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Gain Scheduled Control of Magnetic Suspension System

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ABSTRACT

A gain scheduling approach for the suspension control of a nonlinear MAGLEV System is presented. We show that this technique is very useful for improving not only performance to the operational disturbances originating aerodynamic force but also robustness to the uncertainty of payload. As a scheduling variable, even though the external disturbance need to be estimated in real time, but the additive measurement is not required to do it. Some simulations show that the gain scheduling control system performs very well comparing with other methods using a nonlinear feedback linearization or a fixed gain linear feedback.

1. Introduction

Magnetically Levitated(MAGLEV)System to be considered is a D.C.electromagnetic suspension(EMS)system, which is a highly nonlinear dynamics and open-loop unstable. Different schemes of stabilization and control of single magnet levitation system(SMLS) have been widely studied.

A design method of state feedback control with pole placement[1,2,5] has been mainly applied on the basis of the first-order linear approximate model corresponding to only an operating point.

However this approach has some disadvantages that are difficult to compensate the design constraints due to nonlinearity subject to existing various disturbances.

Because the operating point depends on the change of the suspended mass and external disturbances, the dynamics of linear approximate model should be also varied.

Hence some caution is essential for stability and performance in detailed design.

In recent years, according to development of digital computer technology, nonlinear control schems for MAGLEV have been researched. Sinha[3] suggested a digital implementation of model reference adaptive controller for EMS and Jin et al.[4] proposed a SMLS controller using nonlinear feedback linearization. They make stability and performance of EMS systems improve considerably.

By the way, it is very significant problems that to what extend the controller designed for EMS has the stiffness to the change of mass and the performance against the practical disturbances such as aerodynamic force or movement of payload etc.

To increase the stiffness (that is, to compensate the steady-state error), make the loop gain to be large or an error integral feedback can be introduced in position control loop. But the integrator may give rise to aggravate the transient response.

From this point of view, this paper deals with a design method of gain scheduled controller for SMLS. The gain scheduling is a nonlinear feedback control of special type ; it has a linear regulator whose parameters are changed as a operating conditions. The model is usually arranged so that the operating condition is specified by the values of exogeneous signals or variables, so called "scheduling variable".

The main advantages of the gain scheduling is summarized;

(1) the wealth of linear control theories can be applied
(2) the modern methods of robust deisign for linear systems are available to counter uncertainty in the plant parameters.
(3) the regulator with gain schedulinghas the potential

to respond rapidly to changing operating conditions.

However, the major difficulty in gain scheduling design is to find the appropriate scheduling variables which must reflect the operating conditions.

This is normally done based on the phisics of a system and on the good insight. Besides since the gain scheduling is inherently local, the overall performance and the stability are typically evaluated by simulation studies.

While the gain scheduling approach is used widely in practice[6], it seldom appear in the literature. In this situation the recent work by Rugh[7,8] is remarkable for the gain scheduling procedure.

Some simulations show that the proposed gain scheduling controller for SMLS has much better performance than other two methods using a nonlinear feedback linearization and a fixed-gain linear feedback.

2. Gain Scheduled Control for SMLS

2.1 Modeling of a SMLS

The modelling of a SMLS is used for reasons that in control system design for multi-magnet system, it is possible to express each degree of freedom as an equivalent sigle-magnet system, and that the response of system to external disturbances can be studied using the SMLS.

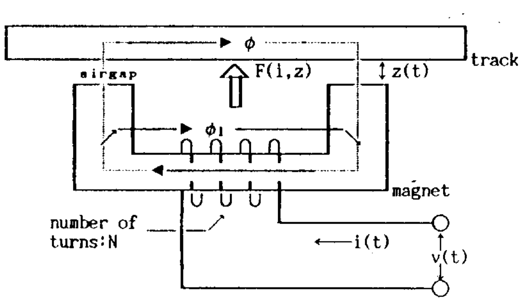

Fig. 1. single magnet levitation system.

For the SMLS shown in Fig.1, the vertical motion is described by the following nonlinear dynamic equation[2];

$$\ddot{z}(t) = -\frac{c}{4m}\left[\frac{i(t)}{z(t)}\right]^2 + \frac{1}{m} f_d(t) + g \qquad (1)$$

$$\dot{i}(t) = \frac{\dot{z}(t)i(t)}{z(t)} + \frac{2z(t)}{c}\left[v(t) - Ri(t) \right] \qquad (2)$$

where, $z(t)$: gap distance between magnet and track

$i(t)$: current fo electromagnet

$f_d(t)$: disturbance input

$v(t)$: applied voltage to the magnet

R : total resistance of electromagnet circuit

g : gravity

c : constant ($c = \mu_0 N^2 A$), permeability μ_0, number of turns of coil N, section area of core A.

For simplicity we write eq.(1) and (2) as a nonlinear state equation ;

$$\dot{x}(t) = f[x(t), u(t), f_d(t)] \qquad (3)$$

$$y(t) = x_1(t)$$

where, $x^T(t) = [x_1(t)\ x_2(t)\ x_3(t)] = [z(t)\ \dot{z}(t)\ i(t)]$

$u(t) = v(t)$

Obviously the function f is smooth except for $x_1 = 0$.

2.2 Design of Gain Scheduling Controller

The overview of gain scheduling approach is as follows : the first step is to linearize the nonlinear model about several operating points. Then linear design methods are applied to the linear approximate model at each operating point so that the closed-loop system perform satisfactorily when operated near the individual operating condition. The final step is the gain scheduling in which the parameters(gains) of linear control law are changed according to the monitored operating condition. First, the disturbance $f_d(t)$ is assumed to be bounded.

$$\mathcal{F} := \{\ f_d(t)\ |\ f_d(t) \in [f_d^-\ ,\ f_d^+],\ t \geq 0\ \} \qquad (4)$$

where, $\mathcal{F}$ is open neighborhood of the origin in $\mathbb{R}$.

At each constant disturbance, $f_d(t) = \overline{f_d}$, the balance equation of dynamic model for an operating point (z_o, i_o) becomes

$$F(i_o, z_o) = \frac{c}{4}\left[\frac{i_o}{z_o}\right]^2 = mg + \overline{f_d} \qquad (5)$$

$$v_o = R\ i_o$$

A family of operating point vector $\bar{x}(\overline{f_d})$ rewritten in the state space form is expressed by

$$\bar{x}(\overline{f_d}) = [\ z_o\ \ 0\ \ \sqrt{4(mg + \overline{f_d})/c}\cdot z_o\ ] \qquad (6)$$

As the operating condition is a function of external disturbance $\overline{f_d}$, we select $f_d(t)$ as a scheduling variable.

Here it is said to be " output trim condition ", $y(\overline{f_d})$ that for each constant $\overline{f_d}$ of the scheduling variable $f_d(t)$, represents the desired output.

Then the output trim condition of SMLS is $y(\overline{f_d}) = z_o$. Now the objective is to design a control law of the form

$$u(t) = K(x(t), f_d(t)) \qquad (7)$$

where, $K(\cdot,\cdot)$ is a smooth function such that at each $\overline{f_d}$

the closed-loop system should have a constant $x(\overline{f_d})$ and the linearized closed-loop system about each constant $\overline{f_d}$ should be asymptotically stable. In other words, the existance of such a closed-loop constant $\bar{x}(\overline{f_d})$ implies that for $\overline{f_d} \in \mathcal{F}$ the smooth function $\bar{x}(\overline{f_d})$ and $\bar{u}(\overline{f_d})$ must satisfy the followings

$$f(\bar{x}(\overline{f_d}),\ \bar{u}(\overline{f_d}),\ \overline{f_d}) = 0 \tag{8·a}$$

$$\bar{u}(\overline{f_d}) = K(\bar{x}(\overline{f_d}),\overline{f_d}) = R\, z_o\sqrt{4(mg + \overline{f_d})/c} \tag{8·b}$$

$$y(\overline{f_d}) = z_o \tag{8·c}$$

Then for each scheduling variable $\overline{f_d} \in \mathcal{F}$ the corresponding linearized closed loop system can be expressed in the form

$$\begin{aligned} \Delta\dot{x}(t) &= A(\overline{f_d}) \cdot \Delta x(t) + B \cdot \Delta u(t) + E \cdot \Delta f_d(t) \\ \Delta u(t) &= K_1^T(\overline{f_d}) \cdot \Delta x(t) + K_2(\overline{f_d}) \cdot \Delta f_d(t) \end{aligned} \tag{9}$$

where,

$$\begin{aligned} \Delta x(t) &= x(t) - \bar{x}(\overline{f_d}) \\ \Delta u(t) &= u(t) - \bar{u}(\overline{f_d}) \\ \Delta f_d(t) &= f_d(t) - \overline{f_d} \end{aligned} \tag{10}$$

$$A(f_d) = \left.\frac{\partial f}{\partial x}\right|_{\bar{x}(\overline{f_d}),\bar{u}(\overline{f_d}),\overline{f_d}} = \begin{bmatrix} 0 & 1 & 0 \\ \dfrac{2(mg+\overline{f_d})}{mz_o} & 0 & -\dfrac{\sqrt{c(mg+\overline{f_d})}}{mz_o} \\ 0 & \sqrt{4(mg+\overline{f_d})/c} & -2Rz_o/c \end{bmatrix}$$

$$B = \left.\frac{\partial f}{\partial u}\right|_{\bar{x}(\overline{f_d}),\bar{u}(\overline{f_d}),\overline{f_d}} = [\ 0 \quad 0 \quad 2z_o/c\ ] \tag{11}$$

$$E = \left.\frac{\partial f}{\partial x}\right|_{\bar{x}(\overline{f_d}),\bar{u}(\overline{f_d}),\overline{f_d}} = [\ 0 \quad 1/m \quad 0\ ]$$

and the linearized control law coefficients are given by

$$K_1^T(\overline{f_d}) = \left.\frac{\partial K}{\partial x}\right|_{\bar{x}(\overline{f_d}),\overline{f_d}} = [K_{11}(\overline{f_d})\ K_{12}(\overline{f_d})\ K_{13}(\overline{f_d})] \tag{12}$$

$$K_2^T(\overline{f_d}) = \left.\frac{\partial K}{\partial f_d}\right|_{\bar{x}(\overline{f_d}),\overline{f_d}}$$

Summing up the design problem for gain scheduling, it is to obtain a nonlinear control law (7) such that satisfy eq.(8) and (12) subject to (9). Here in order to achieve pole-assignment to the closed-loop linearization, eq.(9) is assumed to be controllable for any $\overline{f_d} \in \mathcal{F}$.

Now the control law construction proceeds as follows[7];

step 1: compute a smooth $K_1(\overline{f_d})$ such that $A(\overline{f_d})+BK_1(\overline{f_d})$ should have desired characteristic roots for each $\overline{f_d}$. The local stability problem including uncertainty of mass will be handled later in section 2·3.

step 2: In order to be able to satisfy eq. (8·b) and (12), $K_2(\overline{f_d})$ must be chosen as

$$K_2(\overline{f_d}) = \frac{\partial\bar{u}(\overline{f_d})}{\partial\overline{f_d}} - K_2(\overline{f_d})\frac{\partial\bar{x}(\overline{f_d})}{\partial\overline{f_d}} \tag{13}$$

Eq.(13) is proved easily as differentiating eq. (8·b) with respect to $\overline{f_d}$.

step 3: Once the linear gains $K_1(\overline{f_d})$ and $K_2(\overline{f_d})$ have been fixed, choose a nonlinear control law of the form (7) that satisfy (8·b) and (12).

As a simple form in step 3, We get a structure of the gain scheduling control law as follows ;

$$u(t) = K_1^T(f_d(t))[x(t) - \bar{x}(f_d(t))] + \bar{u}(f_d(t)) \tag{14}$$

The eq.(14) is derived from (9) and (10) when setting $\overline{f_d} = f_d(t)$. It is easy to see that (14) satisfies the conditions in step 3 using (13) even if $K_2(\cdot)$ does not appear in (14).

Now, before the linear gain $K_1(\overline{f_d})$ in step 1 is computed, two problems which are the uncertainty of suspended mass and the acquisition of scheduling variable $f_d(t)$ on real time will be discussed. The payload of MAGLEV is changed according to the number of passengers. Hence the mass can be expressed in the form $m \in [m^-, m^+]$, where m^- and m^+ indicate the minimum and the maximum value of m. Let m_o denote the nominal value of mass. Therefore at the design stage, only m_o is given. Next an estimator is in introduced to obtain the scheduling variable $f_d(t)$.

The estimator of disturbance $\hat{f}_d$ is defined from the SMLS model (1) as follows

$$\hat{f}_d(t) = m_o\ddot{z}(t) + \frac{c}{4}\left[\frac{i(t)}{z(t)}\right]^2 - m_o g \tag{15}$$

Since the state vector $[\ z\ \dot{z}\ i\ ]^T$ in the control law (14) can be constructed with measurements $[\ddot{z}\ \dot{z}\ i]^T$, the eq.(15) is available without measuring another variables in addition. It is now possible to carry out the pole-placement scheme in step 1 for computing K_1. The characteristic equation of the closed-loop linearization system is given by

$$\Delta(s) = \det[sI-(A+BK_1^T)] = s^3 + \delta_2 s^2 + \delta_1 s + \delta_o \quad (16)$$

where,

$$\delta_o = 2K_{11}\sqrt{\frac{mg+f_d}{cm^2}} - \frac{4}{cm}(R-K_{13})(mg+f_d)$$

$$\delta_1 = 2K_{12}\sqrt{\frac{mg+f_d}{cm^2}} \quad (17)$$

$$\delta_2 = \frac{2z_o}{c}(R-K_{13})$$

and the desired characteristic equation is defined as

$$\Delta^*(s) = (s+\alpha)(s^2+2\zeta\omega_n s+\omega_n^2) = s^3 + d_2 s^2 + d_1 s + d_o \quad (18)$$

where,

$$d_o = \alpha\omega_n^2$$
$$d_1 = 2\alpha\zeta\omega_n + \omega_n^2 \quad (19)$$
$$d_2 = \alpha + 2\zeta\omega_n$$

If $f_d(t)$ and m in eq.(17) are replaced by $\hat{f}_d(t)$ and m_o respectively, then the linear gain $K_1(\hat{f}_d(t)) = [K_{11}(t)\ K_{12}(t)\ K_{13}(t)]$ is given from (16)-(18)

$$K_{11}(t) = \frac{d_o}{2}\sqrt{\frac{cm_o^2}{m_o g+\hat{f}_d(t)}} + \frac{d_2}{z_o}\sqrt{c(m_o g+\hat{f}_d(t))}$$

$$K_{12}(t) = \frac{d_1}{2}\sqrt{\frac{cm_o^2}{m_o g+\hat{f}_d(t)}} \quad (20)$$

$$K_{13}(t) = R - \frac{cd_2}{2z_o}$$

At last, a gain scheduled controller for SMLS is proposed as follows, modifying (14) slighty,

$$u(t) = K_1^T(\hat{f}_d(t))\,\Delta\hat{x}(t) + \bar{u}(\hat{f}_d(t)) \quad (21)$$

where,

$$\Delta\hat{x}(t) := x(t) - \bar{x}(\hat{f}_d)$$

$$\bar{x}(\hat{f}_d(t)) = \left[z_o\ 0\ z_o\sqrt{(4(m_o g+\hat{f}_d(t))/c}\right]^T \quad (22)$$

$$\bar{u}(\hat{f}_d(t)) = Rz_o\sqrt{(4(m_o g+\hat{f}_d(t))/c}$$

The block diagram of SMLS with gain scheduling control is shown in Fig.2.

Remark 1: If the disturbance force is disregarded, that is $\hat{f}_d(t) = 0,\ t \geq 0$, then the eq.(21) is the same as just a fixed gain linear feedback controller which is based on a linear approximate model corresponding to only one operating point.

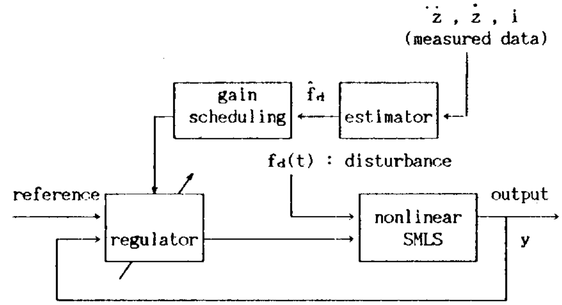

Fig.2. Block Diagram of SMLS with Gain Scheduling Control.

2.3 Local stability to both mass change and disturbance

As mentioned early, the gain scheduling approach is local in nature. Thus the overall performance and the stability must be checked by extensive simulations. However, under the condition the scheduling variable is slowly varying in a time average sense, it has been proved in [7, 9] that the gain scheduled control system provides stability.

In this section, the local stability problems to both the uncertainty of suspended mass and the constant disturbance over $\mathcal{F}$ will be discussed. Substituting eq.(20) into eq.(16), the actual characteristic equation of closed-loop linearization system becomes

$$\Delta a(s) := s^3 + a_2 s^2 + a_1 s + a_o = s^3 + d_2 s^2 + d_1\frac{m_o}{m}\sqrt{\frac{w(t)}{\hat{w}(t)}}\,s + \left[d_o\frac{m_o}{m}\sqrt{\frac{w(t)}{\hat{w}(t)}} + \frac{2d_o}{mz_o}\sqrt{w(t)\hat{w}(t)} - w(t)\right] \quad (23)$$

where $w(t) := mg+f_d(t)$

$$\hat{w}(t) := m_o g+\hat{f}_d(t) \quad (24)$$

from (1),(14) and (24),

$$w(t)-\hat{w}(t) = (m-m_o)\dot{x}_2(t) \quad (25)$$

Rewriting (25),

$$\hat{w}(t) = w(t) - (m-m_o)\dot{x}_2(t) \quad (26)$$

Thus while the disturbance is slowly varying as much as it does not make the velocity of lifting body change, it is clear from(23) and (26) that the closed-loop roots are independent on such a disturbance, and are affected by only the change of mass.

3. Simulations and Discussion

Simulations are performed on the well-known laboratory model[2] whose data are as follows;

$i_o = 2[A]$

$z_o = 1.5[mm]$

$m_o g = 44[Kg]$

$c = \mu_o N^2 A = 9.9 \times 10^{-5}$

The design specifications for above SMLS are considered such that satisfies.

(i) damping ratio ≥ 0.707

(ii) suspension natural frequency $\simeq$ 10 Hz

In (19), we are choose a set of design parameters which are given by $\alpha=100$, $\zeta=0.707$, $\omega_n=50/0.707$ (i.e., $d_0=500 \times 10^3$, $d_1=15\times10^3$ and $d_2=200$). The bounds of mass uncertainty are regarded as $m^-=0.8m_o$ and $m^+=1.3m_o$. Only two cases of $m=m^-$ and $m=m^+$ are examined.

In particular, we have considered several types of disturbance such as parabola, ramp, step, sinusoid, various general functions etc., whose magnitudes are all equal to or less than 40 % of nominal weight($m_o g$). The following three cases out of them will be shown in Fig.3 ~ Fig.7.

Type 1(parabola) : $f_d(t) = 17.6\sin(\pi t),\ 0 \leq t < 1$

Type 2(step) : $f_d(t) = \begin{cases} 0 & 0 \leq t < 0.5 \\ 8.8 & 0.5 \leq t \leq 1 \text{ (20\% of } m_o g) \end{cases}$

Type 3(general function) :

$$f_d(t) = \begin{cases} 8.8t+4.4t^2\sin(\exp(1.2\pi t)) & 0 \leq t < 1 \\ 8.8 & 1 \leq t \leq 2 \end{cases}$$

Now, three kinds of controller are compared with together ; the first one is a gain scheduled controller (GSC), the second is a nonlinear feedback linearizing controller(NFC) proposed by Jin et al.[4], and the third is a conventional linear approximation feedback controller(LFC)[2].

It is shown from Fig. 3 to Fig. 6 that the gain scheduling control have very nice performances in all the cases. Despite the large magnitude of disturbance and mass uncertainty are applied to, the excellent transient response and very little steady state errors are shown.

Note that no integral action is introduced in this gain scheduling controller.

In some cases(when the mass increases 30% of nominal), a linear approximate feedback controller (LFC) have the system to be unstable. An integrator added in LFC loop make the transient behaviour and steady state error not to improve against general disturbances as shown in Fig.7.

A nonlinear feedback linearization controller[4] has good robust stability to almost all the initial conditions. However, simulations show that its control performance is very sensitive to both mass change and disturbances.

4. Conclusions

One of the most important problems in EMS systems is how much robust the designed controller is against practical disturbances and uncertainty. With a view to solve it, a gain scheduling controller for SMLS has been proposed. As the operating condition is a function of disturbance, we choose the estimated disturbance as a scheduling variable. A simple estimator can be constructed without additive measurements. We showed that the gain scheduling is robust if the disturbance is slowly varying as much as it does not make the velocity of lifting body change (i.e., so that acceleration/deceleration is nearly zero). The gain scheduling is inherently local in nature. So the overall performance and stability of the control system have been demonstrated by different simulations. In most cases, the gain scheduling control designed for a SMLS model showed very good results. Comparing with the gain scheduling, we performed the same simulations for two other control methods ; (a) nonlinear feedback linearizing controller (b) a fixed gain linear approximate controller. Both method does not show the satisfactory performances to the disturbance and the mass change.

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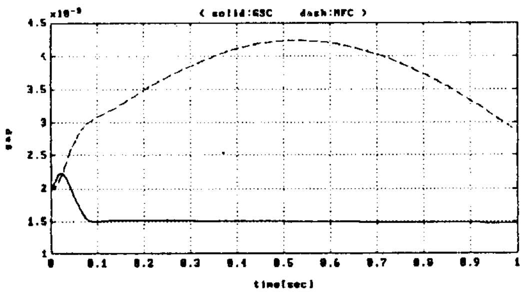


Fig.3 System response for GSC and NFC with $m = 1.3m_o$, f_d = type 1, and $z(0)$ = 2 mm

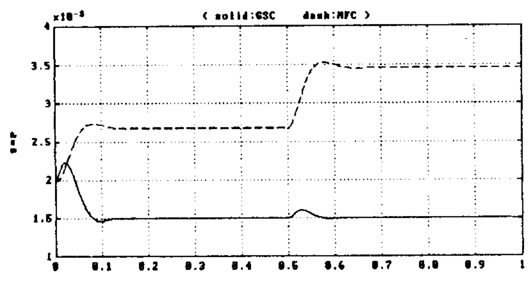
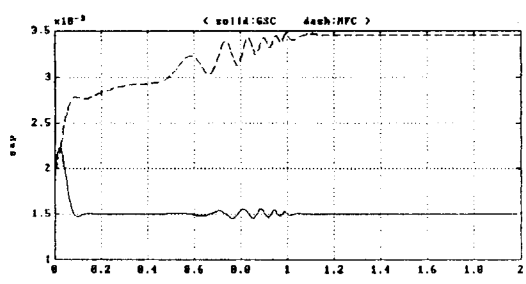
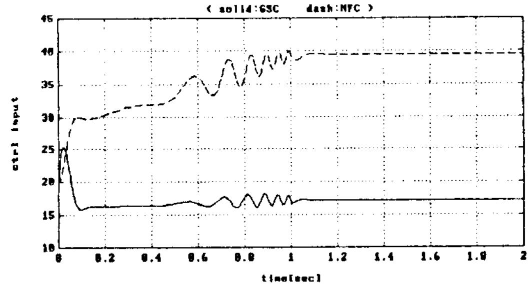


Fig.4 System response for GSC and NFC with $m = 1.3m_o$, f_d = type 2.

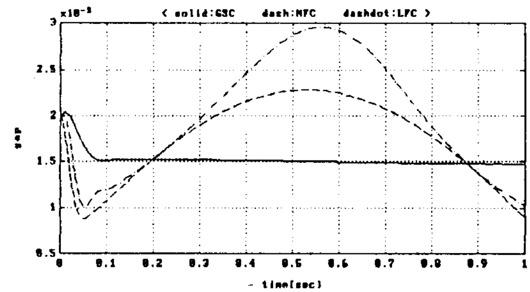


(a) outputs

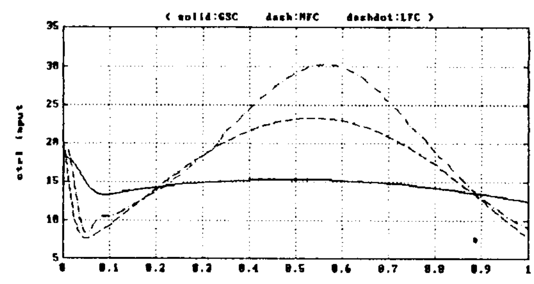


(b) control inputs

Fig.5 System response for GSC and NFC with $m = 1.3m_o$, f_d = type 3 : (a)outputs (b)control inputs[V].



(a) outputs



(b) control inputs

Fig.6 System response for GSC, NFC and LFC with $m = 0.8m_o$, f_d = type 1 : (a) outputs (b) control inputs[V].

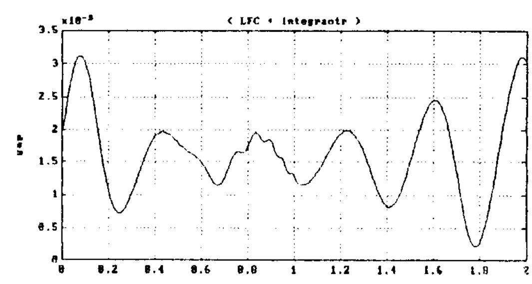


Fig.7 System response for LFC with $m = 0.8m_o$, f_d = type 3, integrator gain 16842.