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Stability Analysis of Piezopolymer Flexible Twisting Micro-actutor with a Linear Feedback Control

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Abstract

A method for the closedloop control of the torsional tip motion of a piezo-polymer actuator is presented. The application of Lyapunov's direct method to the problem is explored. A feedback control of the torsional tip motion of the piezopolymer actuator is derived by considering the time rate of change of the total energy of the system. If the angular velocity of the tip of the actuator is known, all the modes of the actuator can be controlled simultaneously. This approach has the advantage over the conventional methods in the respect that it allows one to directly with the system's partial differential equations without resorting to approximations.

1. Introduction

Many operations in semiconductor manufacturing such as probing and wire bonding require high-precision controlled motion and delicate forces. Micro-actuators which are capable of executing motions in the micron or submicron range have applications in biomedical as well as microrobotics. There exists an ongoing need in the area of microrobotics and micromachines for light fast compact actuators.

The most of piezoelectric actuators are based on bimorph design and are either made of ceramic or polymeric materials. Ceramics have better piezoelectric properties and higher elastic stiffness constant than polymers. This gives ceramics the advantage of producing large forces but small displacements. Polymers have an advantage of producing larger displacements than ceramics but the force generated is greatly reduced. We resolved the stiffness problem with the polymeric material by reducing the actuator in size. In 1969, Kawai discovered the strong piezoelectric effect in PVDF or PVF₂ (polyvinylidenefluoride)[1]. Compared to other piezoelectric materials, PVDF has certain unique properties such as high levels of piezo activity, an extremely wide frequency range, a broad dynamic response, and low acoustic impedance[2,3]. This make it attractive for many sensor and transducer applications. We propose here gluing together two three-layer PVDF-shim metal composites in parallel, and PVDF actuating them with applied electric fields having opposite phases for generating the torsional motion. When a voltage is applied across the elements, one element gets longer, the other shorter-hence deflection is obtained. Therefore opposite fields on diagonal pairs cause a twisting motion of the actuator. To model the piezopolymer micro-actuator under applied electric fields, we applied the classical laminate plate theory[4].

Flexible structures are distributed-parameter systems having a theoretically infinite number of vibrational modes. Often, current design practice is to model the system with a finite number of modes and to design a control system using lumped-parameter control theory. Truncating the model may lead to performance tradeoffs when designing a control system for distributed-parameter systems. In order to realize accurate positioning or tracking of the piezopolymer micro-actuator, the rigidbody motions and elastic behaviors have to be controlled simultaneously. In this case, the stability analysis of the controlled system is very important. In this paper, a method for the closedloop control of the torsional motion of a piezo-polymer actuator is presented.

2. Modeling the Twisting Actuator

From the structural viewpoint, only bending and stretching motion can be actuated or detected by the PVDF alone. We propose gluing together two three-layer PVDF-shim metal composites in parallel, and PVDF actuate them with applied electric fields having opposite phases for generating the torsional motion. To generate a pure bending motion, we PVDF actuate them with applied electric fields having the same phase.

Here, we consider an isotropic lamina with two piezopolymer laminae bonded to the top and bottom surfaces of the plate as shown Fig. 1. The material properties for the isotropic layer are specified by:

$$[\bar{Q}_1]=\left|\begin{array}{lll} E_1/(1-\nu_1^2) & \nu_1 E_1/(1-\nu_1^2) & 0 \\ \nu_1 E_1/(1-\nu_1^2) & E_1/(1-\nu_1^2) & 0 \\ 0 & 0 & E_1/2(1+\nu_1) \end{array}\right|. \quad (1)$$

The material properties for the PVDF are specified by:

$$[\bar{Q}_2]=\left|\begin{array}{lll} E_2/(1-\nu_2^2) & \nu_2 E_2/(1-\nu_2^2) & 0 \\ \nu_2 E_2/(1-\nu_2^2) & E_2/(1-\nu_2^2) & 0 \\ 0 & 0 & E_2/2(1+\nu_2) \end{array}\right|. \quad (2)$$

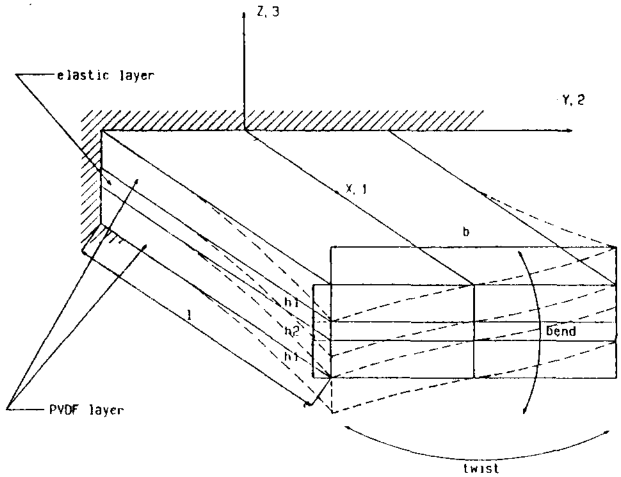

Figure 1. Arrangement and coordinate of piezopolymer flexible twisting micro-actuator

where E_1, ν_1, and E_2, ν_2 are the Young's modulus and Poisson's ratio for the isotropic layer and PVDF respectively.

In this case, the equation of motion of the micro-actuator is written as

$$((\bar{Q}_{66})_1 I_{p1}+(\bar{Q}_{66})_2 I_{p2})\partial^2\theta/\partial x^2=(\rho_1 I_{p1}+\rho_2 I_{p2})\partial^2\theta/\partial t^2, \tag{3}$$

where $((\bar{Q}_{66})_1 I_{p1}+(\bar{Q}_{66})_2 I_{p2})$ is the torsional stiffness and $(\rho_1 I_{p1}+\rho_2 I_{p2})$ is the mass moment of inertia.

The boundary conditions are

$$\theta(0,t)=0, \tag{4}$$

$$((\bar{Q}_{66})_1 I_{p1}+(\bar{Q}_{66})_2 I_{p2})\partial\theta(l,t)/\partial x=b(h_1+h_2)(\bar{Q}_{66})_1 d_{31}V \tag{5}$$

where V is the applied voltage, d_{31} the appropriate static piezoelectric constant, h_1 the thickness of the PVDF layer, h_2 the thickness of the isotropic layer, b the width of the actuator, l the length of the actuator.

3. Deriving a Control Algorithm

Distributed-parameter control theory was used to design a control algorithm for the piezopolymer flexible twisting micro-actuator. This allows one the possibility of controlling all the modes of vibration at once, provided that the system is controllable through the actuator. Hence, one may avoid problems with spillover of the uncontrolled modes.

The control problem is to control the motion of the system described by Eqs. (3)-(5) using the input voltage $V(t)$ to the PVDF as the control variable. We assume there is no restriction on the type of sensors available.

Lyapunov's second or direct method can be used to design control algorithms and can easily deal with distributed-parameter systems. This design method was chosen because it was the most straightforward way to derive an implementable distributed-parameter control law. With this method, one finds a Lyapunov functional of the system and chooses the control to minimize the time rate of change of the functional at every point in time. An appropriate functional for the system described by Eqs. (3)-(5) is the sum of the squares of the angle of twist per unit length and angular velocity, integrated along the length of the actuator[5], or

$$E(t)=1/2\int_0^l(\rho_1 I_{p1}+\rho_2 I_{p2})(\partial\theta/\partial t)^2dx$$

$$+1/2\int_0^l((\bar{Q}_{66})_1 I_{p1}+(\bar{Q}_{66})_2 I_{p2})(\partial\theta/\partial x)^2dx, \tag{6}$$

where the first term in equation (6) is relative measure of its kinetic energy and the second term is related to the torsional strain energy in the structure. Hence the chosen Lyapunov function resembles the total energy of the system. A control law is sought which will minimize the time derivative of this functional. Taking the time derivative of equation (6) yields:

$$\partial E(t)/\partial t=\int_0^l(\rho_1 I_{p1}+\rho_2 I_{p2})(\partial\theta/\partial t)(\partial^2\theta/\partial t^2)dx$$

$$+\int_0^l((\bar{Q}_{66})_1 I_{p1}+(\bar{Q}_{66})_2 I_{p2})(\partial\theta/\partial x)(\partial^2\theta/\partial x\partial t)dx. \tag{7}$$

Substituting equation (3) into (7) gives:

$$\partial E(t)/\partial t=\int_0^l((\bar{Q}_{66})_1 I_{p1}+(\bar{Q}_{66})_2 I_{p2})(\partial^2\theta/\partial x^2)(\partial\theta/\partial t)dx$$

$$+\int_0^l((\bar{Q}_{66})_1 I_{p1}+(\bar{Q}_{66})_2 I_{p2})(\partial\theta/\partial x)(\partial^2\theta/\partial x\partial t)dx. \tag{8}$$

The second term of equation (8) is then integrated by parts and the boundary conditions are substituted into the resulting equation. The result is

$$\partial E(t)/\partial t=\int_0^l((Q_{66})_1 I_{p1}+(Q_{66})_2 I_{p2})(\partial^2\theta/\partial x^2)(\partial\theta/\partial t)dx$$

$$+((\bar{Q}_{66})_1 I_{p1}+(\bar{Q}_{66})_2 I_{p2})(\partial\theta/\partial x)(\partial\theta/\partial t)\Big|_{x=l}$$

$$-((\bar{Q}_{66})_1 I_{p1}+(\bar{Q}_{66})_2 I_{p2})(\partial\theta/\partial x)(\partial\theta/\partial t)\Big|_{x=0}$$

$$-\int_e^l ((\bar{Q}_{\ell\ell})_1 I_{p1}+(\bar{Q}_{\ell\ell})_2 I_{p2})(\partial^2\theta/\partial x^2)(\partial\theta/\partial t)dx$$

$$=((\bar{Q}_{\ell\ell})_1 I_{p1}+(\bar{Q}_{\ell\ell})_2 I_{p2})(\partial\theta/\partial x)(\partial\theta/\partial t)|_{x=l}$$

$$=(\partial\theta/\partial t)|_{x=l}(N_1 V), \tag{9}$$

where

$N_1=b(t_1+t_2)(\bar{Q}_{\ell\ell})_1 d_{31}$.

The feedback control law for $V(t)$ which will minimize equation (6) (i.e., make it as negative as possible given a input) is given as;

$$N_1 V(t)=-K(\partial\theta/\partial t)|_{x=l} \quad ; \qquad K>0 \tag{10}$$

$$\partial E(t)/\partial t=-K((\partial\theta/\partial t)|_{x=l})^2\le 0, \tag{11}$$

where $(\partial\theta/\partial t)|_{x=l}$ is the angular velocity at the tip of the actuator. This means that control voltage should be chosen with as large a magnitude as possible and should generate a torque that ensures asymptotic stability of the tip angular motion of the actuator. This control law has several desirable characteristics. First, no modes have been truncated. This control law will theoretically work with any and all modes of torsional vibration of a actuator, since every mode has some angular at the tip of the actuator. Second, the control law depends only on the angular velocity at the tip of the actuator, not an integral along its length. This means that only one spatially discrete sensor is needed to implement this distributed-parameter control law.

Let θ_d be the desired tip angle. Then a constant voltage V_d is needed to hold the tip position at θ_d.

Let $\bar{\theta}_d(x)$ $(\bar{\theta}_d(l)=\theta_d)$ be the solution corresponding to:

$$((\bar{Q}_{\ell\ell})_1 I_{p1}+(\bar{Q}_{\ell\ell})_2 I_{p2})\partial^2\bar{\theta}_d(x)/\partial x^2=0, \tag{12}$$

$$\bar{\theta}_d(0,t)=0, \tag{13}$$

$$((\bar{Q}_{\ell\ell})_1 I_{p1}+(\bar{Q}_{\ell\ell})_2 I_{p2})\partial\bar{\theta}_d(l,t)/\partial x=N_1 V. \tag{14}$$

Here we define the error $\varepsilon(x,t)=\bar{\theta}_d(x)-\theta(x,t)$. Then $\varepsilon(x,t)$ satisfies:

$$((\bar{Q}_{\ell\ell})_1 I_{p1}+(\bar{Q}_{\ell\ell})_2 I_{p2})\partial^2\varepsilon/\partial x^2=(\rho_1 I_{p1}+\rho_2 I_{p2})\partial^2\varepsilon/\partial t^2, \tag{15}$$

with the boundary conditions

$$\varepsilon(0,t)=0, \tag{16}$$

$$((\bar{Q}_{\ell\ell})_1 I_{p1}+(\bar{Q}_{\ell\ell})_2 I_{p2})(\partial\theta_d/\partial x-\partial\varepsilon(l,t)/\partial x)=N_1 V. \tag{17}$$

To apply Lyapunov's direct method to this problem, consider the following functional:

$$e(t)=1/2\int_e^l(\rho_1 I_{p1}+\rho_2 I_{p2})(\partial\varepsilon/\partial t)^2dx$$

$$+1/2\int_e^l((\bar{Q}_{\ell\ell})_1 I_{p1}+(\bar{Q}_{\ell\ell})_2 I_{p2})(\partial\varepsilon/\partial x)^2dx. \tag{18}$$

This functional can be thought of as a measure of how far the error is from its equilibrium position or as a measure of the energy in the error system. Minimizing the time derivative of this functional is then equivalent to trying to bring the error system to equilibrium as fast as possible or removing as much energy possible from the system at each point in time. Differentiating E(t) with respect to t and integrating by parts lead to

$$\partial e(t)/\partial t=\int_e^l(\rho_1 I_{p1}+\rho_2 I_{p2})(\partial^2\varepsilon/\partial t^2)(\partial\varepsilon/\partial t)dx$$

$$+\int_e^l((\bar{Q}_{\ell\ell})_1 I_{p1}+(\bar{Q}_{\ell\ell})_2 I_{p2})(\partial\varepsilon/\partial x)(\partial^2\varepsilon/\partial x\partial t)dx \tag{19}$$

which, in view of Eqs. (15)-(17), immediately reduces to

$$\partial e(t)/\partial t=-K((\partial\varepsilon/\partial t)|_{x=l})^2\le 0. \tag{20}$$

Therefore, desired feedback control for enhancing stability is

$$\delta V(t)=V_d-V(t)=-K(\partial\varepsilon/\partial t)|_{x=l})/N_1, \tag{21}$$

or

$$V(t)=V_d+K(\partial\varepsilon/\partial t)|_{x=l})/N_1. \tag{22}$$

Equation for ε of feedback system is

$$((\bar{Q}_{\ell\ell})_1 I_{p1}+(\bar{Q}_{\ell\ell})_2 I_{p2})\partial^2\varepsilon/\partial x^2=(\rho_1 I_{p1}+\rho_2 I_{p2})\partial^2\varepsilon/\partial t^2, \tag{23}$$

with the boundary conditions

$$\varepsilon(0,t)=0, \tag{24}$$

$$\partial\varepsilon(l,t)/\partial x=-K(\partial\varepsilon/\partial t)|_{x=l})/((\bar{Q}_{\ell\ell})_1 I_{p1}+(\bar{Q}_{\ell\ell})_2 I_{p2}). \tag{25}$$

Taking the Laplace transform of eqs. (23)-(25) with respect to t gives

$$d^2\varepsilon/dx^2-\beta^2\varepsilon=0, \tag{26}$$

$$\varepsilon(0,s)=0, \tag{27}$$

$$d\varepsilon(l,s)/dx=-Ks\varepsilon(l,s)/((\bar{Q}_{\ell\ell})_1 I_{p1}+(\bar{Q}_{\ell\ell})_2 I_{p2}), \tag{28}$$

where

$\beta^2=(\rho_1 I_{p1}+\rho_2 I_{p2})s^2/((\bar{Q}_{\ell\ell})_1 I_{p1}+(\bar{Q}_{\ell\ell})_2 I_{p2})$.

A general solution of equation (26) is

$$\varepsilon(x,s)=A\cosh\beta x+B\sinh\beta x \tag{29}$$

where A and B are unknown constants determined from the boundary conditions. Substitution of eq. (29) into eqs. (27)-(28) leads to

$$A\{\beta\cosh\beta l+Ks\sinh\beta l/((\bar{Q}_{\ell\ell})_1 I_{p1}+(\bar{Q}_{\ell\ell})_2 I_{p2})\}=0. \tag{30}$$

The characteristic equation is thus given by

$$\beta+Ks\tanh\beta l/((\bar{Q}_{\ell\ell})_1 I_{p1}+(\bar{Q}_{\ell\ell})_2 I_{p2})=0. \tag{31}$$

4. Numerical Results and Discussion

In what follows, some numerical results are presented for a typical piezopolymer flexible micro-actuator undergoing twisting motions. The micro-actuator is assumed to be a composite with parameter values $E_1=2.0\times10^9[N/m^2]$, $E_2=76.9\times10^9[N/m^2]$, $\rho_1=1.78\times10^3[kg/m^3]$, $\rho_2=2.84\times10^3[kg/m^3]$, $\nu_1=1/3$, $\nu_2=1/3$, $h_1=0.0001[m]$, $h_2=0.0001[m]$, $b=$

$0.0003[m]$, $l=0.01[m]$ and $d_{31}=2.3\times10^{-11}[c/N]$. We consider the case with structural damping factor $\delta=2.73\times10^{-3}$ [6,7]. Here, the effects of air drag and etc., are not included in the model. Figure 2 and 3 show the step response for the error system. Figure 2 is the result with the feedback gain $K=100.0$ and Figure 3 that with the feedback gain $K=1.0$. It is seen that the error decreases monotonically with a time. It becomes zero at $0.04[seconds]$ when $K=100.0$. After then the steady-state error is zero. The error becomes zero at $0.025[seconds]$ when $K=1.0$. Figure 4 and 5 illustrate the output response together with the unit step input. Figure 4 is the result for $K=100.0$ and Figure 5 that for $K=1.0$. Notice that the output responses exhibit no overshoots. However, when $K=100.0$ the output response takes longer to reach its final value than when $K=1.0$. This response is described as being overdamped. Micro-Actuator are becoming smaller in size. Comparing with the inertia term, the stiffness term, and the damping term, the effect of the damping factor affects the most in this case.

5. Conclusions

A method for the closedloop control of the torsional tip motion of a piezo-polymer actuator is presented. The application of Lyapunov's direct method to the problem is explored. A feedback control of the torsional motion of the piezopolymer actuator is derived by considering the time rate of change of the total energy of the system. This approach has the advantage over the conventional methods in the respect that it allows one to deal directly with the system's partial differential equations without resorting to approximations. Since no modes were truncated in the analysis, this control law will theoretically control all of the modes of torsional vibration. This avoids any structural problems with uncontrolled modes. The simulation results verify that the proposed control law is effective at controlling twisting motion of the actuator. The approach outlined here has been proved to be useful also for the bending type piezopolymer flexible micro-actuator[8].

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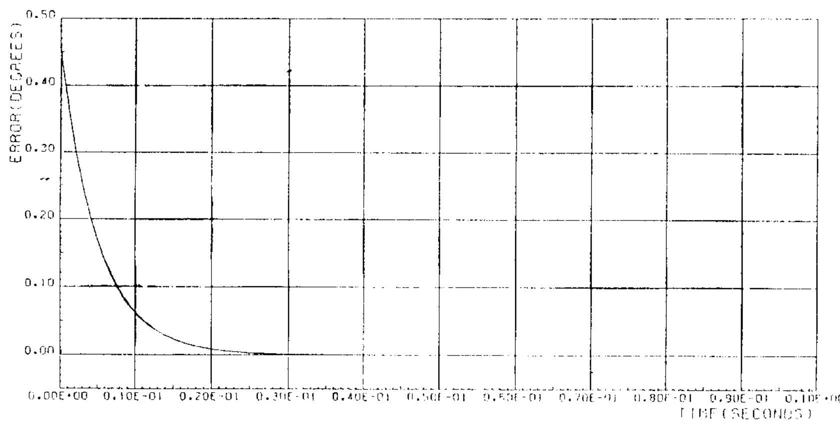

Figure 2. Transient response of a error system ($K=100.0$)

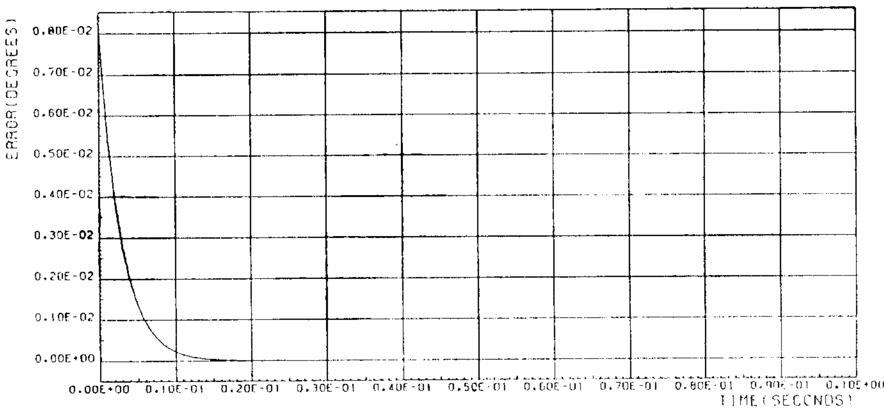

Figure 3. Transient response of a error system (*K=1.0*)

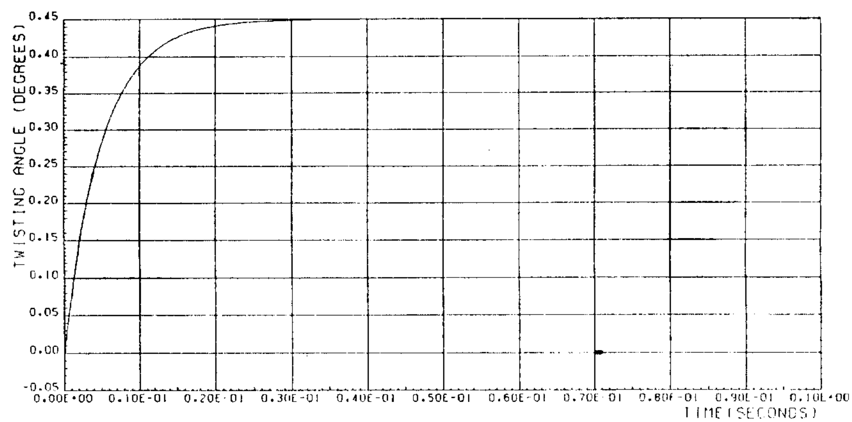

Figure 4. Transient response of the tip twisting angle (*K=100.0*)

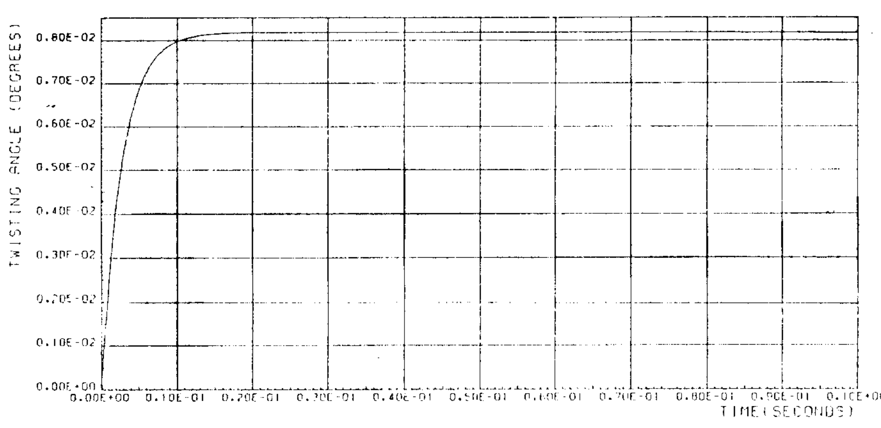

Figure 5. Transient response of the tip twisting angle (*K=1.0*)