'93 KACC (1993.10.20~22)

Optimal Motion Control for Robot Manipulators

°Jin-Ho Shin and Ju-Jang Lee
* Department of Electrical Engineering
Korea Advanced Institute of Science and Technology

Abstract

In this paper, an optimal motion control scheme is proposed for robot manipulators. A simple explicit solution to the Hamilton-Jacobi equation is presented. The optimization of motion control is based on the minimization of the torque term affecting the kinetic energy and the augmented error which has the first-order stable dynamics for the position and velocity tracking error. In the presence of parametric uncertainty, an adaptive control scheme using the optimal principle is proposed. The global stability of the closed-loop system is guaranteed by the Lyapunov stability approach. The effectiveness and feasibility of the proposed control schemes are shown by simulation results.

1 Introduction

In the general robot manipulators, the motion control is required to maintain a prescribed motion for the control object by applying compensating corrective torques. Motion controlled systems with autonomous operation or repetitive motion need usually optimization. Mostly used approach to the optimal control problem is the linear optimal control based on linearized equations of motion with regard to slowly changing operating conditions.

The linear optimal control of the nonlinear system sometimes may be very difficult due to the nonlinear dynamics with motion constraints and rapidly changing operating conditions. From the standpoint of linear quadratic control theory, it is natural to include the control input in the performance index. In the case of robot manipulators, the nonlinear dynamics of the robot system make the linear quadratic control difficult.

Luo and Saridis [1] proposed the linear quadratic design of PID controllers for robot arms. They included the position, velocity and acceleration tracking errors in the performance index and thus proposed an explicit solution for robot arm controller, optimal in the linear quadratic sense, using the Hamilton-Jacobi equation. But the controller needed the inversion of the inertia matrix. Also, the minimization of the applied torques was not considered in their paper [1]. Johansson [2] proposed an explicit solution to the Hamilton-Jacobi equation for the quadratic optimization of motion control. The paper [2] considered the minimization of the torque term affecting the kinetic energy and the position and velocity errors. Also, an adaptive control with self-optimizing adaptation was presented.

In this paper, an optimal motion control for robot manipulators is proposed by using the performance index that has the torque term affecting the kinetic energy and the augmented error term which has the first-order stable dynamics for the position and velocity tracking error. A simple explicit solution to the Hamilton-Jacobi equation is proposed. Under the parameter uncertainty, an adaptive control scheme using the optimal principle is presented. The global stability of the overall closed-loop system is guaranteed by using the Lyapunov stability approach.

2 Dynamic Equation of Robot Manipulators

Using the Lagrangian formulation, the equations of motion of an n-degree-of-freedom manipulator can be written as

$$M(q)\,\ddot{q} + C(q,\dot{q})\,\dot{q} + G(q) = \tau \qquad (1)$$

where $q \in \Re^n$ is the generalized position coordinate; $M(q) \in \Re^{n\times n}$ is the symmetric, bounded, positive definite inertia matrix; $C(q,\dot{q})\dot{q} \in \Re^n$ is the centrifugal and Coriolis torques; $G(q) \in \Re^n$ is the vector of gravitational torques; and $\tau \in \Re^n$ is the vector of applied joint torques.

The robot model (1) has the following structural properties which are useful to the design of controller and the stability analysis.

Property 1: Linear parameterization property

$$M(q)\,\ddot{q} + C(q,\dot{q})\,\dot{q} + G(q) \;=\; Y(q,\dot{q},\ddot{q})\,\theta \qquad (2)$$

where $Y \in \Re^{n\times r}$ is called the *regressor* matrix, and $\theta \in \Re^r$ is a vector of inertia parameters [3].

Property 2: Skew-symmetric property

A suitable definition of $C(q,\dot{q})$ makes matrix $(\dot{M} - 2C)$ skew-symmetric [3].

We define the useful signals to design the controller in next section. First, the tracking error e is as follows.

$$e = q_d - q \qquad (3)$$

where $q_d \in \Re^n$ is a given bounded twice continuously differentiable reference trajectory.

The augmented error s is defined as:

$$s = \dot{e} + \Lambda\, e \qquad (4)$$

where $\Lambda \in \Re^{n\times n}$ is a diagonal positive definite matrix.

The auxiliary signal η is

$$\eta = s + \dot{q} = \Lambda\, e + \dot{q}_d. \qquad (5)$$

From Property 1, the following equation can be obtained.

$$M(q)\,\dot{\eta} + C(q,\dot{q})\,\eta + G(q) \;=\; Y(q,\dot{q},\eta,\dot{\eta})\,\theta \qquad (6)$$

Now, the error dynamics for the augmented error s is

$$\dot{s} \;=\; -M^{-1}\,C\,s + M^{-1}\,Y\,\theta - M^{-1}\,\tau \qquad (7)$$

The transformation of the state is performed by

$$z \;=\; T_s\, s \qquad (8)$$

where $T_s \in \Re^{n\times n}$ is a nonsingular state transformation matrix and $z \in \Re^n$ is a state vector transformed by T_s.

Defining the auxiliary control input u as the control action to be minimized,

$$u \;=\; M(q)\,T_s\,\dot{s} + C(q,\dot{q})\,T_s\,s \qquad (9)$$

u may be considered to be the forces or torques term affecting the kinetic energy.

Substituting (7) into (9), the actual torque is as follows:

$$\tau \;=\; -C\,s + Y\,\theta + M\,T_s^{-1}M^{-1}C\,T_s\,s - M\,T_s^{-1}M^{-1}u \qquad (10)$$

Rewritting (7) as the equation for u, the error dynamics for s and u is obtained as follows.

$$\dot{s} \;=\; A\,s + B\,u \qquad (11)$$

where

$$A \;=\; -T_s^{-1}\,M^{-1}\,C\,T_s \qquad (12)$$
$$B \;=\; T_s^{-1}\,M^{-1} \qquad (13)$$

3 Optimal Motion Control for Robot Manipulators

In this section, an optimal motion control schemes for robot manipulators is proposed by using the Hamilton-Jacobi equation. Here, the optimal control problems of two cases are studied. One is the known parameter case and another is the unknown parameter case (adaptive case).

3.1 Known Parameter Case

To derive the optimal control law, the performance index is given as follows.

$$J = \int_{t_o}^{\infty} L(s,u)\,dt \qquad (14)$$

where $L(s,u)$ is the Lagrangian of linear quadratic form:

$$L(s,u) = \frac{1}{2}s^T Q s + \frac{1}{2}u^T R u \qquad (15)$$

where $Q = Q^T$ and $R = R^T$ are positive definite matrices.

Now, the Hamiltonian is defined as follows.

$$H\Big(s,u,\frac{\partial J^*(s,u,t)}{\partial s}\Big) = L(s,u) + \Big(\frac{\partial J^*(s,u,t)}{\partial s}\Big)^T \dot{s} \qquad (16)$$

An optimal motion control scheme for robot manipulators is proposed by the following theorem.

Theorem 1: If the optimal value J^* satisfying the Hamilton-Jacobi equation is defined as the quadratic form,

$$J^* \;=\; J(s,u^*,t) \;=\; \frac{1}{2}z^T M(q) z \;=\; \frac{1}{2}s^T T_s^T M(q) T_s s \qquad (17)$$

then the optimal control law minimizing the performance index (14)(15) is as follows:

$$u^* \;=\; -R^{-1}\,T_s\,s \qquad (18)$$

Here, T_s is obtained from the following matrix equation

$$\frac{1}{2}s^T(Q \;-\; T_s^T\,R^{-1}\,T_s)s = 0 \quad \forall\; s \qquad (19)$$

Proof: Substituting (17)(11) into (16),

$$H = \frac{1}{2}\,s^T\,Q\,s + \frac{1}{2}\,u^T\,R\,u + s^T\,T_s^T\,M\,T_s\,(As + Bu) \qquad (20)$$

From the Hamiltonian (20),

$$\left.\frac{\partial H}{\partial u}\right|_{u=u^*} = 0 \qquad (21)$$

Equation (21) is calculated as follows:

$$R\,u^* + (s^T\,T_s^T\,M\,T_s\,B)^T = 0 \qquad (22)$$

Since $B = T_s^{-1}\,M^{-1}$, the optimal control law for the system (11) is as follows

$$u^* \;=\; -R^{-1}\,T_s\,s \qquad (23)$$

Also, the optimal Hamiltonian is

$$\begin{aligned} H^* &= H(s, u^*, \frac{\partial J^*(s,u^*,t)}{\partial s}) \\ &= L(s,u^*) + (\frac{\partial J^*(s,u^*,t)}{\partial s})^T \dot{s} \\ &= \frac{1}{2} s^T Q s - \frac{1}{2} s^T T_s^T R^{-1} T_s s - s^T T_s^T C T_s s \quad (24) \end{aligned}$$

Then the Hamilton-Jacobi (H-J) equation is

$$\frac{\partial J^*}{\partial t} + H^* = 0 \quad (25)$$

where

$$\frac{\partial J^*}{\partial t} = \frac{1}{2} z^T \dot{M} z = z^T C z = s^T T_s^T C T_s s \quad (26)$$

Then the H-J equation is

$$s^T T_s^T C T_s s + \frac{1}{2} s^T Q s - \frac{1}{2} s^T T_s^T R^{-1} T_s s - s^T T_s^T C T_s s = 0 \quad (27)$$

Finally,

$$\frac{1}{2} s^T (Q - T_s^T R^{-1} T_s) s = 0 \qquad \forall\, s \quad (28)$$

Therefore, T_s is obtained from the above matrix equation.

The proof is complete. □

Remark: The actual control torque τ is obtained from (10) by replacing u with u^*.

$$\tau^* = -Cs + Y\theta + MT_s^{-1} M^{-1} C T_s s - MT_s^{-1} M^{-1} u^* \quad (29)$$

Here, the calculation of τ^* according to a matrix T_s is discussed in the following statements.

1. If T_s is a general nonsingular matrix except for the case of the diagonal matrix having the same components, then the actual control torque requires M^{-1} as shown in (29). This causes the complicate actual torque and thus it requires heavy computation burden. And T_s affects the actual control torque.

2. If T_s is a diagonal matrix having the same diagonal components, which is obtained for a special choice of Q and R, namely, $T_s = t_s I_n$, $t_s \in \Re$, $I_n \in \Re^{n \times n}$: identity matrix, then the actual control torque don't require M^{-1}.

$$\begin{aligned} \tau^* &= Y\theta - \frac{1}{t_s} u^* \quad (30) \\ &= Y\theta + R^{-1} s \quad (31) \end{aligned}$$

This has the simple actual torque and thus the computation burden is smaller than that of the above case 1. But the actual control torque is independent of T_s.

Now, the following theorem is presented to show the stability of the closed-loop control system.

Theorem 2: If the control input (29) is applied to the robot system (1), and the symmetric positive definte weighting matrices Q and R are chosen such that

$$Q = Q_1^T Q_1, \quad R = R_1^T R_1 \quad (32)$$

where Q_1 and R_1 are nonsingular matrices, and so T_s is chosen such that

$$T_s = R_1^T Q_1 \quad (33)$$

then the closed-loop system is globally asymptotically stable.

Proof: Consider a Lyapunov function candidate as follows.

$$V = J^* = \frac{1}{2} s^T T_s^T M(q) T_s s \quad (34)$$

The Hamilton-Jacobi equation is

$$\frac{\partial V}{\partial t} + L(s,u^*) + (\frac{\partial V}{\partial s})^T \dot{s} = 0 \quad (35)$$

Therefore, the time derivative of V along the solution of the system (1) is

$$\begin{aligned} \dot{V} &= -L(s,u^*) \\ &= -\frac{1}{2} s^T (Q + T_s^T R^{-1} T_s) s \quad (36) \end{aligned}$$

Since $T_s^T R^{-1} T_s = Q$ by (32)(33),

$$\dot{V} = -s^T Q s \le 0 \quad (37)$$

$\dot{V}$ is negative definite and thus $s \to 0$ as $t \to \infty$. Therefore, $\dot{e}, e \to 0$ as $t \to \infty$. The closed-loop system is globally asymptotically stable.

The proof is complete. □

3.2 Unknown Parameter Case : Adaptive Case

The actual control torque (10) is

$$\tau = -Cs + Y\theta + MT_s^{-1} M^{-1} C T_s s - MT_s^{-1} M^{-1} u \quad (38)$$

If $T_s = t_s I_n$, then

$$\tau = Y\theta - \frac{1}{t_s} u \quad (39)$$

Here, θ is an unknown constant parameter vector. Therefore, the control torque is modified as follows.

$$\tau = Y\hat{\theta} - \frac{1}{t_s} \bar{u} \quad (40)$$

where $\hat{\theta} \in \Re^r$ is a vector of parameter estimates. And,

$$\bar{u} = t_s Y \phi + u \quad (41)$$

where $\phi = \hat{\theta} - \theta \in \Re^r$ is a parameter deviation vector. Then, the error dynamics is as follows.

$$\dot{s} = -M^{-1}Cs + \frac{1}{t_s}M^{-1}u \quad (42)$$

$$= -M^{-1}Cs + \frac{1}{t_s}M^{-1}\bar{u} - M^{-1}Y\phi \quad (43)$$

Here, the performance index is defined as follows.

$$J = \int_{t_o}^{\infty} L(s,\bar{u})\,dt \quad (44)$$

where

$$L(s,\bar{u}) = \frac{1}{2}s^T Q s + \frac{1}{2}\bar{u}^T R \bar{u} \quad (45)$$

where $Q = Q^T$ and $R = R^T$ are positive definite.

Now, an adaptive control scheme using the optimal principle under the parameter uncertainty is proposed by the following theorem.

Theorem 3: If the optimal value J^* satisfying the Hamilton-Jacobi equation is defined as the quadratic form,

$$J^* = \frac{1}{2}t_s^2 s^T M(q)s + \frac{1}{2}\phi^T\Gamma^{-1}\phi = \frac{1}{2}X^T P X \quad (46)$$

where

$$X = \begin{pmatrix} t_s s \\ \phi \end{pmatrix}, \quad P = \begin{pmatrix} M & O \\ O & \Gamma^{-1} \end{pmatrix}$$

then the optimal control law minimizing the performance index (44)(45) is as follows:

$$\bar{u}^* = -t_s R^{-1} s \quad (47)$$

Also, t_s is obtained from the following matrix equation

$$\frac{1}{2}s^T(Q - t_s^2 R^{-1})s = 0 \quad \forall\ s \quad (48)$$

The actual control torque is

$$\tau^* = Y\hat{\theta} - \frac{1}{t_s}\bar{u}^* \quad (49)$$

$$= Y\hat{\theta} + R^{-1}s \quad (50)$$

If the above control input (50) and the following adaptation law are applied to the robot system (1), then the tracking errors converge to zero asymptotically.

- *Adaptation law:*

$$\dot{\hat{\theta}} = t_s^2\,\Gamma\,Y^T s \quad (51)$$

Proof: Consider a Lyapunov function candidate as follows.

$$V = J^* = \frac{1}{2}t_s^2 s^T M(q)s + \frac{1}{2}\phi^T\Gamma^{-1}\phi \quad (52)$$

The Hamiltonian is calculated as follows.

$$H(s,\bar{u},\frac{\partial V}{\partial X}) = L(s,\bar{u}) + (\frac{\partial V}{\partial X})^T\dot{X}$$

$$= \frac{1}{2}s^TQs + \frac{1}{2}\bar{u}^TR\bar{u} + t_s^2 s^T M\dot{s} + \phi^T\Gamma^{-1}\dot{\phi}$$

$$H = \frac{1}{2}s^TQs + \frac{1}{2}\bar{u}^TR\bar{u} + t_s^2 s^T M(-M^{-1}Cs + \frac{1}{t_s}M^{-1}u) + \phi^T\Gamma^{-1}\dot{\phi}$$

$$= \frac{1}{2}s^TQs + \frac{1}{2}\bar{u}^TR\bar{u} - t_s^2 s^TCs + t_s s^Tu + \phi^T\Gamma^{-1}\dot{\phi}$$

$$= \frac{1}{2}s^TQs + \frac{1}{2}\bar{u}^TR\bar{u} - t_s^2 s^TCs + t_s s^T(\bar{u} - t_sY\phi) + \phi^T\Gamma^{-1}\dot{\phi} \quad (53)$$

The optimal control law is obtained as:

$$\left.\frac{\partial H}{\partial \bar{u}}\right|_{\bar{u}=\bar{u}^*} = R\bar{u}^* + t_s s = 0 \quad (54)$$

$$\bar{u}^* = -t_s R^{-1}s \quad (55)$$

To obtain the matrix equation for t_s, the optimal Hamiltonian is found as follows:

$$H^* = \frac{1}{2}s^TQs - \frac{1}{2}t_s^2 s^TR^{-1}s - t_s^2 s^TCs - t_s^2 s^TY\phi + \phi^T\Gamma^{-1}\dot{\phi} \quad (56)$$

The Hamilton-Jacobi equation is given as follows.

$$\frac{\partial V}{\partial t} + H^* = 0 \quad (57)$$

By some manipulations,

$$\frac{1}{2}s^TQs - \frac{1}{2}t_s^2 s^TR^{-1}s - t_s^2 s^TY\phi + \phi^T\Gamma^{-1}\dot{\phi} = 0 \quad (58)$$

Here, substituting the adaptation law $\dot{\phi} = \dot{\hat{\theta}} = t_s^2\Gamma Y^T s$ into the above equation (58), Finally,

$$\frac{1}{2}s^T(Q - t_s^2R^{-1})s = 0 \quad \forall\ s \quad (59)$$

Therefore, the control gain t_s is obtained from the above equation for given Q and R. Now, the actual control torque is

$$\tau^* = Y\hat{\theta} - \frac{1}{t_s}\bar{u}^* = Y\hat{\theta} + R^{-1}s \quad (60)$$

Now, the time derivative of V along the solution trajectory of the system (1) is as follows.

$$\dot{V}(X,t) = \frac{\partial V}{\partial t} + (\frac{\partial V}{\partial X})^T\dot{X} \quad (61)$$

$$= -L(s,\bar{u}^*) \quad (62)$$

$$= -\frac{1}{2}s^T(Q + t_s^2R^{-1})s \le 0 \quad (63)$$

$\dot{V}(X,t)$ is negative semidefinite and then by Barbalat's lemma, $s \to 0$ as $t \to \infty$. Therefore, $\dot{e}, e \to 0$ as $t \to \infty$. The proof is complete. □

Remark: If ϕ converges to zero, then the solution reaches the optimal solution.

$$\bar{u} = u^* + t_sY\phi$$

$$= u^*$$

At this time, the actual torque is as follows.

$$\tau^* = Y\theta - \frac{1}{t_s}u^* \quad (64)$$

Here, let's compare with the performance index for two cases,

1. Known parameter case :

$$J(s,u) = \int_{t_o}^{\infty} \frac{1}{2}(s^TQs + u^TRu)dt \quad (65)$$

2. Unknown parameter case (Adaptive case):

$$J(s,\bar{u}) = \int_{t_o}^{\infty} \frac{1}{2}(s^TQs + \bar{u}^TR\bar{u})dt \quad (66)$$

$$= J(s,u) + \Delta J(t_s, R, u, Y, \phi) \quad (67)$$

where

$$\Delta J(t_s, R, u, Y, \phi) = \int_{t_o}^{\infty} \frac{1}{2}(2t_s\phi^TY^TRu + t_s^2\phi^TY^TRY\phi)dt \quad (68)$$

If $\phi \to 0$, then $J(s,\bar{u}) \to J(s,u)$.

4 Simulation Results

To show the feasibility of the proposed control scheme, simulation results for a two-link planar robot manipulator are presented. The numerical parameters for this robot are given as follows.

$$m_1 = 2\ kg; \quad L_1 = 1\ m; \quad J_1 = 0.1\ kgm^2;$$
$$m_2 = 1\ kg; \quad L_2 = 0.8\ m; \quad J_2 = 0.01\ kgm^2.$$

where m_1, J_1 and L_1 are defined as mass, inertia and length of link 1, respectively, and the parameters of link 2, m_2, J_2 and L_2, are also defined as those of link 1.

Here, the matrices R and Q are selected as follows.

$$R = 0.02\, I_2, \quad Q = 15.0\, I_2$$

where $I_2 \in \Re^{2\times 2}$ is the identity matrix.
The state-space transformation matrix is $T_s = t_s\, I_2$.

From the following matrix equation of the Hamilton-Jacobi equation,

$$\frac{1}{2}s^T(Q - t_s^2R^{-1})s = 0 \quad \forall\ s$$

the sufficient condition satisfying the above equation is

$$t_s = \pm\sqrt{15.0 \times 0.02}$$

Here, it is chosen that $t_s = \sqrt{0.3}$.

Simulation results include two cases of the known parameter case and the adaptive case. For the known parameter case, the results of the proposed optimal motion control are presented in Figure 1. For the unknown parameter case, the results of the proposed adaptive control using the optimal principle are given in Figure 2. The augmented errors and the tracking errors converge to zero in all the results. From the simulation results, the performance index of the known parameter case is smaller than that of the unknown parameter case. This difference is caused by the parameter deviations.

In (a) and (b) of Figure 1 and Figure 2, the meanings of lines are as follows. (a) : solid line (——) and dashed line (------) represent the tracking error (rad) of joint 1 and joint 2, respectively, and dashdot line (-----) and dotted line ( ) represent the augmented error (rad/sec) of joint 1 and joint 2, respectively. (b) : solid line and dashed line represent the optimal control input of joint 1 and joint 2, respectively, and dashdot line and dotted line represent the actual control torque of joint 1 and joint 2, respectively.

5 Conclusions

In this paper, an optimal motion control scheme has been proposed for robot manipulators. The optimization of motion control is based on the minimization of the torque term affecting the kinetic energy and the augmented error. A simple explicit solution to the Hamilton-Jacobi equation has been presented for the robot system with nonlinear dynamics. The proposed controller does not need the inversion of the inertia matrix, and it also requires the measurement of only position and velocity of joint variable. Under the parameter uncertainty, an adaptive control scheme has been proposed by using the optimal control scheme which was presented for the known parameter case. If the parameter deviations converge to zero, the solution reaches the optimal value. The global stability of the closed-loop system has been guaranteed by the Lyapunov stability approach. It can be found that the optimal control gain is more simply obtained by using the augmented error in design procedures than Johansson's paper [2]. Simulation results have shown that the proposed control schemes are effective and feasible.

References

[1] G. L. Luo and G. N. Saridis, "L-Q design of PID controllers for robot arms," *IEEE J. Robot. Automat.*, Vol. RA-1, No. 3, pp. 152–159, 1985.

[2] R. Johansson, "Quadratic Optimization of Motion Coordination and Control," *IEEE Trans. on Automatic Control*, Vol. AC-35, No. 11, pp. 1197–1208, 1990.

[3] J. J. E. Slotine and W. Li, "On the adaptive control of robot manipulators," *Int. J. Robotics Res.*, Vol. 6, No. 3, pp. 49–59, 1987.

[4] J. J. Craig, *Introduction to Robotics: Mechanics and Control*, 2nd Edition, Addison-Wesley Publishing Co., Inc., 1989.

[5] F. L. Lewis, *Optimal Control*, John Wiley and Sons, Inc., 1986.

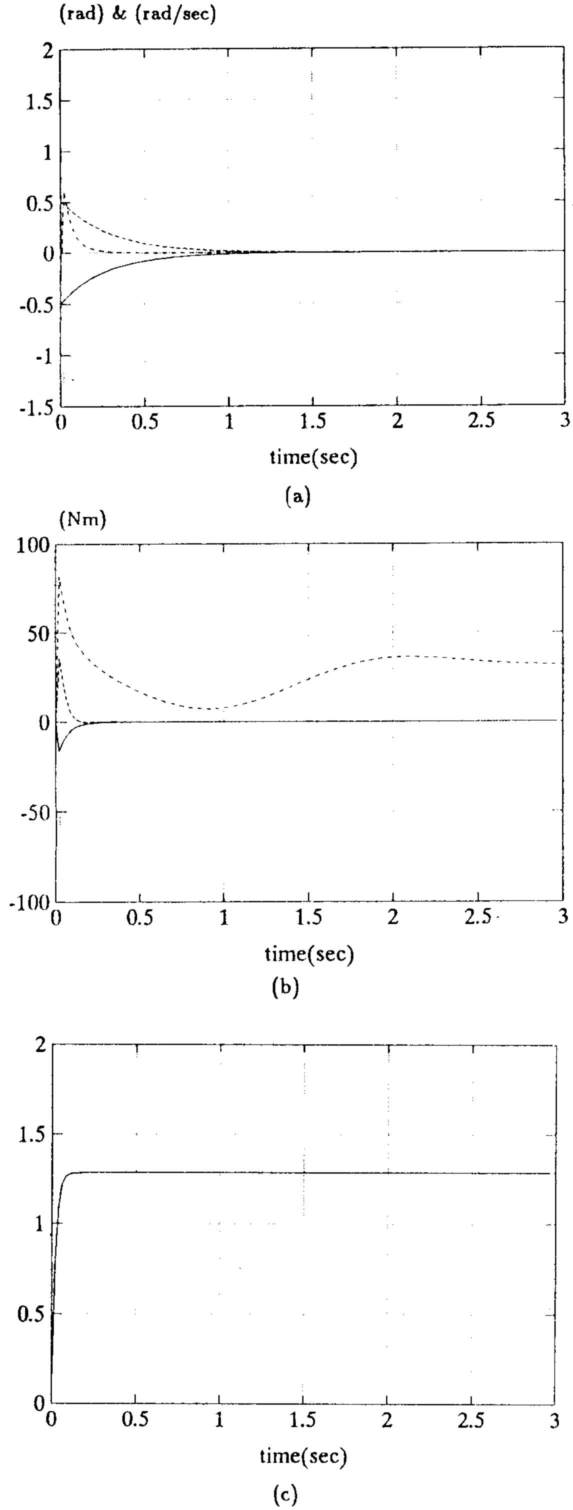

Figure 1. Optimal motion control for known parameter case :
(a) Tracking error (e) and augmented error (s); (b) Optimal control input (u^*) and actual control torque (τ^*); (c) Performance index (J).

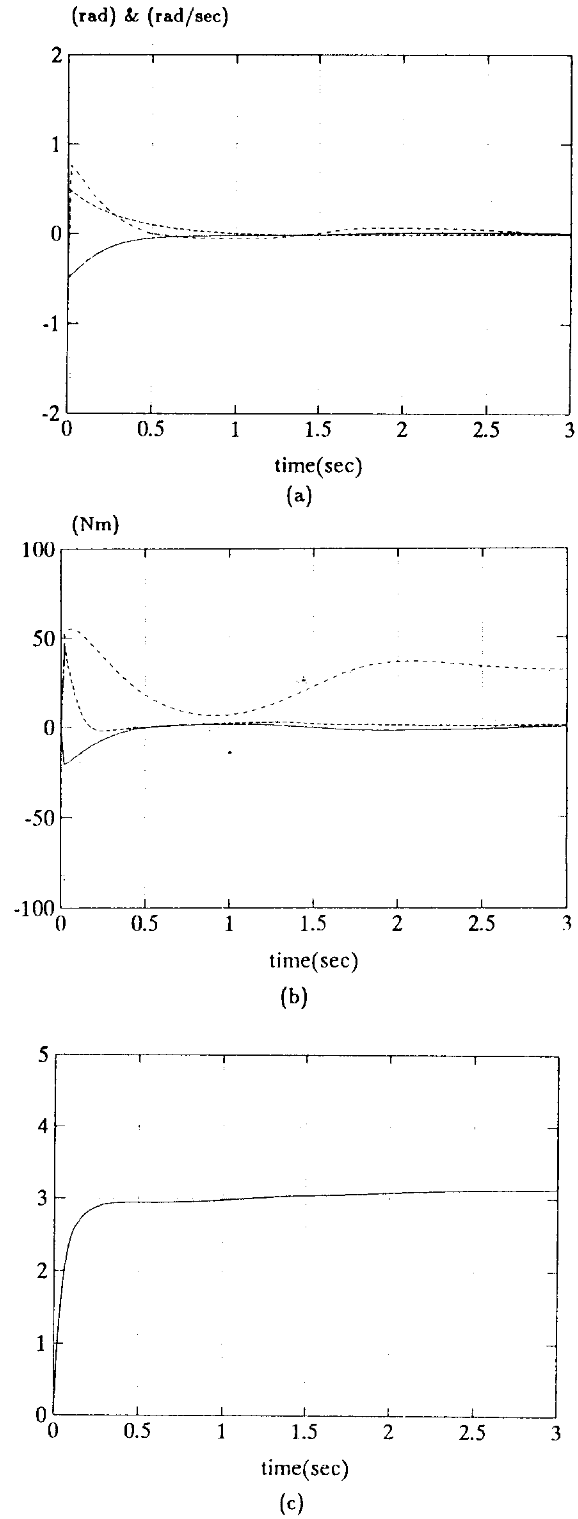

Figure 2. Adaptive control using optimal principle for unknown parameter case :
(a) Tracking error (e) and augmented error (s); (b) Optimal control input ($\bar{u}^*$) and actual control torque (τ^*); (c) Performance index (J).