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Relationship between Motion Speed and Working Accuracy of Industrial Articulated Robot Arms

Satoru Goto*, Masatoshi Nakamura*, Nobuhiro Kyura**
* Department of Electrical Engineering, Saga University, Honjomachi, Saga 840, JAPAN
** Yaskawa Electric, Co., Ltd., Mechatro Laboratory,
480 Kamifujisawa, Iruma, Saitama 358, JAPAN

Abstract

This paper described a relationship between motion speed and working accuracy of industrial articulated robot arms. Working accuracy of the robot arm deteriorates at high speed operation caused by a nonlinear transformation of the kinematics and the time delay of the robot arm dynamics. The deterioration of the following trajectory was expressed as a linear function of the squares of the robot arm motion speed, depending upon a posture of the robot arm and division interval of the objective trajectory.

1. Introduction

The contour control of robot arms is widely implemented in production lines and assembly lines as for cutting, grinding, sealing and welding work. The articulated structure of the robot arms has a high nonlinearity in the transformation between the working coordinates and the joint coordinates. There are many investigations of control for the articulated robot arms such as adaptive control[1], robust control[2], repetitive control[3] and neural control[4]. In industrial robot arms, the actuators of the servo motors are controlled by servo controller, independently. Performances of both high speed and high accuracy are required for the industrial robot arms. However, we experienced that the following trajectory could coincide with an objective trajectory only at slow speed operation of the robot arm. Generally speaking, the higher the speed of the robot arm is required, the less accurate the control performance is obtained. The error between the objective trajectory and the following trajectory of the robot arms is a big problem because the error defects in the working accuracy, directly. Hence, we previously had investigated the deterioration of contour control performance by using computer simulations and also proposed a compensation method for deteriorated control performance[5]. We had found the causes of the deterioration in the nonlinear transformation of the kinematics and the delay of the robot arm dynamics. The previous work however had been done under a limited situation in a continuous domain.

In this paper, we further investigated the previous problems and analyzed the deteriorated control performance of contour control theoretically. We paid attention to the evaluation of the error between the objective trajectory and the following trajectory of the robot arms, which depends on the motion speed, the dynamics, the kinematics, the posture of the robot arm and the division interval of the objective trajectory. We mainly focused on the following three points: posture of the robot arm, division interval of the objective trajectory based on simulation studies, and theoretical derivation of the deteriorated performance. By using the derived relationship between the robot arm motion speed and the working accuracy, we can obtain a suitable motion speed of the robot arm, which accomplishes the desired working accuracy.

2. Problem Statement

2.1 Deteriorated result of an industrial robot arm motion

Figure 1 shows the contour control result of the industrial articulated robot arm (Motoman K6S). The objective trajectory was straight line and its velocity was 1[m/s]. As shown in Fig. 1, the following trajectory slightly deteriorates from the objective trajectory. The deterioration of the trajectory is a big problem because this defects the working accuracy, directly. We focus on the deterioration of the following trajectory when the objective trajectory is the straight line, because the straight line is the most important in industrial applications. We shall investigate the cause of the deterioration and obtain a relationship between the velocity of the motion and the deterioration.

2.2 Contour control strategy

The block diagram of Fig. 2 shows the contour control structure of an industrial articulated robot arm. In or-

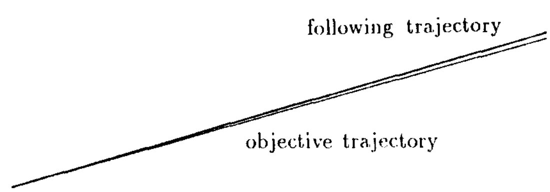


Figure 1 Deteriorated contour control result of industrial articulated robot arm

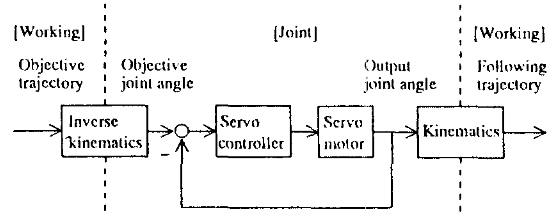


Figure 2 Contour Control of industrial articulated robot arm

der to decrease the effect of the load inertia change in the industrial robot arm, the reduction gear ratio is designed large and the parallelogram mechanism is adopted. Hence, the servo motor of the actuators of the robot arm is controlled independently for each axis[6]. The contour control of the industrial robot arms is usually implemented as follows.

(i) An objective trajectory of the robot arm motion is given in the working coordinates.

(ii) The objective trajectory of the robot arm in the working coordinates is transformed into the objective joint angle in the joint coordinates at each division interval by using the inverse kinematics.

(iii) The actuators of the servo motors are controlled to pursue the objective trajectory based on a linear dynamics in the joint coordinates.

(iv) The output joint angle of the robot arm in the joint coordinates has a time delay which is caused by the dynamics of the servo motor.

(v) The following trajectory of the robot arm in the working coordinates is given by the coordinate transformation of the delayed output joint angle.

3. Mathematical Model of Industrial Articulated Robot Arms

3.1 Kinematics of the articulated robot arm

We focus on the articulated robot arm of two-degree-of-freedom as shown in Fig. 3. In Fig. 3, Tip(x, y) means the position of a tip of the robot arm in the working coordinates and (α, β) means that in the joint coordinates. The length of the link 1 and link 2 are l_1 and l_2, respectively.

The input trajectory $(u_x(t), u_y(t))$ in the working coordinates is transformed into the input trajectory $(u_\alpha(t), u_\beta(t))$ in the joint coordinates as

$$\begin{aligned} u_\alpha(t) &= \sin^{-1}\left(\frac{u_y(t)}{\sqrt{u_x^2(t)+u_y^2(t)}}\right) \\ &\quad - \sin^{-1}\left(\frac{l_2 \sin u_\beta(t)}{\sqrt{u_x^2(t)+u_y^2(t)}}\right) \\ u_\beta(t) &= \pm\cos^{-1}\left(\frac{u_x^2(t)+u_y^2(t)-l_1^2-l_2^2}{2l_1 l_2}\right) \end{aligned} \quad (1)$$

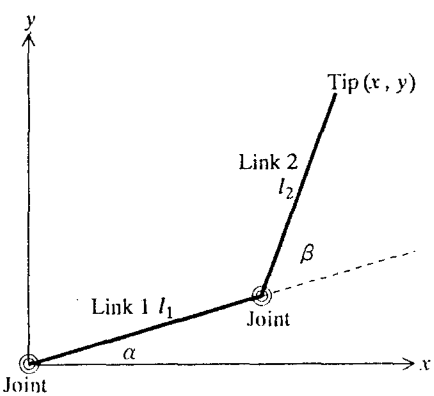


Figure 3 Articulated robot arm of two-degree-of-freedom

We can calculate the input trajectory $(u_\alpha(t), u_\beta(t))$ in the joint coordinates for the input trajectory $(u_x(t), u_y(t))$ in the working coordinates.

3.2 Dynamics of the articulated robot arm

In industrial robot arm, if the motion speed of the robot arm is under 1.0[m/s], the nonlinear term like gravity and Coriolis, and the interference term are neglected,and the dynamics of the servo motors and these controllers is represented by the first order model[7]. Hence, the robot arm dynamics in the joint coordinates is described by the first order model as

$$\begin{aligned} \dot\alpha(t) &= -K_p\alpha(t) + K_p u_\alpha(t) \\ \dot\beta(t) &= -K_p\beta(t) + K_p u_\beta(t), \end{aligned} \quad (2)$$

where $(\alpha(t), \beta(t))$ means the following trajectory in the joint coordinates and K_p means the position loop gain. Figure 4 shows the block diagram of the robot arm dynamics in the joint coordinates. The following trajectory in the joint coordinates deteriorates from the objective trajectory.

3.3 Following trajectory

To derive the following trajectory in the joint coordinates, we solve (2) for $(\alpha(t), \beta(t))$ as

$$\begin{aligned} \alpha(t) &= \alpha(0)e^{-K_p t} + \int_0^t u_\alpha(\tau)e^{K_p(\tau-t)}d\tau \\ \beta(t) &= \beta(0)e^{-K_p t} + \int_0^t u_\beta(\tau)e^{K_p(\tau-t)}d\tau, \end{aligned} \quad (3)$$

where $(\alpha(0), \beta(0))$ means the initial value of the position of the robot arm in the joint coordinates.

The following trajectory $(\alpha(t), \beta(t))$ in the joint coordinates is transformed into the following trajectory $(x(t), y(t))$ in the working coordinates by using the kinematics as

$$\begin{aligned} x(t) &= l_1\cos\alpha(t) + l_2\cos(\alpha(t)+\beta(t)) \\ y(t) &= l_1\sin\alpha(t) + l_2\sin(\alpha(t)+\beta(t)). \end{aligned} \quad (4)$$

Consequently, the following trajectory $(x(t), y(t))$ is calculated by using (1), (3), (4) for given input trajectory $(u_x(t), u_y(t))$.

4. Computer Simulation Studies of the Deteriorated Trajectory

4.1 Objective velocity

We investigate the following trajectory errors for some input trajectories based on computer simulation studies. The link length of the simulated robot arm is l_1 = 700[mm], l_2 = 900[mm] and the objective path is the straight line from the start point (−500[mm],500[mm])

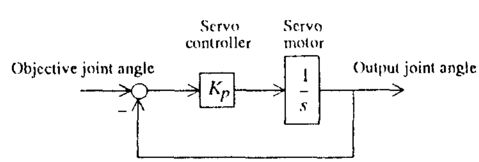


Figure 4 Block diagram of robot arm dynamics

to the end point (500[mm],500[mm]) as shown in Fig. 5. Division interval of the objective trajectory $\Delta s = 1$[ms].

The following trajectory depends on both input trajectory and the robot arm dynamics (2). Then, we introduce relative velocity which depends on the dynamics characteristics. The change of the input velocity is equivalent to that of the time scale. If a time scale changes from t [s] to μt [s], then the position loop gain K_p [1/s] in (2) changes to μK_p [1/s]. Hence, we define the relative velocity V/K_p [m], which is invariant to the characteristics of the robot arm dynamics (2), instead of the absolute velocity V [m/s].

The computer simulation results under V/K_p = 0.1, 0.05, 0.02, 0.01 [m] are shown in Fig. 5. It is noted that the scale of Y-axis is much enlarged compared with the X-axis. The results show that the trajectory error depends on the relative velocity and the higher the relative velocity of the robot arm is required, the less accurate the control performance is obtained.

The locus error, the deviation of the following path from the objective path, is an important measure for the working precision of the contour control of the industrial robot arm. In Fig. 6, we plot the maximum locus error during an operation versus the squared relative velocity $(V/K_p)^2$ for simulation studies of 100 operations and approximate the points by a linear function $y = 0.964x$ based on the least squares error method. According to the computer simulation results, the maximum locus error E_p is in proportion to the squared relative velocity V/K_p as

$$E \approx 0.964 \left(\frac{V}{K_p}\right)^2 \qquad (5)$$

The experimental relationship is proved theoretically in chapter 5.

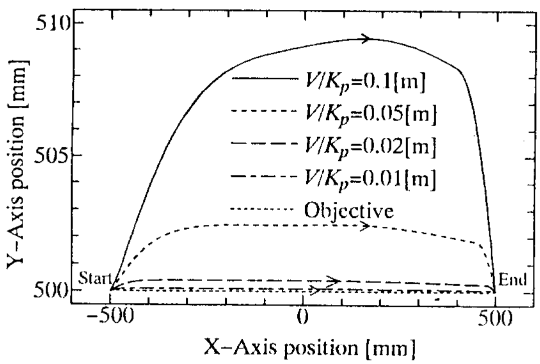


Figure 5 Trajectory error for different relative velocities V/K_p

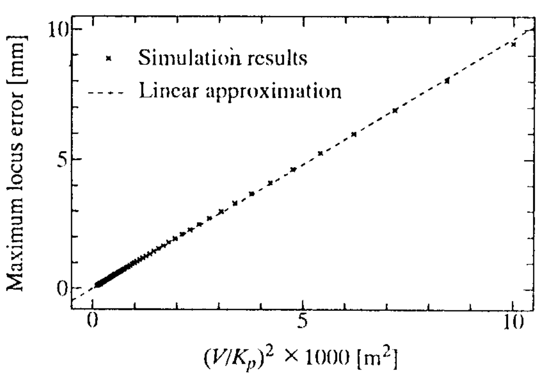


Figure 6 Linearity between the maximum locus error and the relative velocity V/K_p based on computer simulation studies

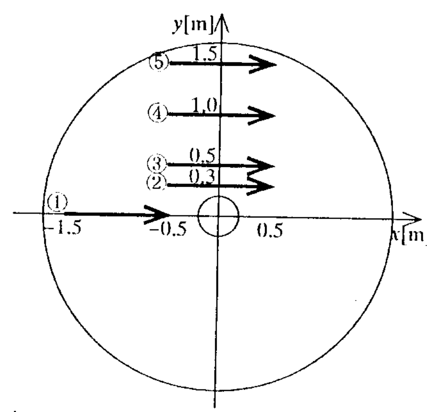
4.2 Posture of robot arms

Next, we investigated the contour control performance dependency on a posture of the robot arm. Several positions of the objective trajectories in the working coordinates are shown in Fig. 7(a). The computer simulation results for the objective velocity being V = 1.0 [m/s] are shown in Fig. 7(b). The locus errors illustrate a dependency on the posture of the robot arm. According to the computer simulation results, the locus error becomes larger when the position of the objective trajectory is closer to the origin of the working coordinate axes.

4.3 Division interval of objective trajectory

In the industrial articulated robot arm, the objective trajectory $(u_x(t), u_y(t))$ is transformed into the joint coordinates at each division interval. Then the actuators of the servo motors are controlled to pursue the divided objective trajectory in the joint coordinates. We investigated the contour control performance dependency on the division interval of the objective trajectory based on

(a) The position of the objective trajectory



(b) Trajectory error

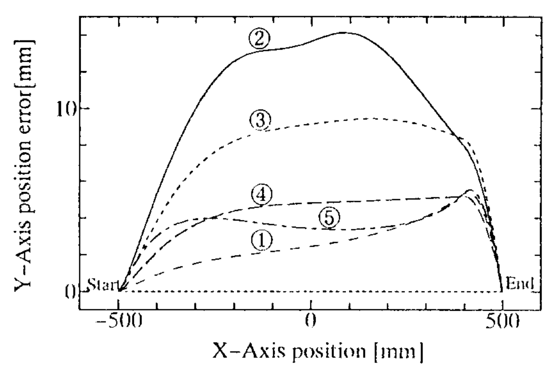


Figure 7 Trajectory error for different postures of the robot arm

computer simulation studies. The simulation conditions were the same as in section 4.1. The division intervals were $\Delta s = 1, 10, 20, 50, 100$[ms], which meant that the division intervals of the objective trajectory were varied at $V\Delta s = 1, 10, 20, 50, 100$[mm] because of the objective velocity being $V = 1$[m/s]. The computer simulation results, depicted in Fig. 8, showed that the division interval Δs is permissible within 20[ms] ($V\Delta s \leq 20$[mm]) because oscillation of the following trajectory is invisible.

5. Theoretical Derivation of Velocity Dependency

We shall derive the relationship (5) between the maximum locus error E_p and the relative velocity V/K_p theoretically in this chapter.

We introduce following assumptions as follows:

(i) Objective velocity V in the working coordinates is approximately in proportion to the angular velocity ω in the joint coordinates.

(ii) The deterioration of the amplitude of the frequency transfer function in the joint coordinates is transformed into the deterioration of the maximum locus error in the working coordinates.

(iii) The maximum locus error is a steady state error.

The transfer function of the robot arm dynamics (2) is calculated by the Laplace transformation as

$$G(s) = \frac{K_p}{s + K_p}. \tag{6}$$

Then, the frequency transfer function is obtained by substituting $s = j\omega$ into (6) as

$$G(j\omega) = \frac{K_p}{j\omega + K_p}. \tag{7}$$

The gain of (7) is calculated as

$$|G(j\omega)| = \frac{1}{\sqrt{1 + (\omega/K_p)^2}} \tag{8}$$

and the amplitude of a unit sinusoidal input for angular frequency ω is reduced as

$$1 - |G(j\omega)| = 1 - \frac{1}{\sqrt{1 + (\omega/K_p)^2}} \approx \frac{1}{2}\left(\frac{\omega}{K_p}\right)^2 \tag{9}$$

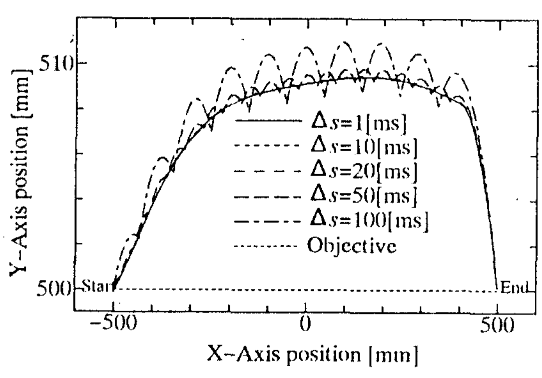


Figure 8 Trajectory error for different division intervals of objective trajectory

By substituting the assumption (i) $V = R\omega$, (R: curvature radius) into equation (9), the maximum locus error E_p is derived

$$E_p \propto \left(\frac{V}{K_p}\right)^2 \tag{10}$$

The equation (10) is equivalent to the relationship (5).

To achieve the desired accuracy of the following trajectory, the experiment of arbitrary objective velocity is carried out and measure the maximum locus error. Then, the suitable objective velocity is calculated from the relationship (10) such that the maximum locus error is the desired one.

6. Conclusions

We investigated the deteriorated performance of the following trajectory of the robot arm caused by the nonlinear transformation and the time delay based on both simulation studies and theoretical studies. The deterioration of the following trajectory was proved to be in proportion to the square of the input velocity. The derived relationship can be use effectively in determining appropriate motion speed so that is achieved a desired accuracy in the following trajectory.

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