'93 KACC (1993.10.20~22)

Optimal Control Approach to Resolve the Redundancy of Robot Manipulators

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Abstract

Most of the control problems for the redundant manipulators use the pseudo-inverse control, that is, the redundancy is resolved by the pseudo-inverse of the Jacobian matrix and then the controller is designed based on this resolution. However, this pseudo-inverse control has some problems when the redundant robot repeats the cyclic tasks. This is because the pseudo-inverse resolution is a local solution that generates the different configurations of the robot arm for the same hand position. Therefore it is necessary to find the global solution that maintains the optimal configuration of the robot for the repetitive tasks.

In this paper, we want to propose a redundancy resolution method by the optimal theory that uses the calculus of variation. The problem formulations are: first to convert the optimal resolution problem to an optimal control problem and then to resolve the redundancy using the necessary conditions of optimal control.

I. Introduction

The redundant manipulators which have more degree-of-freedoms than they need for the specified tasks are the keys of dexterity. Due to the extra degree-of-freedoms, they do perform the multi-purpose tasks, For example, they can track the desired trajectory while avoiding the obstacles.

In general, the kinematics and the inverse kinematics of the robots are necessary to control them. For the redundant manipulators this inverse kinematics problems become very complicated so that a lot of researches on the redundant robots concentrated on resolving the inverse kinematics. Almost all of the past researches on resolution of inverse kinematics have used the pseudo-inverse of the Jacobian matrix and this pseudo-inverse resolution has succeeded in many applications. However for the cyclic tasks, this resolution does not have the closed results, that is, it generates the different configurations for the same end-effector position and orientation. This property is because the pseudo-inverse resolution is a kind of local solution of the inverse kinematics [4].

Therefore it is necessary to find the global solution that maintains the optimal configuration of the robot for the repetitive tasks. Some research efforts have been directed for the global redundancy resolution of the robot manipulators. For example, Nakamura and Hanafusa [9],[10] used Pontryagin's maximum principle, and Kazerounian and Wang [2], Martin, Baillieul and Hollerbach [8], and Suh and Hollerbach [11] resolved the redundancy by using the calculus of variation. Won, Choi and Chung [12],[13] also investigated the optimal resolution using the variation approach.

In this paper, we deal the optimal resolution problem as an optimal control problem as proposed by Nakamura and Hanafusa [10]. The difference between ours and their approach is that they used Pontryagin's maximum principle but we do not use this principle. As known generally, Pontryagin's principle is the extension of the calculus of variation so that it can be applied when the admissible control input is restricted. However in their problem formulation, there are no constraints for the input. In other words, this problem can be solved by the conventional variation approach.

Thus the approach in this paper is the one using the calculus of variation for the optimal control problems. The problem formulations are: first to convert the optimal resolution problem to an optimal control problem and to resolve the redundancy using the necessary conditions of optimal control.

As listed in the past researches, the variation approach has also done in some papers, but they have not used the optimal control technique. One more points to be noted is that their resolution is acceleration level, defined by the second order differential equation, but ours is resolved in velocity level and defined by two first order differential equations. Since we use the necessary conditions of the optimal control problem, this redundancy resolution problem becomes two ordinary differential equations that the boundary values are given at the each end of trajectory. The effect of boundary conditions for this resolution is to be considered.

In the simulation, it can be verified that the cyclic motion becomes conservative and the drift-away of the motion is avoided if we use the proposed redundancy resolving method.

II. Formulations for Optimal Redundancy Resolution

In this section, we briefly review the past formulations and solutions, and we propose a new formation that considers the optimal resolution as an optimal control problem.

A. Variation Approaches

Many researchers have directed their efforts to solve the optimal resolution problem by the calculus of variation [2],[8],[10],[11],[12],[13]. Their formulation first begin by defining the performance index

$$\int_{t_0}^{t_f} G(q,\dot{q},t)dt \tag{1}$$

subject to the kinematic constraints,

$$x(t) = f(q(t)). \tag{2}$$

The function $G(q,\dot{q},t)$ might be selected differently according to the designer's needs, except to note that one reasonable choice in such a problem is

$$G(q,\dot{q},t) = \frac{1}{2}\dot{q}'W(q,t)\dot{q} + p(q) \tag{3}$$

where $W(q,t)$ is a nonsingular symmetric matrix whose elements reflect the relative weight of each joint axis and the function $p(q)$ denotes the configuration of the redundant robot, for example, the *manipulability measure* [11] or the distance to some obstacles.

The necessary conditions for optimal trajectories that minimize the integral criterion are now developed. First the following Lagrangian is introduced.

$$L(q,\dot{q},\lambda,t) = G(q,\dot{q},t) + \lambda'(x(t) - f(q)). \tag{4}$$

Then the necessary conditions for optimality are given by the Euler-Lagrange equations

$$\frac{\partial L}{\partial q} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} = 0, \tag{5}$$

and the kinematic constraints are

$$x(t) - f(q(t)) = 0. \tag{6}$$

After some mathematical manipulation [8], the resolution becomes

$$\ddot{q} = J_W^+(\ddot{x} - \dot{J}\dot{q}) + (I - J_W^+ J)W^{-1}[(-\dot{W} + \tfrac{1}{2}B)\dot{q} + \partial p/\partial q] \tag{7}$$

where J_W^+ is the weighted pseudo inverse of the Jacobian matrix $J(q) = \partial f/\partial q$, given by

$$J_W^+ = W^{-1}J'(JW^{-1}J')^{-1} \tag{8}$$

for nonsingular $JW^{-1}J'$

Using the resolution of (7), the redundancy is resolved at the acceleration level.

B. Approach using Pontryagin's Minimum Principle

Nakamura and Hanafusa used Pontryagin's maximum principle [9],[10]. To formulate the optimal control problem, differentiation of (2) with respect to time generates the following linear equation is derived:

$$\dot{x}(t) = J(q)\dot{q}(t). \tag{9}$$

Suppose that $\dot{q}(t)$ satisfying (9) exists, then (3) can be written by

$$\dot{q} = J^+\dot{x} + (I - J^+J)u \tag{10}$$

where J^+ is the pseudo inverse of Jacobian matrix and u is n-dimensional arbitrary vector.

Equation (10) can be regarded as a dynamic system by considering q as a state vector and u as an input vector with minimizing the following cost function

$$\dot{q} = J^+\dot{x} + (I - J^+J)u \equiv g(q,u,t) \tag{11a}$$

$$\int_{t_0}^{t_f} G_1(q,\dot{q},t)dt \tag{11b}$$

where $G_1(q,\dot{q},t) = \frac{1}{2}\dot{q}'\dot{q} + p(q)$. Thus equation (11) consists an ordinary optimal control problem so that Pontryagin's principle can be applied.

According to the Pontryagin's principle, define the Hamiltonian as follows:

$$H(q,\dot{q},\lambda,t) = G_1(q,\dot{q},t) + \lambda' g(q,u,t). \tag{12}$$

For the function $G_1(q,\dot{q},t)$, the Hamiltonian becomes

$$\begin{aligned} H &= \tfrac{1}{2}\dot{q}'\dot{q} + p(q) + \lambda' g(q,u,t) \\ &= \tfrac{1}{2}(g+\lambda)'(g+\lambda) - \tfrac{1}{2}\lambda'\lambda + p(q) \end{aligned} \tag{13}$$

In (13), the second and third terms of the right-hand side have no relation to u, since u is included only in g. Therefore the input u minimizing the first term of (13) also minimizes the Hamiltonian. Such u is as follows:

$$u = -(I - J^+J)^+[J^+\dot{x} + \lambda] = -(I - J^+J)\lambda \tag{14}$$

where the properties of pseudo inverse $(I - J^+J)^+ = I - J^+J$ and $(I - J^+J)^+ J^+ = 0$ are used.

Thus the optimal trajectory $q(t)$ for the cost function of (11b) is governed by the following two differential equations.

$$\dot{q} = g(q,u,t) \tag{15}$$

$$\dot{\lambda} = -(\partial g/\partial q)'(g + \lambda) - (\partial p/\partial q)' \tag{16}$$

*C. Optimal Control Approach -- **Proposed***

We propose an optimal control approach for the optimal redundancy resolution. The problem formulation is same as in the Section 2-B. That is, the systems (11), and the Hamiltonian (12) is considered, but we do not use the Pontryagin's principle. Alternately the optimal trajectory is governed by the following three necessary conditions.

$$\dot{q} = \frac{\partial H}{\partial \lambda} \tag{17a}$$

$$\dot{\lambda} = -\frac{\partial H}{\partial q} \tag{17b}$$

$$\frac{\partial H}{\partial u} = 0. \tag{17c}$$

The first and second condition of (17) yield (15) and (16), respectively. Since the third equation can be written as follows

$$\begin{aligned} \frac{\partial H}{\partial u} &= \left(\frac{\partial g}{\partial u}\right)(g + \lambda) \\ &= (I - J^+J)'\left[J^+\dot{x} + (I - J^+J)u + \lambda\right] = 0 \end{aligned} \tag{18}$$

the optimal u minimizing the Hamiltonian becomes

$$u = -(I - J^+J)^+(I - J^+J)\lambda = -(I - J^+J)\lambda. \tag{19}$$

Comparing (19) with (14), we can see that the proposed approach yields the same results as those of [10]. This is due to the fact that Pontryagin's minimum principle generates the same result as the ordinary optimal control problem when there exist no constraints on the control input or the state values.

III. Numerical Example

We consider a planar 3-link redundant manipulator [Fig. 1]. The length of each link is $\ell_1 = 0.5m, \ell_2 = 0.4m, \ell_3 = 0.3m$, respectively. Now we repeat the formulation in Section 2-C for this robot.

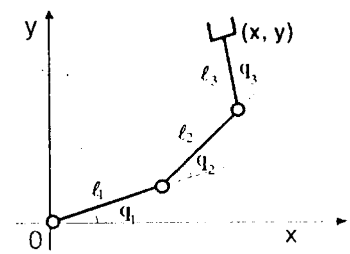


Fig. 1. A 3-dof redundant robot

A. Formulation for an Optimal Control Problem

The system and the cost function in (11) are rewritten here.

System:

$$\dot{q} = J^+\dot{x} + (I - J^+J)u \equiv g(q,u,t) \tag{20a}$$

Cost function:

$$\int_{t_0}^{t_f} \tfrac{1}{2}\dot{q}'\dot{q}\, dt \tag{20b}$$

At this stage, the configuration function $p(\cdot)$ in (3) of the robot arm is not considered. The above cost function is related to the joint kinetic energy.

Then the Hamiltonian becomes:

$$H = \tfrac{1}{2}\dot{q}'\dot{q} + \lambda' g(q,u,t) = \tfrac{1}{2}g'g + \lambda' g \tag{21}$$

and the necessary conditions are

$$\dot{q} = g(q,u,t) \tag{22a}$$

$$-\dot{\lambda} = \left(\frac{\partial g}{\partial q}\right)(\lambda + g) \tag{22b}$$

$$u = -(I - J^+J)\lambda \tag{22c}$$

To solve the above nonlinear ordinary differential equations, the boundary conditions must be given. For the robot manipulator to have the conservative motion, the initial and the final conditions are to be given and are equal, i.e.,

$$q(t_0) = q_0, \text{ and } q(t_f) = q_0 \tag{23}$$

but the conditions of λ are free at both ends so that

$$\lambda(t_0), \lambda(t_f) = \mathit{free}. \tag{24}$$

After inserting u in (22c) into (22a) and (22b), we get the following two nonlinear first order ordinary boundary value equation.

$$\dot{q} = g(q, -(I - J^+J)\lambda, t) \tag{25a}$$

$$-\dot{\lambda} = \left(\frac{\partial g}{\partial q}\right)' (\lambda + g(q, -(I - J^+J)\lambda, t)) \quad (26b)$$

The two equations in (25) are nonlinear so that it can not be solved by a closed-form solution. Therefore a numerical algorithm to solve the nonlinear ordinary boundary value differential equation must be considered.

B. Numerical Solution -- Steepest Descent Algorithm [6]

The procedure used to solve optimal control problems is the method of steepest descent which is

1. Select discrete approximations to the nominal control history $u^0(t), t \in [t_0, t_f]$, and store this in the memory of the digital computer. This can be done, for example, by subdividing the interval $[t_0, t_f]$ into N subintervals and considering the control $u^0(t)$ as being piecewise-continuous during each of these subintervals; i.e.,

   $$u^0(t) = u^0(t_k), \quad t \in [t_k, t_{k+1}), \quad k = 0, 1, \ldots, N-1$$

2. Using the nominal control history $u^i(t)$, integrate this system equation with $q(t_0) = q_0$, and store the resulting state trajectory $q^i(t_k)$.

   $$q^i(t_k) = \int_{t_0}^{t_k} [J^+\dot{x} + (I - J^+J)u^i(t)]dt$$

3. Calculate $\lambda^i(t_f)$ by substituting $q^i(t_f)$ from step 2 into (25b). Using this value of $\lambda^i(t_f)$ as the "initial condition" and the stored $q^i(t_k)$ in step 2, backward integrate (25b), evaluate $\partial H^i(t_k)/\partial u$ and store this function.

4. If $\left\|\frac{\partial H^i}{\partial u}\right\| \le \gamma$ where γ is a positive constant and

   $$\left\|\frac{\partial H^i}{\partial u}\right\|^2 = \int_{t_0}^{t_f} \left(\frac{\partial H^i}{\partial u}\right)' \left(\frac{\partial H^i}{\partial u}\right) dt$$

   terminate the iterative procedure, and output the state and control. If the stopping criterion is not satisfied, generate a new piecewise-constant control function given by

   $$u^{i+1}(t_k) = u^i(t_k) - \tau \frac{\partial H^i}{\partial u}(t_k)$$

   and return to step 2.

Remark: This steepest descent method must know the final value of $\lambda^i(t_f)$, so the modification is required for fixed end point problems. One way is to use the *penalty function approach*. For example, if the desired final state is denoted by q_f, we add a term to the performance index of the form

$$\tfrac{1}{2}[q(t_f) - q_f]' M[q(t_f) - q_f]$$

where M is a diagonal matrix with large positive elements, and treat $q(t_f)$ as if it were free. Doing this, we find that the boundary conditions become

$$\lambda_f = M[q(t_f) - q_f].$$

By this technique, fixed and free end point problems can be treated with the same computer program.

C. Simulation

A circular cyclic motions is considered.

Fig. 2(a) shows the drift-away of the robot when the redundancy is resolved using the minimum norm pseudo-inverse solution, that is

$$\dot{q} = J^+\dot{x}.$$

Since the null space of Jacobian matrix is not utilized, the motion is not conservative. Fig 2(b) shows the joint angle during the first cycle. It denotes that the initial joint values are not equal to the final values. These differences cause the drift motion.

Fig. 3(a) and 3(b) show the joint values and the corresponding control input u, respectively when the proposed global resolution is used. We see that the initial and final joint values are equal so that the motion is conservative.

Fig. 4 depicts the performance index evaluation of the cyclic motion.

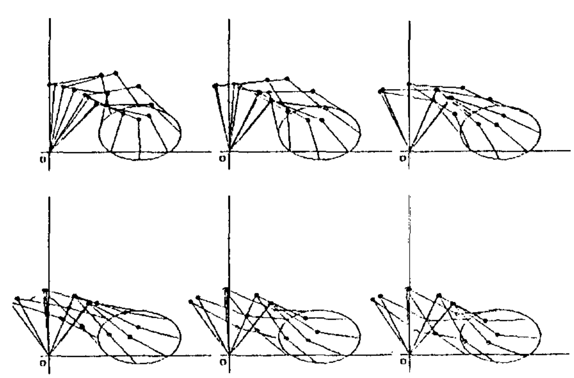


Fig. 2(a). Drift-away of the robot when the local redundancy resolution is used

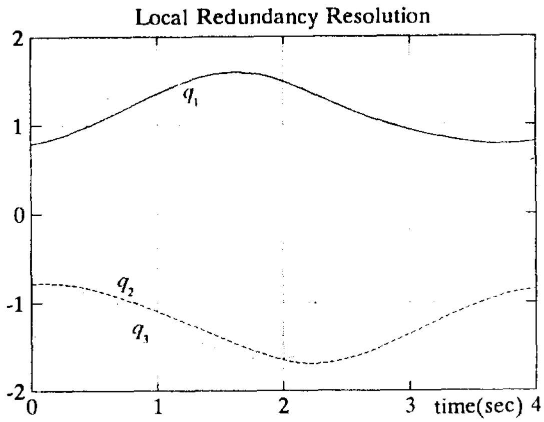


Fig. 2(b). Joint angle during the first cycle

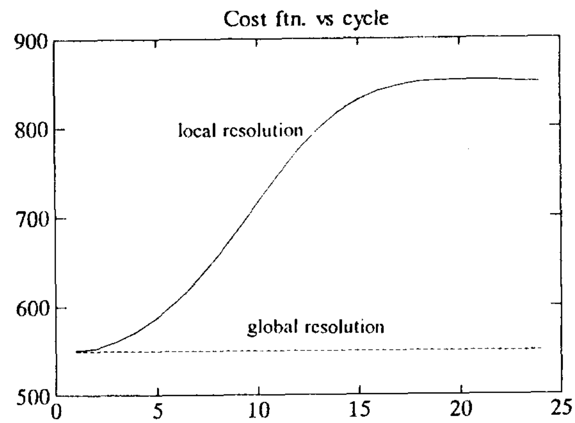
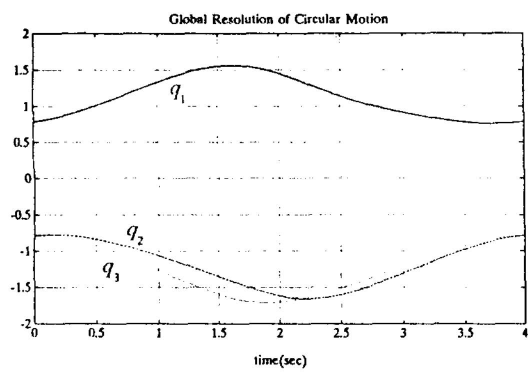
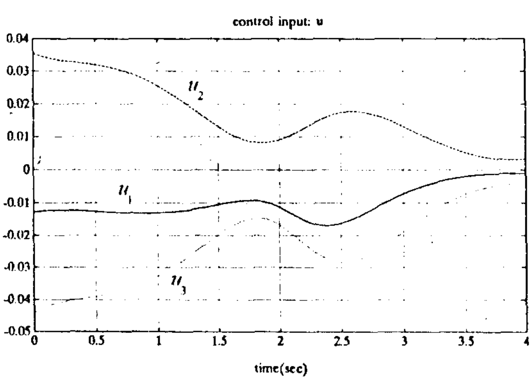


Fig. 4. Comparison of the evaluation of the performance index



(a) Joint angles



(b) Control inputs

Fig. 3. Global redundancy resolution during the first cycle

IV. Discussion

We obtained the global resolution in the previous sections, but there may be a question, "Why the local resolution is not the global?". The answer can be achieved by analyzing the cost function.

In the previous numerical example, the cost function was

$$\int_{t_0}^{t_f} \tfrac{1}{2}\dot{q}'\dot{q}\, dt \tag{26}$$

Expanding (26) by inserting (20a), we get

$$\int_{t_0}^{t_f} \tfrac{1}{2}\dot{q}'\dot{q}\, dt = \int_{t_0}^{t_f} \|\dot{q}\|^2\, dt$$

$$= \int_{t_0}^{t_f} \left(\|J^+\dot{x}\|^2 + 2\|(J^+\dot{x})'(I - J^+J)u\| + \|(I - J^+J)u\|^2\right)dt$$

$$= \int_{t_0}^{t_f} \left(\|J^+\dot{x}\|^2 + \|(I - J^+J)u\|^2\right)dt. \tag{27}$$

If we use the local resolution of the minimum norm, then $\dot{q} = J^+\dot{x}$ and the cost function becomes

$$\int_{t_0}^{t_f} \tfrac{1}{2}\dot{q}'\dot{q}\, dt = \int_{t_0}^{t_f} \|J^+\dot{x}\|^2\, dt. \tag{28}$$

One might think that (28) has smaller value of the cost function than (27), but that is not true. This is because J^+ is a function of q, and q is dependent on u. Therefore the value of cost function can be made smaller by proper manipulation of u. In other words, the *null space* of Jacobian matrix is utilized to minimize the cost function.

One more thing to discuss is that the good results shown as in the previous simulations are not guaranteed for all the possible cases, that is, for some robot arm configurations the numerical solutions of optimal resolution have never converged. Why? Have we done mistakes in simulation or does the optimal resolution not exist for each case?

The problem formulation shown in (10) for the optimal resolution is nonlinear with respect to the control input, u. In that stage, we didn't verify that there always exists a solution satisfying the optimal redundancy resolution and whether the solution is unique or not. I think this is closely related to the *nonlinear controllability*.

V. Conclusion and Further Study

We obtain a global resolution of redundancy of robot manipulator by the optimal control approach. The main idea is that the kinematic resolution problem is considered as an optimal control problem and by solving this optimal control problem, the redundancy is resolved. As a result, in cyclic motion, the kinematic drift of redundant manipulator is avoided and the conservative motion is achieved. This is because the null space of Jacobian matrix is utilized by the necessary conditions of the optimality.

From the necessary conditions of the optimal control problems, we get two nonlinear ordinary differential equations with the boundary values. Since these boundary value problems are nonlinear, they were solved numerically by the steepest descent method.

For further study, to reduce the computational burden, the constant optimal control law, u will be considered. The existence condition check for the optimal resolution could be a further study.

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