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Robust Control System Design for a Flexible arm by a Two-Degree-of-Freedom Compensator

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ABSTRACT

This paper is concerned with a two-degree-of-freedom control system design for a flexible arm. a two-degree-of-freedom control system can achieve a robust stability specification and a control performance spesification independently. By this property we improve the control performance with maintaining the same robust stability level as that of the one-degree-of-freedom control system. At First we design a two-degree-of-freedom control system which includes a feedforward controller and a feedback controller. The feedforward controller can be given by specifying a transfer function of a desired closed-loop model. We obtain a feedback controller by solving a mixed sensitivity problem. Several numerical results show that two-degree-of-freedom control systems acheive a better control performance than that of one-degree-of-freedom control systems.

1. INTRODUCTION

In the design for servo control systems one of the most fundamental specifications is to track the desired trajectry corecctly. Even in the presence of modelling errors the servo control systems have to achieve a tracking specification. A design problem of servo control systems in the presence of modelling error is called a robust servo control system design problem.

One of the important robust control systems is a two-degree-of-freedom control system[1]. a two-degree-of-freedom control system can achieve a robust stability specification and control performance specification independently. This property indicates that a two-degree-of-freedom control system can improve the control performance with maitaining the same robust stability level as that of a one-degree-of-freedom control system.

In this report the usefulness of a two-degree-of-freedom control system is numerically demondtrated on a flexible arm. In the control system design for a flexible arm many papers have paid attention to only robust stabilization[2],[3]. We consider not only robust stabilization but also improvement in control performances.

This report is organized as follows. In section 2 a mathematical model of the flexible arm is derived. Section 3 is devoted to the control problem setup. We fomulate robust stabilization problem as a mixed sensitivitiy problem in which the argumented plant is constructed with frequency weighting functions. In section 4 several numerical simulations are carried out in order to evalute the robustness and control performance of two-degree-of-freedom control system. These numerical results show that the two-degree-of-freedom control system improve the control perfomance with maintaining the same robust stability level as that of one-degree-of-freedom control system.

We use following notations in this report.

$\|\cdot\|_\infty$ senotes the H_∞-norm. Let G(s) be any proper transfer function. A State-space realization of G(s) is denoted by

$$G(s)=[A, B, C, D] \qquad (1)$$

2. PLANT DESCRIPTION

In this section a mathematical model of a flexible arm is obtained. A schematic manipulator is shown in Fig. 1. The notations in Fig. 1. are shown in Table 1. Parameters of the system are given as Table 2. The flexible arm is attached at a satellite body which is located at the origin of the coordinate. The lumped mass is attached at the tip of the arm.

By several assumptions about the arm, a mathematical model of the arm can be obtained.

$$J_r\ddot{\theta}(t) + \rho A \int_0^L x\ddot{y}(x,t)dx + M_cL\ddot{y}(L,t) = u(t) \quad (2)$$

$$EI\frac{\partial^4 y(x,t)}{\partial t^4} + \rho A\{x\theta(t) + y(x,t)\} = 0 \quad (3)$$

where $J_r = J_0 + \frac{\rho A L^3}{3} + M_e L^2$

The boundary conditions are given as follows.

$$EI\frac{\partial^3 y(L,t)}{\partial t^3} = M_e\{L\theta(t) + y(L,t)\} \quad (4)$$

$$\frac{\partial^2 y(L,t)}{\partial t^2} = 0 \quad (5)$$

By eigenfunctions corresponding to the boundary conditions (4) and (5), (2) and (3) can be rewritten as following infinite number of ordinary differential equations[2].

$$q_i(t) + 2\zeta\omega_i q + \omega_i^2 q = b_i u(t) \quad (6)$$

$$(i = 1, 2, \cdots)$$

where q_i is the mode coordinate of the i-th oscillation mode. ζ denotes the damping coefficient and ω_i denotes the natural frequency of the i-th oscillation mode.

The Angle of the arm is obtained by (7).

$$\theta(t) = b^T q \quad (7)$$

where $q = [q_1, q_2, \cdots]^T$ and $b = [b_1, b_2, \cdots]^T$.

By (6) and (7) the transfer function from the applied torque to the angle of the arm is given as follows.

$$G(s) = \frac{A_0}{s^2} + \sum_{i=1}^{\infty} \frac{A_i}{s^2 + 2\zeta\omega_i s + \omega_i^2} \quad (8)$$

The Table 3 shows the first six parameter of the transfer function (8).

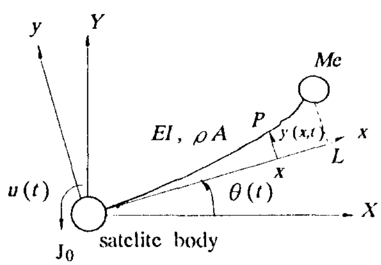

Fig.1 Schematic Diagram of the arm

Table 1 Notations in Fig. 1

J_0	Moment of inatia of the satelite body
L	Length of the arm
ρA	Linear density of the arm
EI	Flexurel rigidity
M_e	Lumped mass at the tip of the arm
J_e	Moment of inatia of the lumped mass at the tip of the arm
$u(t)$	Applied torque
$\theta(t)$	Angle of the arm

Table2 Parameter of Plant

L [m]	ρA [kg/m]	EI [Nm]	M_e [kg]
1.0	0.77	3.4	0.1

Table 3 Parameters of transfer function

i	$2\zeta\omega_i$	ω_i^2	A_i
0	0	0	2.4336
1	0.054	182.25	8.6436
2	0.1772	1962.5	1.21
3	0.492	15129	0.1444
4	0.988	61009	0.0361
5	1.66	172230	0.0121

3. CONTROLLER DESIGN

For the flexible arm discussed in the previous section, our control objective is to achieve a good control performance in the presence of model uncertainty. In this section control problem is formulated.

In general a two-degree-of-freedom control system is obtained as shown in Fig. 2[3].

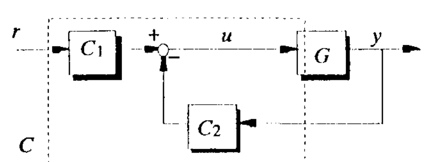

Fig. 2 Two-degree-of-freedom control system

The plant input u can be described by using the plant output y and the reference input r as follows.

$$u = C_1 r - C_2 y \quad (9)$$

where C_1 represents a feedforward controller and C_2 represents a feedback controller.

In this report these controllers C_1 and C_2 are given by (10) and (11).

$$C_1 = (G_{nom}^{-1} + C_B)\,G_m \qquad (10)$$

$$C_2 = C_B \qquad (11)$$

where G_{nom} denotes a transfer function of a nominal plant. G_m is a desired transfer function of a closed loop system, which can regard as a free parameter of the feedforward controller C_1. C_B is a robust stabilizing controller. From (10) and (11) the two-degree-of-freedom control system is constructed as shown in Fig. 3. In Fig. 3 G_{per} represents the perturbed plant.

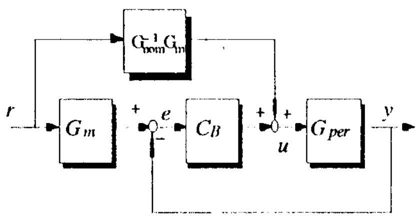

Fig. 3 Configuration of two-degree-of-freedom control system

In order to design the robust stabilizing controller C_B we solved the mixed sensitivity problem. The design specification of the mixed sensitivity problem is given by (12)[4].

$$\left\| \begin{bmatrix} W_1(s)\,S(s) \\ W_2(s)\,T(s) \end{bmatrix} \right\|_\infty < \gamma \qquad (12)$$

where $S(s) = (I + G_{nom}(s)C_B(s))^{-1}$ and $T(s) = C_B(s)\,(I + G_{nom}(s)C_B(s))^{-1}$. γ is a pre-specified positive number. In (12) $W_1(s)$ and $W_2(s)$ are weighting functions which depend on frequency. $W_1(s)$ is usually chosen so that its gain is relatively large in a low frequency range and $W_2(s)$ is usually chosen so that its gain is relatively large in a high frequency range. For a given nominal plant the mixed sensitivity problem is to find a controller C_B such that the closed loop system is internally stable and specification (12) is satisfied.

4. NUMERICAL RESULTS

We demonstrate numerical simulations for two types of a nominal plant model. One nominal plant model includes a rigid body mode and the first two flexible mode (Case I). The other nominal plant model only includes the rigid body mode (Case II). In Fig. 4 and Fig. 5 we use the solid line to show the results of a two-degree-of-freedom control system and dashed line to show those of a one-degree-of-freedom control system.

(I) Case I

The weighting functions in the specification (12) $W_1(s)$ and $W_2(s)$ are chosen as

$$W_1(s) = \frac{0.7}{s + 0.00001} \qquad (13)$$

$$W_2(s) = \frac{170\,s + 500}{s + 20000} \qquad (14)$$

For these weighting functions the robust stabilizing controller C_B can be obtained by the H_∞ control theory[4]. The desired transfer function of the closed loop system G_m is given by

$$G_m(s) = \frac{s + 13}{s^3 + 4s^2 + 11.9s + 13} \qquad (15)$$

From (10) we can get the feedforward controller We apply these controllers to the flexible arm described in (8). Fig. 4 shows the step response of the closed-loop system. From Fig. 4 it can be seen that the two-degree-of-freedom control system achieves better control performance than one-degree-of-freedom control system. We also confirm that two-degree-of-freedom control system rejects effect of ignored dynamics by Fig. 4. Fig. 5 shows the plant input. As shown in Fig. 5 the plant input is oscillating. This oscillation is caused by the feedforward controller . The feedforward controller includes the flexible modes which are contained in the nominal plant model. The oscillational response of plant input is caused by the effect of these flexible modes. Next we consider removing this oscillation.

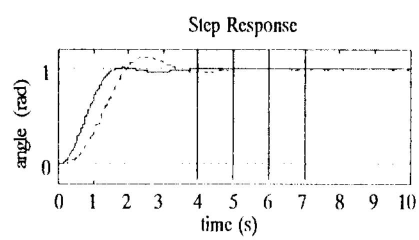

Fig. 4 Step response (Case I)

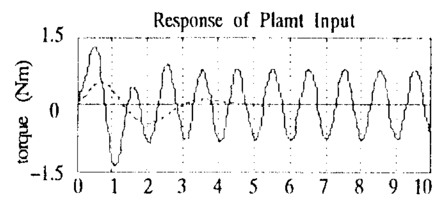

Fig. 5 The plant input (Case I)

(II) Case II

As mentioned before, the plant input is oscillated by the effect of flexible modes included in the nominal plant model. In order to remove this oscillation we consider the other nominal plant model which only includes the rigid body mode. The weighting functions in the specification (12) $W_1(s)$ and $W_2(s)$ are chosen as

$$W_1(s) = \frac{0.85}{s + 0.00001} \quad (16)$$

$$W_2(s) = \frac{2200s^2 + 2200s + 1100}{s^2 + 200s + 20000} \quad (17)$$

The desired transfer function of the closed loop system G_m is give by

$$G_m(s) = \frac{5.5s + 10}{s^5 + 6s^4 + 20s^3 + 30s^2 + 27s + 10} \quad (18)$$

For these $W_1(s)$, $W_2(s)$ and G_m we design robust stabilizing controller C_B and feedforward controller C_1.

Fig. 6 shows the step response of the closed loop system. From Fig. 6 we can see that the two-degree-of-freedom control system rejects the effect of ignored dynamics. Fig. 7 shows the response of the plant input. As shown in Fig. 7 it can be seen that the oscillational plant input is removed.

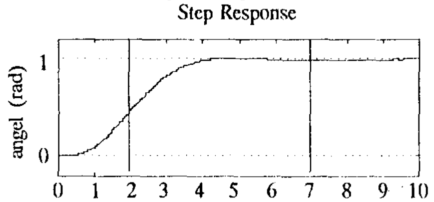

Fig. 6 Step response (Case II)

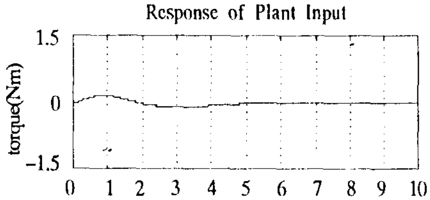

Fig. 7 Response of plant input (Case II)

5. CONCLUSION

In this report two-degree-of-freedom control system design for a flexible arm has been presented.

At First we consider a nominal plant model which includes a rigid body mode and the first two flexible modes. For this nominal plant model we design a two-degree-of-freedom control system. Numerical results show that the two-degree-of-freedom control system achieves better control performance than a one-degree-of-freedom control system. However The plant input oscillates by the effect of flexible modes which are included in the nominal plant.

Secondly in order to remove the oscillation the of the plant input we consider the other nominal plant model which only includes the rigid body mode. For this nominal plant model a two-degree-of-freedom control system also achieves better control performance. Further more, the oscilational the plant input is removed.

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