H_∞ Control Approach to a Magnetic Levitation System with Two Poles on $j\omega$–Axis

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Abstract

An H_∞ control system design for a magnetic levitation system is presented. In the control system design, we consider the influence of both disturbances and uncertainties in the model. The main disturbances stem from the position sensors. The uncertainties are divided into electromagnetic and mechanical ones: the former are due to the gain change in the current amplifier, the influence of leakage flux and modelling error in the magnetic circuit and the latter are due to the changes of the mass and the moments of inertia of the vehicle. Therefore, the designed controller is indispensable to guarantee the robustness of this system for both stability and performance. The controller design is based on the standard H_∞ optimal control problem. As the novel features in this paper: (1) there are two poles on $j\omega$−axis in the control model; (2) an integrator is included in the controller so that equivalently there are three poles on $j\omega$−axis in the model. Finally, several experiments and simulations are carried out to verify the high performance and robustness of the designed control system.

Keywords: Magnetic levitation system, H_∞ control, Robust control, Two poles on $j\omega$−axis, Integral type controller.

1 Introduction

A request has been rising for control performance as the industrial world gets more and more progressive. It is necessary to make the processing environments ultraclean during microscopic processings of a high density in the field of semiconductors. Because the magnetic levitation system has contactlessness, its application is favorable for realizing superclean environments as a conveyance.

However, there are various modeling errors and disturbances in the magnetic levitation transportation system considered in this paper. Because they impair the stability of the system, a design of a controller to avoid the instability of the control system is required. For this purpose, a H_∞ control method which is robust is considered. The H_∞ control theory has been established as one of the control system design methods, such that it is able to design a control system which reduces the sensitivity of a system to the influence of disturbances and has robust stability for the change of parameters in the control model. Recently, its effectiveness is reported in several papers [1],[2],[3] which include magnetic levitation systems.

In this paper, we design a controller which is based on the H_∞ control theory in that there are two poles on the origin in the transfer function of a control model. The controller also includes an integrator. The effectiveness and robustness of this design are verified by computer simulations theoretically for the system and further by experimental results on the equipment. In particular, this paper gets a high performance for an application which does not allow overshooting in the transient response and has robust stability for the disturbances.

2 Equipment and its modelling

Magnetic levitation equipment as a control object is shown in Fig.1.

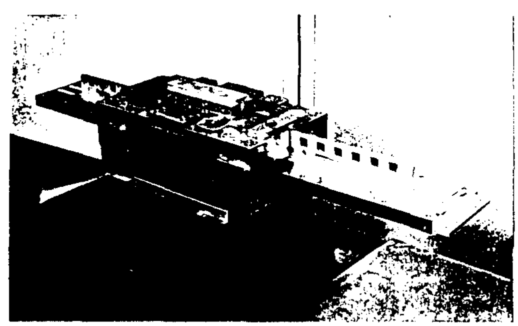

Fig.1. Exterior view of the magnetic levitation equipment.

There are four pairs of magnets, which are used for pulling up and down a movable vehicle, and two pairs of magnets, which are used for guiding the vehicle, in a stator. The movable vehicle of a rectangular sheet, which runs as the secondary in a reluctance–type linear motor, is inserted into the stator. The relationships of positions between the movable vehicle and magnets and their sizes are represented in Fig.2. The size ratio $a : b : h : k$ is approximately equal to $28 : 4 : 9 : 2(cm)$. The horizontal distance between the center of magnets M_1 and M_2 and the center Q of gravity of the movable vehicle is denoted by ℓ and is called the position of the center of gravity in the following discussion. In this paper, we consider the system designs for the directions of levitation, pitching and rolling according to the H_∞ optimal control theory. For the control system design of the guide direction, it is enough to use the simple feedback control design that is confirmed by experiments.

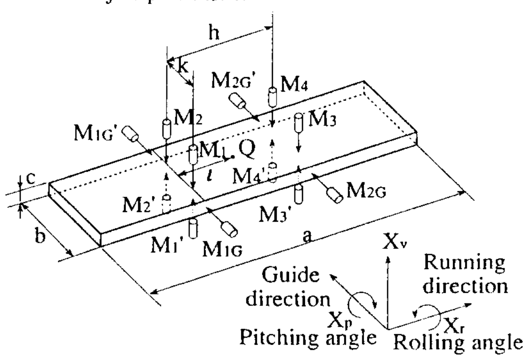

Fig.2. The movable vehicle and the positions of the magnets.

Assume the dynamics of the movable vehicle as a rigid body movement, then the dynamic equation in the levitation direction is

$$m\ddot{x} = f \tag{1}$$

Where, m is the mass of the movable vehicle, $\ddot{x}$ is the acceleration of the center Q of gravity of the movable vehicle on the levitation direction, $f = f' - mg$, f' is the attraction force of magnets on the levitation direction, g is the acceleration of gravity. Therefore, the transfer function is given as

$$p_v(s) = \frac{1}{ms^2} \tag{2}$$

Similarly, the transfer function for both directions of pitching and rolling are

$$p_p(s) = \frac{\ell}{I_p s^2} \tag{3}$$

$$p_r(s) = \frac{k}{2I_r s^2} \tag{4}$$

Where, ℓ : the position of the center of gravity,

k : the horizontal distance between M_1 and M_2 or M_3 and M_4 ,

I_p : the moment of inertia of the vehicle in the pitching direction,

I_r : the moment of inertia of the vehicle in the rolling direction.

The formulas (2), (3), and (4) become equivalently

$$P(s) = \frac{1}{s^2} \tag{5}$$

by suitable changes of inputs. There are also two poles at the origin in $P(s)$. The following control conditions are required for this system.

(1). Reduce the influence of disturbances for the system.

(2). Make the system robustly stable for the modelling error and the changing parameters of the control model. Therefore, the foundation of the control system design is to reduce the sensitivity and increase the robust stability of the system.

Remark: As the characteristics of magnets, we can consider their transfer functions to be 1, because the inverse of transfer functions from the currents to the magnetic forces are added on the output of controllers.

3 Controller design by H_∞ control theory

A controller that is going to be designed is added on $P(s)$ to form a closed loop feedback system in Fig.3.

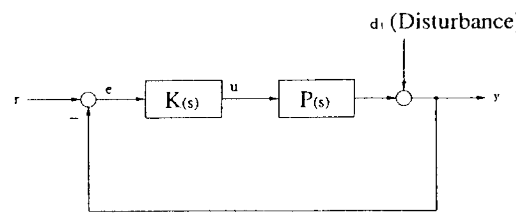

Fig.3. Closed loop feedback control system 1.

According to the control requisitions stated above, the design must make gains ($\| S(j\omega) \|$ and $\| T(j\omega) \|$) of the sensitivity function $S(s)=(1+ P(s) K(s))^{-1}$ and the complementary sensitivity function $T(s)=P(s) K(s) (1+ P(s) K(s))^{-1}$ of the closed loop control system as low as possible together, but it is clear that both functions are not independent because of $S+T=1$, namely two requisitions cannot be satisfied simultaneously. We try to obtain the low sensitivity on the low frequency band and consider the robust stability firstly on the high frequency band, where the modelling errors arise up generally. Choosing suitable weighted functions separately, $W_s(s)$ and $W_t(s)$, for the functions of sensitivity and complementary sensitivity, we consider a mixed sensitivity problem that satisfies

$$\left\| \begin{matrix} W_s(s)S(s) \\ W_t(s)T(s) \end{matrix} \right\|_\infty < \gamma \tag{6}$$

3.1 Transfer function G of generalized control model

Referring to the method [4] , we propose weighted functions as follows

$$W_s(s) = \frac{\rho}{(s+\eta)^2} \tag{7}$$

$$W_t(s) = \frac{(s+\alpha)^2}{\alpha^2\beta} \tag{8}$$

If the parameter ρ is bigger, the sensitivity of the system becomes lower. It is possible to keep the robustness of the system and realize a hopeful response characteristics of the closed loop system, because a working frequency band can be regulated by choosing parameters, α and β.

An extended control system is shown in Fig.4 which is used for finding a H_∞ controller.

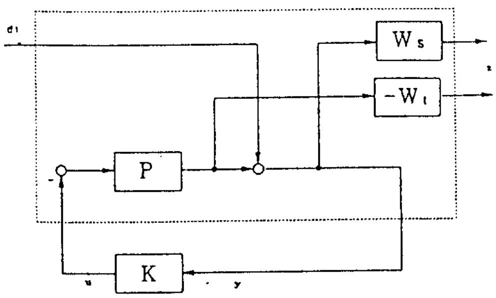

Fig.4. Extended control system.

Where, d_1 is the disturbance, z is the controlled output, u is the control, and y is the measured output. Stabilizable and detectable state space realization of the control model $P(s)$ in the formula (5) and the weighted function $W_s(s)$ in the formula (7) are

$$P(s) = \left[\begin{array}{c|c} A_p & B_p \\ \hline C_p & 0 \end{array}\right] = \left[\begin{array}{cc|c} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \hline 1 & 0 & 0 \end{array}\right] \tag{9}$$

$$W_s(s) = \left[\begin{array}{c|c} A_s & B_s \\ \hline C_s & 0 \end{array}\right] = \left[\begin{array}{cc|c} 0 & 1 & 0 \\ -\eta^2 & -2\eta & 1 \\ \hline \rho & 0 & 0 \end{array}\right] \tag{10}$$

For $W_t(s)$, because $W_t(s)P(s)$ is chosen as proper, a state space realization of $W_tP(s)$ becomes (by formulas (5) and (8))

$$W_t(s)P(s) = \left[\begin{array}{c|c} A_p & B_p \\ \hline C_t & D_t \end{array}\right] = \left[\begin{array}{cc|c} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \hline 1/\beta & 2/\alpha\beta & 1/\alpha^2\beta \end{array}\right] \tag{11}$$

Therefore, the state space realization of the extended control model G is

$$G(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & 0 \end{array}\right]$$

$$= \left[\begin{array}{cc|cc} A_s & B_sC_p & B_s & 0 \\ 0 & A_p & 0 & -B_p \\ \hline C_s & D_sC_p & D_s & 0 \\ 0 & -C_t & 0 & D_t \\ 0 & C_p & 1 & 0 \end{array}\right] \tag{12}$$

A generalized feedback control system is shown in Fig.5. Let

$$u = K(s)y \tag{13}$$

then the transfer function from d_1 to z is given as

$$z = T_{zd}d_1$$

$$T_{zd} = G_{11}(s) + G_{12}(s)K(s)(1 - G_{22}K(s))^{-1}G_{21}(s) \tag{14}$$

Using the formula above, the formula (6) can be rewritten

$$\|T_{zd}(s)\|_\infty < \gamma \tag{15}$$

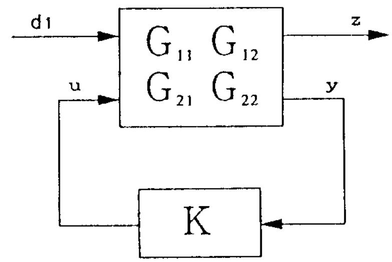

Fig.5. Generalized control system.

3.2 Controller design 1

Using the weighted functions stated above, according to the reference [5] , we check the solvable conditions and then construct a dynamic feedback internally stabilizing controller $K(s)$ to minimize the H_∞ norm of the formula (15).

A1. (A,B_2,C_2) is stabilizable and detectable.

A2. rank $D_{12} = 1$, rank $D_{21} = 1$.

A3. $D_{12} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $D_{21} = [0 \quad 1]$

A4. $D_{22} = 0$.

A5. rank $\begin{bmatrix} A - j\omega I & B_2 \\ C_1 & D_{12} \end{bmatrix} = 5 \quad \forall\, \omega \in R.$

A6. rank $\begin{bmatrix} A - j\omega I & B_1 \\ C_2 & D_{21} \end{bmatrix} = 5 \quad \forall\, \omega \in R.$

It is easily understood that the conditions of A1~A5 are satisfied through a suitable change, but the condition A6 is not satisfied, namely

$$\text{rank} \begin{bmatrix} A - j\omega I & B_1 \\ C_2 & D_{21} \end{bmatrix} = 4 < 5 \quad (\omega = 0)$$

For the magnetic levitation system treated in this paper, then, it is impossible to design a controller with the standard H_∞ optimal control problem just as it is. We use the method of augmenting a system with a small positive constant ε to solve this problem [6] . Use the following formula instead of the formula (12),

$$G_\varepsilon(s) = \left[\begin{array}{c|c|c} A & [\ B_1 \quad \sqrt{\varepsilon}I\ ] & B_2 \\ \hline C_1 & [\ D_{11} \quad 0\ ] & D_{12} \\ C_2 & [\ D_{21} \quad 0\ ] & 0 \end{array}\right] \tag{16}$$

then

$$\text{rank} \begin{bmatrix} A - j\omega I & B_1 & \sqrt{\varepsilon}I \\ C_2 & D_{21} & 0 \end{bmatrix} = 5 \quad \forall\, \omega \in R.$$

the condition A6 is satisfied. The H_∞ control problem of $G(s)$ is solvable if and only if one certain $\varepsilon > 0$ exists and the H_∞ control problem of $G_\varepsilon(s)$ is solvable. In view of this, we design the H_∞ controller and derive a general solution for G_ϵ following the previous algorithm. The central solution is

$$K_1(s) = \frac{k_{10}(s+a)}{(s+b_{11})(s+\bar{b}_{11})} \tag{17}$$

Where

$k_{10} = 7.3037 \cdot 10^6, \quad a = 46.959, \quad b_{11} = 214.89 + j174.62.$

and during the design, let:

$\rho = 2100, \quad \eta = 4.18 \cdot 10^{-12}, \quad \alpha = 120, \quad \beta = 10, \quad \epsilon = 10^{-12}$

By this result, the gain plot of $T_{zd}(s)$ is shown in Fig.6, and gain plots of functions of the sensitivity and the complementary sensitivity are shown in Fig.7 by a solid line and a dashed line respectively. We use the pole–zero mapping method to discretize the obtained H_∞ controller $K_1(s)$ into a digital controller, so that it can be realized by a computer.

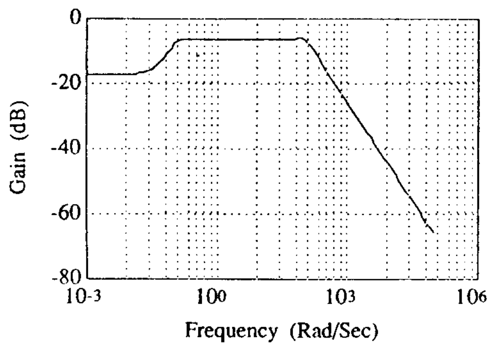

Fig.6. Gain plot of $T_{zd}(s)$.

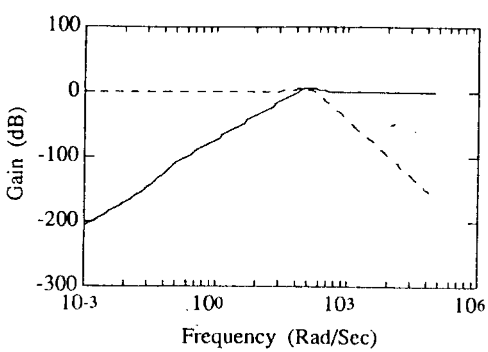

Fig.7. Gain plots of S and T.

3.3 Controller design 2

Using the controller $K_1(s)$ given above can remove the influence of the disturbance d_1 which enters the measured output from a sensor, but produces the steady state error when the disturbance d_2 enters the system from the front of the control object such as Fig.8, or there are modelling errors in the control model. In order to take that influence away, it is necessary to make the H_∞ controller include an integrator namely as an integral type servo system. For this reason, we construct an extended model $P_{ext}(s)$ which attaches an integrator on the control model $P(s)$.

$$P_{ext}(s) = \frac{s+\xi}{s} P(s) = \frac{s+\xi}{s^3} \tag{18}$$

A stabilizable and detectable state space realization of this extended model becomes

$$P_{ext}(s) = \left[\begin{array}{c|c} A_{pext} & B_{pext} \\ \hline C_{pext} & 0 \end{array}\right] = \left[\begin{array}{ccc|c} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \hline \xi & 1 & 0 & 0 \end{array}\right] \tag{19}$$

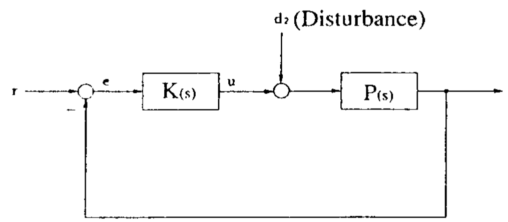

Fig.8. Closed loop feedback control system 2.

We use the same method as 3.2 to design a H_∞ controller for the extended control model. The H_∞ controller for the original model $P(s)$ which corresponds to the central solution of the extended model is given

$$K_2(s) = K_{ext}\frac{s+\xi}{s} = \frac{k_{20}(s+a_1)(s+a_2)}{s(s+b_{21})(s+\bar{b}_{21})} \tag{20}$$

Where

$$k_{20} = 2.5053 \cdot 10^7,$$

$$a_1 = 57.205, \quad a_2 = 16.674, \quad b_{21} = 320.40 + j260.00.$$

and during the design, let:

$\rho = 930, \quad \eta = 1.001 \cdot 10^{-9}, \quad \alpha = 200, \quad \beta = 40, \quad \epsilon = 10^{-8}$

By this result, the gain plot of $T_{zd}(s)$ is shown in Fig.9, and gain plots of functions of the sensitivity and the complementary sensitivity are shown in Fig.10 by a solid line and a dashed line respectively.

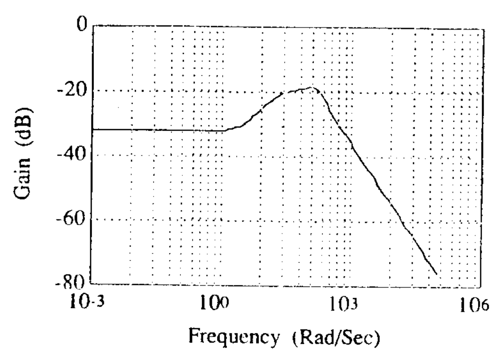

Fig.9. Gain plot of $T_{zd}(s)$.

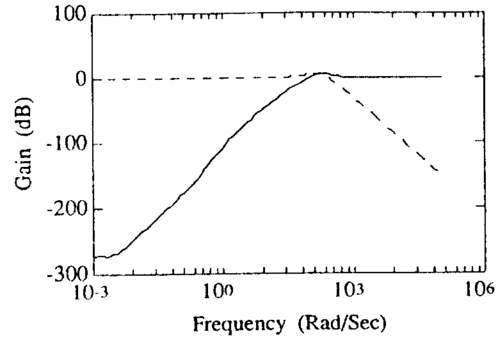

Fig.10. Gain plots of S and T.

Using the pole–zero mapping method to discretize the H_∞ controller $K_2(s)$ in the formula (20), a discretized controller is obtained.

$$K_2(z) = \frac{k_{2z0}(z - e^{-a_1 T})(z - e^{-a_2 T})}{(z-1)(z - e^{-b_{21} T})(z - e^{-\bar{b}_{21} T})} \tag{21}$$

Where

$$K_{2z0} = \frac{k_{20} a_1 a_2 T (1 - e^{-b_{21} T})(1 - e^{-\bar{b}_{21} T})}{b_{21} \bar{b}_{21} (1 - e^{-a_1 T})(1 - e^{-a_2 T})}$$

$T = 0.00075 sec$ is the sampling period.

4 Experiments

To verify the effectiveness of the designed H_∞ controls above, several experiments are carried out on the equipment of the magnetic levitation system.

4.1 Outline of experiment system

We construct a control system such as Fig.11 with the magnetic levitation equipment shown in Fig.1.

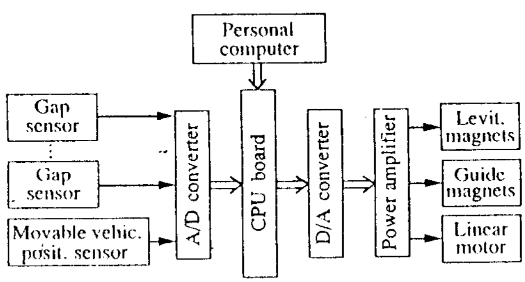

Fig.11. Control system.

Where, $m = 7.0\ kg$, $I_p = 0.373\ Nms^2$, $I_r = 0.0072\ Nms^2$. A microcomputer CPU is a 64 bit processor called a transputer. The distances between the movable vehicle and magnets are measured by transformer type distance sensors and changed into digital signals by a 12 bit A/D converter. The ℓ is measured by a linear resolver. The control parts calculate a control force of each direction with the discrete H_∞ controller, change the forces into electric current order values, and output the order values into the electric current amplifier of each electromagnet through a 12 bit D/A converter.

4.2 Simulation and experiments 1

The results of simulation for $P(s)$ and experiments of the levitation direction on the equipment by the controller $K_1(s)$ are shown in Fig.12, 13, and 14 respectively. There is no artificial disturbance in Fig.13 case. In Fig.14, we change the mass (four $500g$ pieces of clay are put on each corner of the vehicle), and verify the robustness of the system. It is affirmed that the low sensitivity of the system is realized for the disturbance d_1, but there is a steady state error in the stationary state.

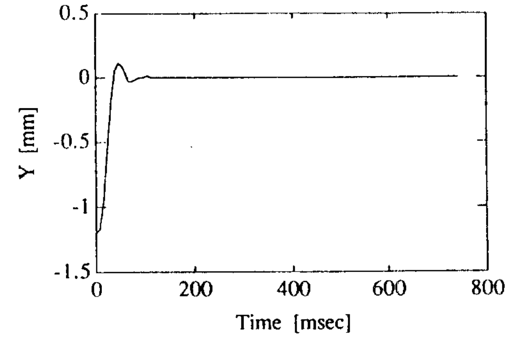

Fig.12. Result of simulation by controller $K_1(s)$.

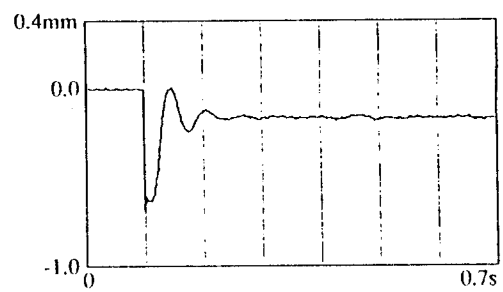

Fig.13. Experiment result by controller $K_1(s)$.

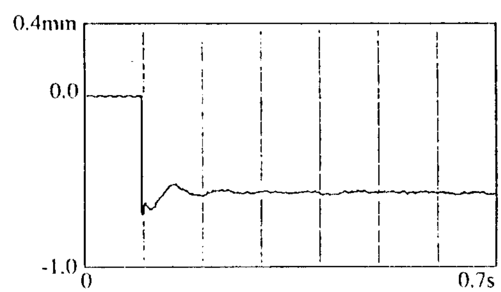

Fig.14. Experiment result by controller $K_1(s)$ with the weight increased by $2kg$.

4.3 Simulation and experiments 2

Now, we do the same simulation and experiments as before by the controller $K_2(s)$. The results are shown in Fig.15, 16, and 17. From them, it is verified as in the preceding case 4.2 that the system has the robustness for the change of model and the low sensitivity for the disturbance d_1 which enters the measured output y . It is also confirmed that the system does not have the steady state error and has the robustness even for the modelling errors and the disturbance shown in Fig.8.

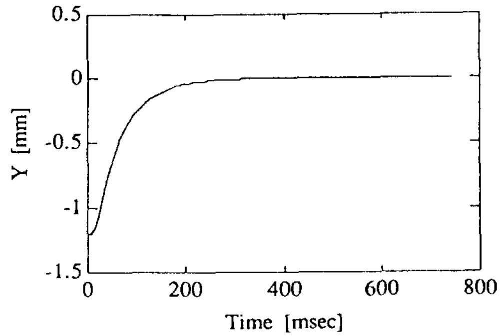

Fig.15. Result of simulation by controller $K_2(s)$.

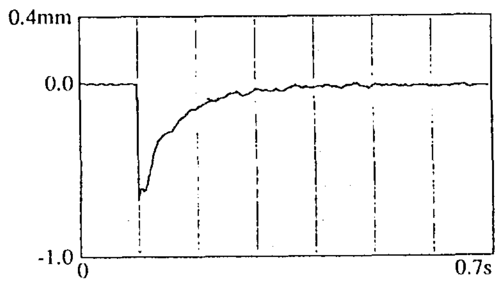

Fig.16. Experiment result by controller $K_2(s)$.

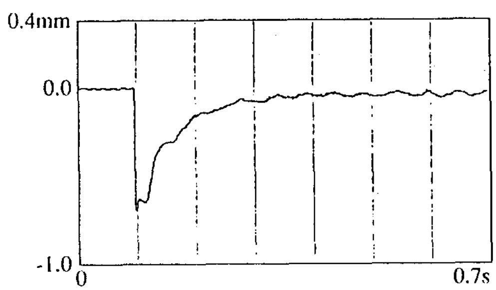

Fig.17. Experiment result by controller $K_2(s)$ with the weight increased by $2kg$.

As for the pitching direction and the rolling direction, their plots may be omitted, because the step responses are almost the same as the levitation direction.

5 Conclusions

In this paper, we apply the controller which is designed by the H_∞ control theory to the magnetic levitation system and verify the high performance and the robustness of this system by simulations and experiments. As the novel features in this paper:

(1). there are two poles at the origin in the magnetic levitation system;

(2). We make an integrator in the H_∞ controller, so that the system changes to have three poles at the origin. One method is established for the H_∞ controller design with the above characteristics that makes the design both satisfy the conditions of reference [5] and accomplish the control effect which was our objective.

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