'93 KACC (1993.10.20~22)

Robust Control of Linear Systems Under Structured Nonlinear Time-Varying Perturbations II: Synthesis via Convex Optimazation

Riyanto T. BAMBANG and Etsujiro SHIMEMURA
Department of Electrical Engineering, Waseda University
3-4-1 Ohkubo, Sinjuku-ku, Tokyo 169, JAPAN

1 Introduction

In Part I, we derived robust stability conditions for an LTI interconnected to time-varying nonlinear perturbations belonging to several classes of nonlinearities. These conditions were presented in terms of positive definite solutions to LMI. In this paper we address a problem of synthesizing feedback controllers for linear time-invariant systems under structured time-varying uncertainties, combined with a worst-case $\mathcal{H}_2$ performance. This problem is introduced in [7, 8, 15, 35] in case of time-invariant uncertainties, where the necessary conditions involve highly coupled linear and nonlinear matrix equations. Such coupled equations are in general difficult to solve.

A convex optimization approach will be employed in this synthesis problem in order to avoid solving highly coupled nonlinear matrix equations that commonly arises in multiobjective synthesis problem. Using LMI formulation, this convex optimization problem can in turn be cast as generalized eigenvalue minimization problem, where an attractive algorithm based on the method of centers has been recently introduced to find its solution [30, 36].

In the present paper we will restrict our discussion to state feedback case with Popov multipliers. A more general case of output feedback and other types of multipliers will be addressed in a future paper.

2 Robust Control Synthesis With Worst-Case $\mathcal{H}_2$ Performance

This section considers the synthesis of feedback control under structured time-varying uncertainties, combined with a worst-case $\mathcal{H}_2$ performance. We will employ robust stability conditions for an LTI system coupled with time-varying nonlinearities, as presented in Part I, but we specializes the results to linear time-varying case. Since robust stability conditions for linear-time invariant nonlinearities can be deduced from those of time-varying uncertainties, the synthesis tools presented in this paper could be employed to handle time-invariant uncertainties as well.

Let $\mathcal{G}$ be dynamics of the plant with the following state space representation,

$$\dot{x}(t) = (A + \triangle A)x(t) + Bu(t) + Dw(t) \tag{2.1}$$
$$y(t) = x(t) \tag{2.2}$$

where $x(t) \in \Re^n$, $u(t) \in \Re^{n_u}$, and $w(t) \in \Re^{n_w}$. $\triangle A$ is uncertainty belonging to a prespecified uncertainty structure $\mathcal{S}$. Assume that for all uncertainty $\triangle A \in \mathcal{S}$, the pair $(A + \triangle A, B)$ is stabilizable. Let the transfer function of the plant $\mathcal{G}$ be denoted by $G(s)$. The state feedback controller is described by

$$u(t) = Kx(t) \tag{2.3}$$

The objective of robust synthesis addressed in this paper is twofold. First, we would like that our controller will render the closed-loop system is asymptotically stable for all uncertainties in the prespecified set $\mathcal{S}$. Secondly, we would like that the same controller will minimize a worst-case $\mathcal{H}_2$ given by [7, 8, 15]

$$J := \sup_{\triangle A \in \mathcal{S}} \limsup_{t\to\infty} \frac{1}{t}\mathcal{E}\left[\int_0^\infty (x(t)'Q_x x(t) + u(t)'Q_u u(t))dt\right] \tag{2.4}$$

where Q_x and Q_u are both positive definite. Assume that $w(t)$ is a white noise disturbance with unit intensity. For each uncertainty $\triangle A \in \mathcal{S}$, the closed-loop system can be written as

$$\dot{\tilde{x}}(t) = (\tilde{A} + \triangle\tilde{A})\tilde{x}(t) + Dw(t) \tag{2.5}$$

where

$$\tilde{A} = A + BK, \ \ \triangle\tilde{A} = \triangle A \tag{2.6}$$

It is well known that, provided $(\tilde{A} + \triangle\tilde{A})$ is asymptotically stable for all $\triangle A \in \mathcal{S}$ for a given controller, the $\mathcal{H}_2$ performance (2.4), can also be written as

$$J = \sup_{\triangle A \in \mathcal{S}} tr(\hat{P}DD') = 0 \tag{2.7}$$

where

$$(\tilde{A} + \triangle\tilde{A})'\hat{P} + \hat{P}(\tilde{A} + \triangle\tilde{A}) + \tilde{R} = 0 \tag{2.8}$$

with $\tilde{R} := Q_x + K'Q_uK$.

In the case of Popov multiplier with m independent scalar uncertainties, the uncertainty set $\mathcal{S}$ be specified as follows [7, 35]

$$\mathcal{S} := \{\triangle A = -B_0F(t)C_0, F \in \mathcal{F}\} \tag{2.9}$$
$$\mathcal{F} := \{F(t) \in \mathcal{D}^m : 0 \le F(t) \le M\}, \tag{2.10}$$

where $\mathcal{F} := \{F(t) \in \Re^{m\times m} : F(t) \ge 0,$ and the elements of $F(.)$ are Lebesgue measurable on $[0,\infty)\}$, and where $B_0 \in \Re^{n\times m}$ and $C_0 \in \Re^{m\times n}$ are fixed matrices denoting the structure of uncertainty, $M \in \Re^{m\times m}$ is a given positive diagonal matrix, and $F(t) \in \Re^{m\times m}$ is time varying uncertain matrix. $\mathcal{D}$ denotes matrix having diagonal entries, while M is upper bounds on the uncertain diagonal matrix $F(t)$. The corresponding Popov multipliers take the form

$$W_i(s) = \alpha_{i0} + \epsilon_i\beta_{i0} + \beta_{i0}s \tag{2.11}$$

3 Optimization Problem: A Convexity Result

In this section we formulate an optimization problem associated with the synthesis problem described in Section 2, in terms of solution to a Riccati equation. Then, using a change of variable techniques, we formulate an equivalent optimization problem with nice convexity properties. In doing so, let us first define

$$\begin{aligned}\hat{R}_0(P) &:= [(H_0M^{-1} + N_0C_0B_0 + N_0Q_0M^{-1}) \\ &\quad +(H_0M^{-1} + N_0C_0B_0 + N_0Q_0M^{-1})'] > 0 \qquad (3.1)\\ \mathcal{R}(P) &:= \tilde{A}'P + P\tilde{A} + \tilde{R} + [H_0C_0 + N_0Q_0C_0 + N_0C_0\tilde{A} \\ &\quad -B_0'P]' \times \hat{R}_0^{-1}[H_0C_0 + N_0Q_0C_0 + N_0C_0\tilde{A} - B_0'P] \\ &= 0 \qquad (3.2)\\ J_u &:= tr[(P + C_0'MN_0C_0)DD'] \qquad (3.3)\end{aligned}$$

with $V_0 - H_0 \geq 0$ and $V_0 := N_0S_0$. Following [7, 8, 15], it can be shown that the closed-loop system (2.5) is asymptotically stable if there exists $P > 0$ that satisfies (3.2), and in this case

$$J_u \geq J \qquad (3.4)$$

Thus, J_u, which is given in terms of a symmetric positive definite solution to the Riccati equation (3.2), is an upperbound to the worst-case performance J. This upperbound J_u will be viewed as a cotst to be minimized in our optimization problem defined later.

Sufficient condition for the existence of solution to the Riccati equation (3.2), can be derived using the result of Willems[2, 3]. See also Theorem 5.2 in [7, 15].

Lemma 3.1 *(Willems[2, 3], How [7, 15]) Let $\hat{G}(s)$ be a transfer function matrix with minimal realization given by*

$$\hat{G}(s) \sim \left[\begin{array}{c|c}\tilde{A} & B_0 \\ \hline \Gamma_1 & \Gamma_2\end{array}\right] \qquad (3.5)$$

where

$$\begin{aligned}\Gamma_1 &:= H_0C_0 + N_0C_0\tilde{A} + N_0Q_0C_0\\ \Gamma_2 &:= H_0M^{-1} + N_0C_0B_0 + N_0Q_0M^{-1}\end{aligned}$$

If $\tilde{A}$ is asymptotically stable and $\hat{G}(s)$ is strongly positive real, then there exists $P > 0$ satisfying (3.2). Conversely, if $\hat{R}_0 > 0$ and there exists $P > 0$ satisfying (3.2) for all $\tilde{R} > 0$, then $\tilde{A}$ is asymptotically stable and $\hat{G}(s)$ is strongly positive real.

The following lemma gives another characterization to the upperbound J_u, which prove useful in formulating a convex optimization problem.

Lemma 3.2 *Consider the system (2.1) and (2.2). Suppose that the conditions stated in Lemma 2.2 hold. Then,*

$$J_u(K) := inf\{tr[(P + C_0'MN_0C_0)DD'] : P \in \mathcal{P}\} \qquad (3.6)$$

where

$$\mathcal{P} = \{P : P > 0, \mathcal{R}(P) < 0, \text{ and } H_0 - V_0 \leq 0\} \qquad (3.7)$$

Proof: Follows from similar argument introduced in the proof of Lemma 2.1 in [26] for mixed H_2/H_∞ design, combined with some results concerning dissipation inequality described in [2, 3]. □.

We call a controller K *admissible* if K internally stabilizes the plant $\mathcal{G}$. Introduce the following sets :

$$\begin{aligned}\mathcal{A}(\mathcal{G}) &:= \{K : K \text{ is admissible}\}\\ \mathcal{A}_{SPR}(\mathcal{G}) &:= \{K \in \mathcal{A}(\mathcal{G}) : \hat{G}(s) \text{ is strongly positive real }\} \qquad (3.8)\end{aligned}$$

Motivated by [26], consider the following synthesis problem **Robust Control Synthesis Problem**: "Compute the performance measure

$$\theta_m(\mathcal{G}) := inf\{J_u : K \in \mathcal{A}_{SPR}(\mathcal{G})\}, \qquad (3.9)$$

and, given any $\theta > \theta_m$, find a controller $K \in \mathcal{A}_{SPR}(\mathcal{G})$ such that $J_u < \theta$".

In this paper we are interested in the computation of constant state feedback matrices for the minimization of $J_u(\mathcal{G}, K)$. The set of such controllers will be denoted by

$$\mathcal{A}_{SPR,s}(\mathcal{G}) := \{K \in \mathcal{A}_{SPR}(\mathcal{G}) : K \in \Re^{n_u \times n}\}. \qquad (3.10)$$

It will be shown that the optimal performance $\theta_m(\mathcal{G})$ defined in (3.9) is the value of a (finite dimensional) convex optimization problem. Further, given any $\theta > \theta_m$, one can find K such that $J_u(\mathcal{G}, K) < \theta$ by solving a convex programming problem.

Let Ξ_1 and Ξ_2 denote the set of $n \times n$ real symmetric matrices, and the set of $m \times m$ diagonal matrices, respectively, and define

$$\begin{aligned}\Omega &:= \{(X, Y, V_0, H_0) \in \Re^{n_u \times n} \times \Xi_1 \times \Xi_2 \times \Xi_2 :\\ &\quad Y > 0, V_0 \geq 0, H_0 \geq 0\}. \qquad (3.11)\end{aligned}$$

Observe that Ω is an open strictly convex subset of $\Re^{n_u \times n} \times \Xi_1 \times \Xi_2 \times \Xi_2$. Given $(X, Y, V_0, H_0) \in \Omega$, define

$$f(X, Y, V_0, H_0) := tr[(Y^{-1} + C_0'MN_0C_0)DD'] \qquad (3.12)$$

and, for $(X, Y, V_0, H_0) \in \Re^{n_u \times n} \times \Xi_1 \times \Xi_2 \times \Xi_2$, let

$$\begin{aligned}\mathcal{R}_Y(X, Y, V_0, H_0) &:= AY + YA' + Y'Q_xY\\ &\quad +X'Q_uX + X'B' + BX\\ &\quad +[H_0C_0Y + N_0Q_0C_0Y + N_0C_0AY + N_0C_0B_0X - B_0']'\hat{R}_0^{-1}\\ &\quad \times[H_0C_0Y + N_0Q_0C_0Y + N_0C_0AY + N_0C_0B_0X - B_0'] \qquad (3.13)\end{aligned}$$

Define also the set of real matrices

$$\begin{aligned}\Phi(\mathcal{G}) &:= \{(X, Y, V_0, H_0) \in \Omega : \mathcal{R}_Y(X, Y, V_0, H_0) < 0\\ &\quad \text{and } \mathcal{H}_0(X, Y, V_0, H_0) \leq 0\}, \qquad (3.14)\end{aligned}$$

with $\mathcal{H}_0(X, Y, V_0, H_0) := H_0 - V_0$ and consider the optimization problem

$$\tau(\mathcal{G}) := inf\{f(X, Y, V_0, H_0) : (X, Y, V_0, H_0) \in \Phi(\mathcal{G})\}. \qquad (3.15)$$

Theorem 3.1 *Consider the plant $\mathcal{G}$ defined in (2.1) and (2.2). Let G denote its transfer matrix, and $\mathcal{A}_{SPR,s}(\mathcal{G})$ denote the set of controllers defined in (3.8). Let $\Phi(\mathcal{G})$ be given by (3.14). Let θ_m and $\tau(\mathcal{G})$ be as defined in (3.9) and (3.15), respectively. Then,*

$$\mathcal{A}_{SPR,s}(\mathcal{G}) \neq \emptyset \qquad (3.16)$$

if, and only if,

$$\Phi(\mathcal{G}) \neq \emptyset \tag{3.17}$$

with $\emptyset$ denote empty set. In this case,

$$\theta_m(\mathcal{G}) = \tau(\mathcal{G}). \tag{3.18}$$

Furthermore, given any $\theta > \theta_m(\mathcal{G})$, there exists $(X, Y, V_0, H_0) \in \Phi(\mathcal{G})$ such that the state feedback gain $K := XY^{-1}$ satisfies

$$K \in \mathcal{A}_{SPR,s}(\mathcal{G}) \text{ and } J_u(G, K) \leq f(X, Y, V_0, H_0) < \theta. \tag{3.19}$$

Proof: The proof can be constructed along the similar arguments introduced in [26]. Unlike those of [26], however, we first convert the Riccati inequality $\mathcal{R}(P) < 0$ to its dual, by pre- and post-multiplying it by P^{-1}, to arrive at

$$\begin{aligned} & P^{-1}\tilde{A}' + \tilde{A}P^{-1} + P^{-1}\tilde{R}P^{-1} \\ & + [H_0C_0P^{-1} + N_0Q_0C_0P^{-1} + N_0C_0\tilde{A}P^{-1} - B_0']'\hat{R}_0^{-1} \\ & \times [H_0C_0P^{-1} + N_0Q_0C_0P^{-1} + N_0C_0\tilde{A}P^{-1} - B_0'] < 0 \end{aligned}$$

Defining $Y := P^{-1} > 0$, and substituting $K = XY^{-1}$ in to the last equation leads to $\mathcal{R}_Y(X, Y, V_0, H_0) < 0$. The rest of the proofs follows from [26] using characterization of the upperbound J_u given in Lemma 3.2, together with existence result of the solution to the Riccati (3.2) stated in Lemma 3.1.

□

From Theorem 4.1, it follows that the computation of $\tau(\mathcal{G})$ involves a search over the set $\Phi(\mathcal{G})$, where X,Y,V_0 and H_0 serve as the decision variables. On the other hand $\theta_m(\mathcal{G})$ is computed by solving nonlinear programming problem with only the real matrix K as the decision variable. Furthermore, the set of feasible static feedback gains, $\mathcal{A}_{SPR,s}(\mathcal{G})$ is not necessarily convex, and therefore the original optimization problem for robust controller synthesis is not necessarily convex. We will show that the optimization problem $\Phi(\mathcal{G})$ defined in (3.14) is indeed a *convex problem.*

Theorem 3.2 *Let f and Φ be as defined in (3.12) and (3.14), respectively, and consider the optimization problem (3.15). Then, the set Φ is convex and the function $f : \Phi \to \Re$ is convex and real analytic. Consequently, the optimization problem $\tau(\mathcal{G})$ defined in (3.15) is convex.*

Proof: The proof can be constructed along the same line as those of [26], using matricial convexity results derived in [1], on noting that here $\Phi(\mathcal{G})$ is given in (3.14). □

Due to the convexity established in this paper, one can employ any advance in convex optimization problem with global optimality properties. In this paper, the optimization problem $\tau(\mathcal{G})$ in (3.15) will be further reduced to the Generalized Eigenvalue Minimization Problem(GEMP) [30] where an effective algorithm based on the method of centers has been introduced to find its solution.

4 Reduction to GEMP Via LMI Formulation

In this section, we will show that the optimization problem defined in (3.6) can be reduced to Generalized Eigenvalue Minimization Problem(GEMP) and describe a method of centers for solving the problem[9]. GEMP is the problem of minimizing the maximum generalized eigenvalue of a (symmetric positive-definite) pair of matrices that depend affinely on a variable x that is subject to some constraints. In [9], a fast and attractive algorithm based on Interior Point Method has been applied to solve efficiently GEMP.

In the general case, GEMP with variables $x \in \Re^m$ and $\lambda \in \Re$ takes the form

$$\begin{aligned} \min_{\substack{\lambda G(x) - F(x) > 0 \\ G(x) > 0 \\ H(x) > 0}} \quad \lambda \end{aligned} \tag{4.1}$$

or equivalently,

$$\min_{\substack{G(x) > 0 \\ H(x) > 0}} \quad \lambda_{max}(F(x), G(x)). \tag{4.2}$$

where λ_{max} denotes the generalized maximum eigenvalue. This is a function defined on a pair of matrices X, Y by $\lambda_{max}(X, Y) := max\{\lambda \in \Re | det(\lambda Y - X) = 0\}$. In (4.1) and (4.2), F ,G and H are symmetric matrices that depend affinely on $x \in \Re^m$:

$$\begin{aligned} F(x) &:= F_0 + \sum_{i=1}^{m} x_i F_i \\ G(x) &:= G_0 + \sum_{i=1}^{m} x_i G_i \\ H(x) &:= H_0 + \sum_{i=1}^{m} x_i H_i \end{aligned} \tag{4.3}$$

where $F_i = F_i'$, $G_i = G_i' \in \Re^{r \times r}$, and $H_i = H_i' \in \Re^{s \times s}$. Matrices $F(x)$ and $G(x)$ may be complex Hermitian.

Let us turn our attention to the optimization problem $\tau(\mathcal{G})$ defined in (3.15), which we rewrite here for convenience,

$$\tau(\mathcal{G}) := inf\{f(X, Y, V_0, H_0) : (X, Y, V_0, H_0) \in \Phi(\mathcal{G})\}.$$

where $f(X, Y, V_0, H_0)$ and $\Phi(\mathcal{G})$ are given by (3.12) and (3.14), respectively. The coefficients of the multipliers will be restricted to the case where $H_0 > 0, V_0 > 0$ and $(H_0 - V_0) < 0$ without loss of generality. Let us express the objective function (3.12) as:

$$f(X, Y, V_0, H_0) = tr(D'Y^{-1}D + D'C_0'MN_0C_0D) \tag{4.4}$$

The first term $\Theta(X, Y, V_0, H_0) := tr(D'Y^{-1}D)$ in the above equation can be equivalently expressed as

$$\Theta(X, Y, V_0, H_0) = \min_{\begin{bmatrix} S & D' \\ D & Y \end{bmatrix} > 0} tr(S).$$

Let us further define

$$\begin{aligned} L_1(\lambda, X, Y, V_0, H_0, S) &:= -tr(D'C_0'MN_0C_0D - tr(S) + \lambda \\ L_2(\lambda, X, Y, V_0, H_0, S) &:= \begin{bmatrix} L_{2a} & L_{2b} \\ L_{2c} & L_{2d} \end{bmatrix} \\ L_3(\lambda, X, Y, V_0, H_0, S) &:= \begin{bmatrix} S & D' \\ D & Y \end{bmatrix} \\ L_4(\lambda, X, Y, V_0, H_0, S) &:= V_0 - H_0 \\ L_5(\lambda, X, Y, V_0, H_0, S) &:= V_0 \\ L_6(\lambda, X, Y, V_0, H_0, S) &:= H_0 \\ L(\lambda, X, Y, V_0, H_0, S) &:= diag(L_1, L_2, L_3, L_4, L_5, L_6), \end{aligned}$$

where

$$
\begin{aligned}
L_{2a} &= -(AY + YA' + X'B' + BX) \\
L_{2b} &= \begin{bmatrix} Y' & X' & \Gamma_Y \end{bmatrix} \\
\Gamma_Y &= (H_0C_0Y + N_0Q_0C_0Y + N_0C_0AY + N_0C_0B_0X - B_0')' \\
L_{2c} &= L_{2b}' \\
L_{2d} &= \begin{bmatrix} Q_x^{-1} & 0 & 0 \\ 0 & Q_u^{-1} & 0 \\ 0 & 0 & \hat{R}_0^{-1} \end{bmatrix}
\end{aligned}
$$

Note carefully that $L_1(\lambda, X, Y, V_0, H_0, S)$, $L_2(\lambda, X, Y, V_0, H_0, S)$ and $L_3(\lambda, X, Y, V_0, H_0, S)$ are affine matrix in the variables $(\lambda, X, Y, V_0, H_0, S)$.

Using the above constructions and employing the Schur complement formula which states that

$$\begin{bmatrix} Z_1 & Z_3 \\ Z_3' & Z_2 \end{bmatrix} > 0 \iff Z_2 > 0, \ \ and \ \ Z_1 - Z_3 Z_2^{-1} Z_3' > 0,$$

our optimization problem $\tau(\mathcal{P})$ above can now be represented as

$$\min_{L(\lambda, X, Y, V_0, H_0, S) > 0} \lambda \tag{4.5}$$

which indeed is of the form (4.1). Represented in the form of (4.1), symmetric affine matrices $F(x)$ and $G(x)$ for optimization problem (4.5) are given by

$$
\begin{aligned}
F(x) &:= -diag([-tr(D'C_0'MN_0C_0D) - tr(S)], \\
&\quad\ L_2, L_3, L_4, L_5, L_6) \\
G(x) &:= diag(1, 0, 0, 0, 0, 0) \\
H(x) &:= Y.
\end{aligned}
$$

Vector x in (4.1) then contains the optimization variables which consist of the independent variables of $(\lambda, X, Y, V_0, H_0, S)$. Note that matrices $F(x), G(x)$ and $F(x)$ are in the form of linear matrix inequalities(LMI).

The GEMP (4.1) can be effectively solved using interior point method. The method is based on the notion of *analytic center* of an affine matrix inequality, say $D(x) = D_0 + \sum_{i=1}^{N} x_i D_i > 0$. Suppose that $\mathbf{X}$ denotes the feasible set

$$\mathbf{X} := \{x \in \Re^N | D(x) > 0\}.$$

The analytic center x^* of the inequality $D(x) > 0$ is defined as

$$x^* = argmin_{x \in \mathbf{X}} log\ det D(x)^{-1}.$$

Starting with any feasible $x^{(0)}$, and a $\lambda^{(0)} = \lambda_{max}(A(x^{(0)}, B(x^{(0)}))$, the algorithm proceed as follows

$$
\begin{aligned}
\lambda^{i+1} &:= (1 - \eta)\lambda_{max}(F(x^{(i)}, G(x^{(i)})) + \eta\lambda^{(i)} \\
x^{(i+1)} &:= \text{analytic center of } \lambda^{(i+1)}G(x) - F(x) > 0.
\end{aligned}
$$

In the above procedure $\eta \in (0, 1)$ is a parameter which is typically small. It enables one to take $x^{(i)}$ as an initial guess for the Newton type method that finds the analytic center of inequality $\lambda(i+1)G(x) - F(x) > 0$. Detailed analysis as well as the proof of convergence can be found in [9].

In the present paper, the definiteness requirement of $G(x)$ in (4.1) is accomplished by simple modification(via the use of variable λ) of the above expression for $G(x)$, as well as by a minor modification to the algorithm of [9]. For related discussion as well as numerical results for mixed H_2/H_∞ design see [27, 28].

5 Conclusion

The problem of analyzing robustness of finite-dimensional linear time-invariant systems under nonlinear time-varying perturbation has been presented via the use of dissipativity and absolute stability theory. The robust stability conditions for several class of nonlinearities have been expressed conveniently in terms of solutions to LMI. These conditions can also be viewed as an extension of mixed μ upperbound to nonlinear time varying perturbations. Based on this result, a synthesis problem is addressed by incorporating the worst-case H_2 performance criterion. It is shown that this synthesis problem can be solved via convex optimization problem and LMI formulation.

ACKNOWLEDGEMENTS: The first author wishes to thank DR. How of MIT for providing his Ph.D. thesis which stimulates the discussion of the present paper and for his helpful comments. Thanks are also due to Prof. Boyd of Stanford Univ for providing many of his recent publications on LMI, to Prof. Khargonekar of Michigan Univ. for early access to a preprint of [26] and to Prof. Safonov of University of Southern California for providing references [12, 13]. Helpful comments from Prof. Uchida of Waseda Univ. are also appreciated.

References

[1] Marshall, A.W. and Olkin, I., *Inequalities: Theory of Majorization and Its Application*, Academic Press, New York, 1979.

[2] Willems, J.C., "Disspative Dynamical Systems Part 1 & Part 2", Archieve Rational Mechanics Analysis, vol. 45, pp. 321-393, 1972.

[3] Trentelman, H.L. and J.C. Willems, "The Dissipation Inequality and the Algebraic Riccati Equation", in *The Riccati Equation*(Bittanti, Laub, Willems, eds.), pp. 197-242, Springer-Verlag, New York, 1991.

[4] Hill, D.J. and P.J. Moylan,"The Stability of Nonlinear Dissipative Systems", IEEE T.A.C., vol. 21, pp. 708-711, 1976.

[5] Willems, J.C., "Mechanisms for the Stability and Instability in Feedback Systems", Proc. IEEE, vol. 64, no. 1, pp. 24-35, 1976.

[6] Willem, J.L., "A General Stability Criterion for Nonlinear Time Varying Feedback Systems", I.J. Control, vol. 11, no.4, pp. 625-631, 1970.

[7] How, J.P., "Robust Control Design with Real Parameter Uncertainty Using Absolute Stability Theory", Ph. D. Thesis, MIT, 1993.

[8] How, J.P. and Hall, S.R., "Connection Between Absolute Stability Theory and the Structured Singular Value", submitted to IEEE T.A.C., May, 1992.

[9] How, J.P. and Hall, S.R., "Connection Between Popov Criterion and Bounds for Real Parameter Uncertainty", 1993 ACC, pp. 1084-1089, 1993.

[10] Doyle, J.C., "Analysis of Feedback Systems with Structured Uncertainties", IEE Proc., vol. 129, Part D, no. 6, pp. 242-250, 1982

[11] Packard, A. and Doyle, J., "The Complex Structured Singular Value", Automatica, vo. 29, no. 1, pp.71-109, 1993.

[12] Safonov, M.G. and P.H. Lee, "A Multiplier Method for Computing Real Multivariable Stability Margin", preprint, 1993.

[13] Safonov, M.G. and R.Y. Chiang, "Real/Complex K_m-Synthesis Without Curve Fitting", preprint, 1993.

[14] Safonov, M.G., "Stability of Interconnected Systems Having Slope Bounded Nonlinearities", 6th Int. Conf. on Analysis and Optimization of Systems, Nice, 1984.

[15] Haddad, W.M., J.P. How, S.R. Hall, and D.S. Bernstein, "Extension of Mixed μ Bounds to Monotonic and Odd Monotonic Nonlinearities Using Absolute Stability Theory", Proc. 31st Conf. on Decision and Control, Tucson, pp. 2813-2823, 1992.

[16] Dahleh M. and J.C. Doyle, "Overview of Robust Stability and Performance Methods of Systems with Structured Mixed Perturbations" Proc. 31st Conf. Decision and Control, Tucson, pp. 3158-3162, 1992.

[17] Narendra, K.S. and J.H. Taylor, "Lyapunov Functions for Nonlinear Time-Varying Systems", Information and Control, vol. 12, pp. 378-393, 1968.

[18] Taylor, J.H. and K.S. Narendra, "The Corduneau-Popov Approach to The Stability of Nonlinear Time-Varying Systems", SIAM J. Appl. Math., vol. 18, no. 2, pp. 267-281, 1970.

[19] Srinath, M.D., M.A.L. Thathachar and H.K. Ramapriyan, "Stability of a Class of Nonlinear Time Varying Systems", I.J. Control, vol. 7, pp. 117-132, 1968.

[20] Thathachar, M.A.L. and M.D. Srinath, "An Improved Stability Criterion for a Systems With Non-monotone Nonlinearity", I.J. Control, vol. 12, no.1, pp. 145-155, 1970.

[21] Thathachar, M.A.L., M.D. Srinath and H.K. Ramapriyan, "On a Modified Lur'e Problem", IEEE T.A.C., vol. 12, pp. 731-739, 1967.

[22] Srinath, M.D., "Passivity of a Class of Non-monotone Nonlinearities", I.J. Control, vol. 16, no.5, pp.-889-896, 1972.

[23] Desoer, C.A. and M. Vidyasagar, *Feedback Systems: Input-Output Properties*, Academic Press, New York, 1975.

[24] Popov, V.M., *Hyperstability of Control Systems*, Springer, New York, 1973.

[25] Narendra, K.S., *Frequency Domain Criteria for Absolute Stability*, Academic Press, New York, 1973.

[26] Khargonekar, P.P. and Rotea, M.A., "Mixed H_2/H_∞ Control : A Convex Optimization Approach", IEEE T.A.C., vol. 36, no. 7, pp. 824-836, 1991.

[27] Bambang, R., Shimemura, E. and Uchida, K., "Mixed H_2/H_∞ Control of Uncertain Systems", Proc. 1993 ACC, 1993.

[28] Bambang, R., Shimemura, E. and Uchida, K., "Mixed H_2/H_∞ Control With Pole Placement : State Feedback Case", Proc. 1993 ACC, 1993.

[29] Doyle, J.C., Packard, A., and Zhou, K., "Review of LFTs, LMIs, and μ", draft, 1991.

[30] Boyd, S., and El Ghaoui, L., "Method of Centers For Minimizing Generalized Eigenvalues", preprint, 1992.

[31] Balakrishnan, V., Feron, E., Boyd, S. and El Ghaoui, L. "Computing Bounds for The Structured Singular Value via Interior Point Algorithm", Proc. ACC., 1992.

[32] Fan, M.K.H., Tits, A.L., and Doyle, J.C., "Robustness in the Presence of Mixed Parametric Uncertainty and Unmodeled Dynamics", IEEE T.A.C., vol. 36, no. 1, pp. 25-38, 1991.

[33] Zames, G., "On the Input-Output Stability of Time Varying Nonlinear Feedback Systems Part I & Part II", IEEE T.A.C., vol. 11, no. 2, pp. 228-238 and no. 3, pp. 465-476, 1966.

[34] Shamma, J.S., "Robustness Analysis for Time Varying Systems", Proc. 31st Conf. Decision and Control, Tucson, pp. 3163-3168, 1992.

[35] Haddad, W.M. and Bernstein, D.S., "Parameter-Dependent Lyapunov Functions, Constant Real Parameter Uncertainty, and the Popov Criterion in Robust Analysis and Synthesis, Parts I and II", Proc. 30th IEEE Conf. Decision and Control, 1991.

[36] Boyd, S., El Ghaoui, L., Feron, E. and Balakrishnan, V., "Linear Matrix Inequalities in Systems and Control Theory", draft, 1993.

[37] Gahinet, P. and Apkarian, P., "A Linear Matrix Inequality Approach to H_∞ Control", to appear in Int. J. Robust and Nonlinear Control, 1993.

[38] Rantzer, A., "Uncertainties With Bounded Rate of Variation", Proc. 1993 ACC, pp. 29-30, 1993.