}_d$ and $\tilde{T}_d$ are thus iteratively calculated by using the above procedures (27)-(30) again and again until the maximum steady state errors $\Delta H_m(i)$, $\Delta T_m(i)$ in the steady state duration of experiment are minimized within a permissible range (± 0.1[cm], ± 0.1[°C]). The final values of $\tilde{G}_d$ and $\tilde{T}_d$ are adopted to the actual experiment of the tanked water system. The compensated manipulated variables are given as

$$\begin{aligned} G_c &= \hat{u}_1(k) + \hat{G}_c^0, \\ G_h &= \hat{u}_2(k) + \hat{G}_h^0. \end{aligned} \tag{31}$$

The $\hat{G}_h^0$ and $\hat{G}_c^0$ are calculated by the equation (25) with the final values of $\tilde{G}_d$ and $\tilde{T}_d$.

5.3 Experimental results

By using the proposed controller based on both the detection delay compensation and the steady state error compensation, we performed an actual experiment of the level and temperature control of the pilot tanked water system. The conditions of the experiment were the same as the previous experiments in 4.3.

Figure 7 illustrates the experimental results. We can see that the steady state errors of the level and the temperature were eliminated almost completely (compare Fig. 7 and Fig. 5(d)). The results show the effectiveness of the proposed compensation method for eliminating both the oscillation and the steady state errors.

6. CONCLUSION

By handling a pilot plant of the tanked water system, we discussed an important idea of controller design for nonlinear dynamical systems and proposed simple compensation technique for oscillations and steady state errors. If we separate nonlinear dynamical system into nonlinear static part and linear dynamical part, the nonlinear components can be eliminated by introducing an inversion characteristics of the nonlinearity, and conventional linear control theories are ideally applicable to the linear dynamical part in constructing the controller. We further proposed simple compensation methods for oscillation caused by detection time delay and for steady state error caused by modelling errors. Combinational use of the two compensation techniques brought satisfactory control performances in the control experiment of the pilot plant of the tanked water system. Those compensation methods will effectively applicable to wide range of systems in actual thermal-flow plants with system time delay.

References

[1] R. E. Kalman, "On the general theory of control systems," Proceedings of the 1st IFAC World Congress, Moscow, pp. 481-492, 1960.

[2] R. Bellman, "Dynamic programming," Princeton Univ. Press, 1957.

[3] L. S. Pontryagin, V. G. Boltyanskii, R. V. Gamkrelidge and E. F. Mishchenko, "The mathematical theory of optimal process," John Willy, 1962.

[4] T. Nishimura, "Application of Kalman filter to space orbit system", Systems and Control, JAACE, vol. 22, 1, pp. 10-19, 1978 (in Japanese).

[5] D. Ernst, "New trends in the application of process computers," Proceedings of the 7th IFAC World Congress, Helsinki, pp. 2327-2336, 1978.

[6] K. J. Aström, U. Borisson, L. Ljung and B. Wittenmark, "Theory and application of self-tuning regulators," Automatica, vol. 13, pp. 457-476, 1977.

[7] Special Issue on Theory of Nonlinear Control, Journal of SICE, vol. 31, 1992 (in Japanese).

[8] Special Issue on Robust Control -Focusing on H^∞ Control," Journal of SICE, vol. 29, 1990 (in Japanese).

[9] M. Nakamura, "Controller design to diminish oscillation and steady state error in water temperature systems with drive delay," Proceedings of '91 KACC, pp. 1888-1893, 1991.