Control of a Magnetic Levitation System via Feedback Error Learning

Shuang-Hui Hao, Zi-Jiang Yang and Teruo Tsuji
Kyushu Institute of Technology
1-1 Sensui-cho, Tobataku, Kitakyushu, 804, JAPAN

Abstract — This paper presents an on-line feedback error learning control algorithm for a magnetic levitation system. It will be shown that even in the case of abrupt changes of the system parameters and disturbanes, the control performance is still very satisfactory.

1 Introduction

Applications of advanced control techniques to magnetic levitation systems have received growing attentions, especially by utilizing robust control theories and techniques(1)~(6). As an alternative approach, in this paper, the authors challenge to apply the recently developed feedback-error learing method to the vertical position control of a 4-point attraction magnetic levitation system. We design the stable closed-loop using a PD controller based on the mechanical model. Then a two-layered linear neural network (LNN) which learns the inverse mechanical model of the system is used to construct a multivariable feedforward controller. The parameters of the LNN are adjusted in an on-line manner by taking the outputs of the pre-designed feedback controller as the error signal. Using the normal inverse model of the electromagnetic system (IMES), we generate the voltage values of the coils as the control signal, from the desired magnetic force vector which is the sum of the outputs of the feedback controller and the feedforward controller. Therefore, if the on-line adjusted LNN acts as the inverse of the mechanical system and compensates the unknown disturbances, the system output tracks the desired value very precisely. Experimental results are included to show the excellent performance of the designed control system and it is verified that even in the case of abruptly large changes of the system parameters and disturbances, the control performance are still very satisfactory.

2 Experimental Equipment

2.1 Mechanical model

The exterior view of the experimental equipment is shown as Fig.1. The shape of the levitated vehicle is like a rectangular sheet. There are the electromagnets, the gap sensors and the linear motor in the stator.

The mechanical differential equations of the levitated vehicle in Fig.1 can be written as:

$$\ddot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{f} + \boldsymbol{g} \tag{1}$$

where

$$\begin{aligned} \boldsymbol{x} &= [x_v, x_p, x_r]^T \\ \boldsymbol{f} &= [f_1, f_2, f_3, f_4]^T \\ \boldsymbol{g} &= [-g, 0, 0]^T \end{aligned} \tag{2}$$

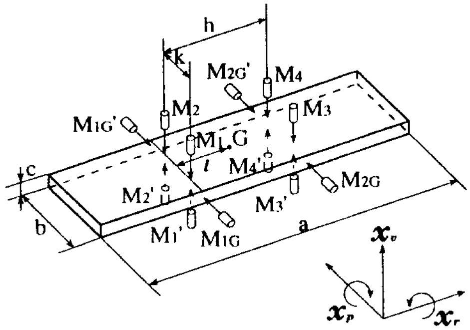

Fig. 1: Exterior view of the magnetic levitation equipment.

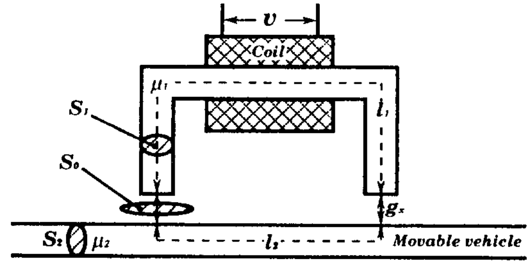

Fig. 2: Typical model of the electromagnet.

$$\boldsymbol{A} = \begin{bmatrix} \frac{1}{m} & \frac{1}{m} & \frac{1}{m} & \frac{1}{m} \\ \frac{\ell}{I_p} & \frac{\ell}{I_p} & -\frac{h-\ell}{I_p} & -\frac{h-\ell}{I_p} \\ \frac{k}{2I_r} & -\frac{k}{2I_r} & \frac{k}{2I_r} & -\frac{k}{2I_r} \end{bmatrix} \tag{3}$$

where x_v is the position of center of gravity; x_p is the pitching angle; x_r is the rolling angle; m is mass of the levitated vehicle; I_p is the moment of inertia in the direction of pitching motion; I_r is the moment of inertia in the direction of rolling motion; g is the acceleration of gravity; $f_1 \sim f_4$ is the electromagnetic force produced by each pair of pulling-up and pulling-down magnets; ℓ is the position of the center of gravity; and h, k is the distance between the electromagnets.

Now the accelerations in the vertical direction, the pitching direction and the rolling direction are defined by

$$\boldsymbol{a} = [a_v, a_p, a_r]^T \tag{4}$$

Then, the motion equation of the system can be described by:

$$\ddot{\boldsymbol{x}} = \boldsymbol{a} \tag{5}$$

Since $\boldsymbol{A}$ is an unsqare matrix, we introduce the following constraint:

$$0 = f_1 + f_3 - f_2 - f_4 \tag{6}$$

Thus, we have the unique solution of the magnetic attractive force $\boldsymbol{f}$ as:

$$\begin{aligned} \boldsymbol{f} &= \overline{\boldsymbol{A}}(\boldsymbol{a} - \boldsymbol{g}) \\ &= \overline{\boldsymbol{A}}\boldsymbol{a} - \boldsymbol{d} \end{aligned} \tag{7}$$

where

$$\boldsymbol{d} = \overline{\boldsymbol{A}}\boldsymbol{g} \tag{8}$$

$$\overline{\boldsymbol{A}} = \begin{bmatrix} \frac{m(h-\ell)}{2h} & \frac{I_p}{2h} & \frac{I_r}{2k} \\ \frac{m(h-\ell)}{2h} & \frac{I_p}{2h} & -\frac{I_r}{2k} \\ \frac{m\ell}{2h} & -\frac{I_p}{2h} & \frac{I_r}{2k} \\ \frac{m\ell}{2h} & -\frac{I_p}{2h} & -\frac{I_r}{2k} \end{bmatrix} \tag{9}$$

Equation (7) shows the inverse model of the mechanical system (IMMS) for the magnetic levitation system.

2.2 Electromagnetic model

Fig.3 shows a typical model of one of the electromagnets. Let μ_1 and μ_2 denote permiabilities ; ℓ_1, ℓ_2 denote the magnetic path lengths; and S_1, S_2 denote the equivalent cross-sectional areas; all of the electromagnet and movable vehicle, respectively. Denoting the gap lenth between the movable vehicle and the magnet by x_j, electromagnetic force f_j exerted on the movable vehicle is given by

$$\left. \begin{aligned} f_j &= \frac{\mu_0 S_0 N^2 i^2}{\left(2x_j + \frac{\mu_0 S_0 l_1}{\mu_1 S_1} + \frac{\mu_0 S_0 l_2}{\mu_2 S_2}\right)^2} \\ & (j = 1,2,3,4) \end{aligned} \right\} \tag{10}$$

Since the permeabilities are as large as 10200 to 15100 in the experimental apparatus, the denominator of the foregoing equation is approximately equal to $4x_j^2$. Therefore, the electromagnetic force is given by:

$$f_j = K\frac{i^2}{x_j^2} \tag{11}$$

The coil inductance of each magnet varies with the distance between the movable vehicle and magnet core.

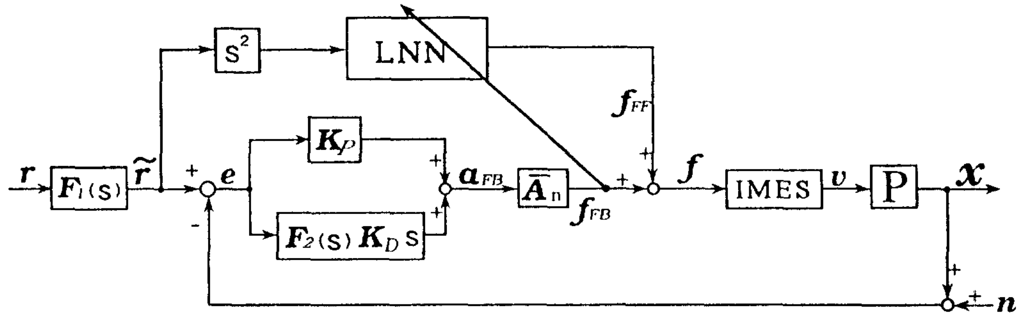

Fig. 3: Block diagram of the controller.

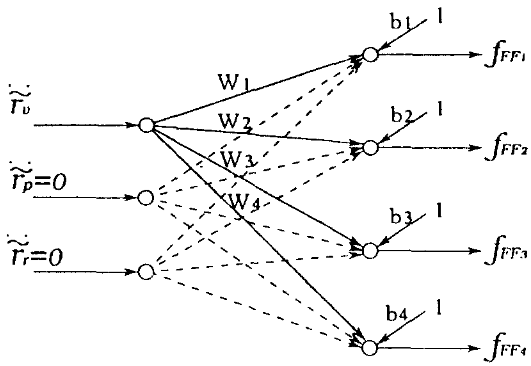

Fig. 4: Structure of LNN.

To simplify discussion, however, it is assumed that the coil inductance is kept constant around the working point. Under this assumption, the electrical equation of a coil can be written as:

$$v_j = R_j i_j + L_j \frac{di_j}{dt} \tag{12}$$

From equations (9) and (10), the inverse model of the electromagnetic system (IMES) for the magnetic levitation vehicle system can be written as the following:

$$v_j = R_j x_j \sqrt{\frac{f_j}{K}} + \frac{L_j}{\sqrt{K}} \frac{d}{dt}\left(\sqrt{f_j} x_j\right) \tag{13}$$

In references (2) and (3) we used the IMMS with $\overline{A}_n$ which is the nominal value of $\overline{A}$, to achieve the desired magnetic force vector from the desired value of the acceleration[(2)(3)]. However, in the case of abruptly large change of the system parameters and disturbances, ex., change of the load, it is necessary to construct a controller which is adaptive to the parameter change.

3 Design of the Controller

The block diagram of the control system via feedback error learning is shown in Fig.3. A PD feedback controller is designed to stabilize the closed-loop of the controlled system, so that the LNN can learn the IMMS in an on-line manner[(7)~(12)].

3.1 Feedback Controller

Based on the motion equation of the system, shown as equation (5), the PD feedback controller is designed to stabilize the closed-loop. The proportion and derivative gains of the PD controller are denoted by matrixes as follows, respectively.

$$\boldsymbol{K}_P = \begin{bmatrix} K_{P_v} & 0 & 0 \\ 0 & K_{P_p} & 0 \\ 0 & 0 & K_{P_r} \end{bmatrix} \tag{14}$$

$$\boldsymbol{K}_D = \begin{bmatrix} K_{D_v} & 0 & 0 \\ 0 & K_{D_p} & 0 \\ 0 & 0 & K_{D_r} \end{bmatrix} \tag{15}$$

In this system, since the sampling period is very small, we use the pseudo-differential operation to construct the D-controller with the filter $\boldsymbol{F}_2$ to reduce the noise effects:

$$\boldsymbol{F}_2(s) = \begin{bmatrix} \frac{1}{\tau_{2_v}s+1} & 0 & 0 \\ 0 & \frac{1}{\tau_{2_p}s+1} & 0 \\ 0 & 0 & \frac{1}{\tau_{2_r}s+1} \end{bmatrix} \tag{16}$$

Therefore, the output of the PD controller is given as the desired value of acceleration:

$$\boldsymbol{a}_{FB} = (\boldsymbol{K}_P + \boldsymbol{K}_D \boldsymbol{F}_2(s)\, s)\,\boldsymbol{e} \tag{17}$$

where

$$\boldsymbol{e} = [e_v, e_p, e_r]^T \tag{18}$$

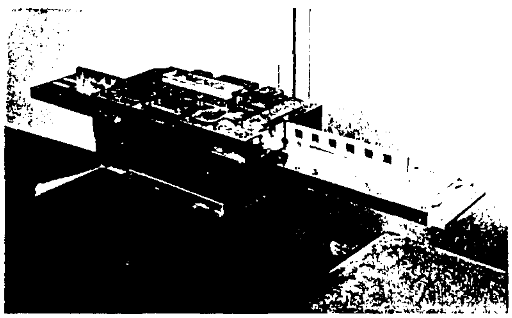

Fig. 5: Picture of the magnetic levitation equipment.

is the feedback error as shown in Fig.2. And then we use the $\overline{\boldsymbol{A}}_n$ to obtain the desired magnetic force vector from the desired value of acceleration.

$$\boldsymbol{f}_{FB} = \overline{\boldsymbol{A}}_n \boldsymbol{a}_{FB} \tag{19}$$

3.2 Feedforward Controller

To improve the learning performance, a third–order $\boldsymbol{F}_1$ is introduced to filter the command signal input $\boldsymbol{r} = [r_v, r_p, r_r]^T$ for generating a continuous reference acceleration signal $\ddot{\tilde{\boldsymbol{r}}}$.

$$\boldsymbol{F}_1(s) = \begin{bmatrix} f_{v1}(s) & 0 & 0 \\ 0 & f_{p1}(s) & 0 \\ 0 & 0 & f_{r1}(s) \end{bmatrix} \tag{20}$$

where

$$\begin{aligned} f_{v1}(s) &= \frac{1}{(\tau_{v11}s+1)(\tau_{v12}s+1)(\tau_{v13}s+1)} \\ f_{p1}(s) &= \frac{1}{(\tau_{p11}s+1)(\tau_{p12}s+1)(\tau_{p13}s+1)} \\ f_{r1}(s) &= \frac{1}{(\tau_{r11}s+1)(\tau_{r12}s+1)(\tau_{r13}s+1)} \end{aligned} \tag{21}$$

Then, the command signal is filtered as:

$$\tilde{\boldsymbol{r}} = \boldsymbol{F}_1(s)\ \boldsymbol{r} \tag{22}$$

where, $\tilde{\boldsymbol{r}}$ is called a desired position value. In this paper, our task is set to perform the vertical position control, therefore we have:

$$\begin{aligned} r_p &\equiv 0 \\ r_r &\equiv 0 \end{aligned} \tag{23}$$

From Fig.3 the output $\boldsymbol{f}_{FF}$ of the feedforward controller, which is the magnetic force vector, can be shown as the following:

$$\begin{aligned} \boldsymbol{f}_{FF} &= \boldsymbol{w}\ddot{\tilde{r}}_v + \boldsymbol{b} \\ &= [f_{FF1}, f_{FF2}, f_{FF3}, f_{FF4}]^T \end{aligned} \tag{24}$$

where, $\boldsymbol{b}$ and $\boldsymbol{w}$ are the bias and weight vector, respectively:

$$\begin{aligned} \boldsymbol{b} &= [b_1, b_2, b_3, b_4]^T \\ \boldsymbol{w} &= [w_1, w_2, w_3, w_4]^T \end{aligned} \tag{25}$$

Then, to control the levitated vehicle, the desired voltage values are generated from the desired magnetic forces vector based on the IMES. Since the IMES is used to generate the desired voltage value of the electromagnet coils from the desired magnetic force vector. When the LNN becomes the mapping of the IMMS, the outputs of the plant can be described as:

$$\boldsymbol{x} = \boldsymbol{F}_1(s)\ \boldsymbol{r} \tag{26}$$

4 Feedback error Learning using LNN

In this section, we will simply give a brief view of the feedback error learning algorthm. From section 3, the input vector $\boldsymbol{f}$ of the IMES is given as

$$\boldsymbol{f}(k) = \boldsymbol{f}_{FF}(k) + \boldsymbol{f}_{FB}(k) \tag{27}$$

where k is the discrete index. The feedback error learning control(7)~(12) is to let the LNN learns the inverse model during the control operation so that the output of the feedback controller $\boldsymbol{f}_{FB}$ decrease to zero•when learning has been finished. Therefore, the error signal $\boldsymbol{\varepsilon}(k)$ is given as

$$\begin{aligned} \boldsymbol{\varepsilon}(k) &= \boldsymbol{f}(k) - \boldsymbol{f}_{FF}(k) \\ &= \boldsymbol{f}_{FB}(k) \end{aligned} \tag{28}$$

and the loss function to minimize is definded as

$$\boldsymbol{J}(k) = \frac{1}{2}\boldsymbol{\varepsilon}(k)^T\ \boldsymbol{\varepsilon}(k) \tag{29}$$

Then the network parameters are updated by the following algorithm :

$$\hat{\boldsymbol{\theta}}(k) = \hat{\boldsymbol{\theta}}(k-1) - \alpha\frac{\partial \boldsymbol{J}(k)}{\partial \boldsymbol{\theta}} + \beta\{\hat{\boldsymbol{\theta}}(k-1) - \hat{\boldsymbol{\theta}}(k-2)\} \tag{30}$$

where α and β, are sufficently small postive numbers, and

$$\boldsymbol{\theta} = [\boldsymbol{w}^T, \boldsymbol{b}^T]^T \tag{31}$$

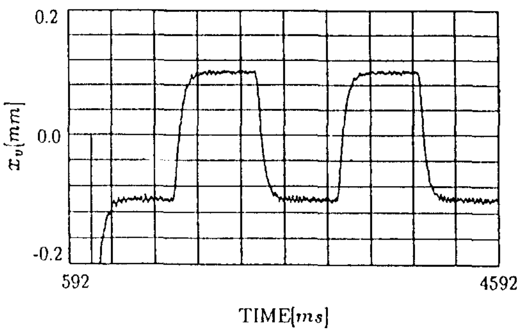

Fig. 6: Response of the vertical position.

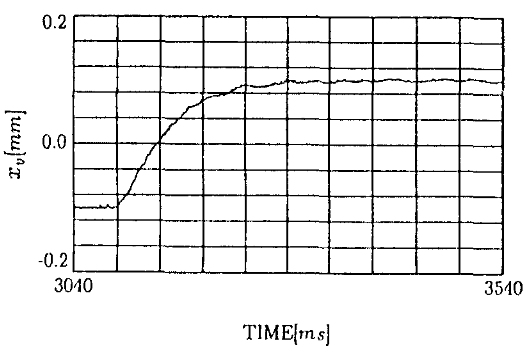

Fig. 7: Zoomed-up curve of a part of the actual output in Fig. 5.

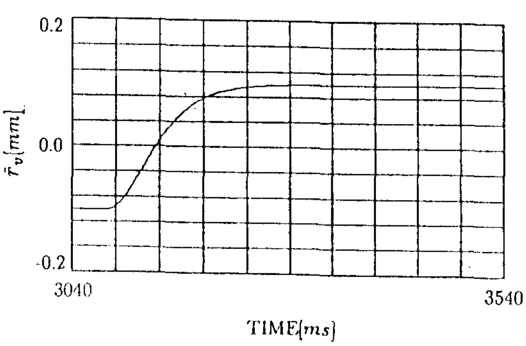

Fig. 8: Zoomed-up curve of a part of the desired output.

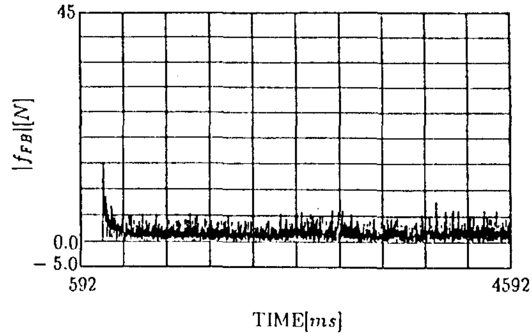

Fig. 9: Norm of the output of the feedback controller.

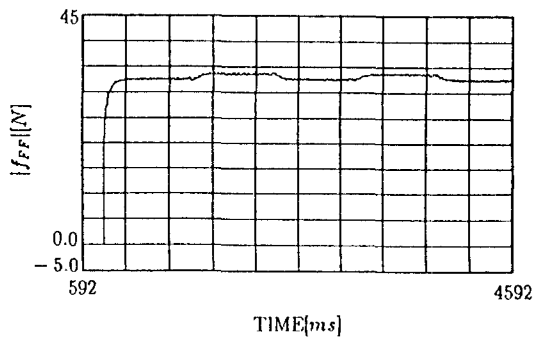

Fig. 10: Norm of the output of the feedforward controller.

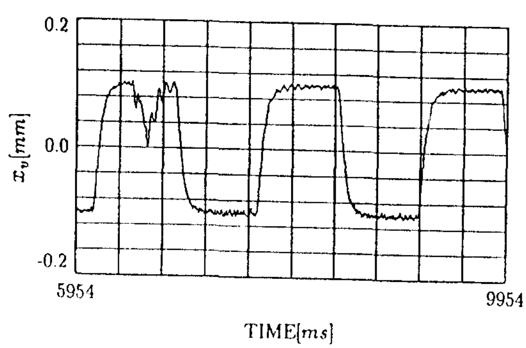

Fig. 11: Response of the vertical position in the case of abrupt system variation.

$$\frac{\partial \boldsymbol{J}(k)}{\partial \boldsymbol{\theta}} = \begin{cases} -\boldsymbol{f}_{FB}(k) & (for \quad \boldsymbol{b}) \\ -\boldsymbol{f}_{FB}(k)\ddot{\boldsymbol{r}}(k) & (for \quad \boldsymbol{w}) \end{cases} \tag{32}$$

The actual control signal, i.e., the output voltage signal of the D/A converter, is calculated by equation (13) with the desired magnetic force vector $\boldsymbol{f}(k)$.

5 Experimental evaluation

The picture of the magnetic levitation equipment is shown in Fig.5. The movable vechicle which has a mass of about 7kg is 80cm long, 11cm wide and 1cm thick. The load mass is about 2.1Kg, say 30% of the movable vechicle. A 64-bit transputer is used.

The sampling period is set to 750 μs. The command signal is a rectangular wave with magnitude $\pm 0.1mm$. In the experiment, the following parameters are chosen:

$$\tau_{v11} = \tau_{v12} = \tau_{v13} = 0.022$$

$$\tau_{v2} = \tau_{p2} = \tau_{r2} = 0.0015$$

$$\boldsymbol{K}_P = \begin{bmatrix} 3.93 \times 10^4 & 0 & 0 \\ 0 & 3.93 \times 10^4 & 0 \\ 0 & 0 & 3.93 \times 10^4 \end{bmatrix}$$

$$\boldsymbol{K}_D = \begin{bmatrix} 1.34 \times 10^3 & 0 & 0 \\ 0 & 1.34 \times 10^3 & 0 \\ 0 & 0 & 1.34 \times 10^3 \end{bmatrix}$$

$$\alpha = 0.06 \qquad \beta = 0.03$$

$$\hat{\boldsymbol{b}}(0) = [10, 10, 10, 10]^T$$
$$\hat{\boldsymbol{w}}(0) = [1, 1, 1, 1]^T$$

The experimental results of the vertical position control are shown in Fig.6 ~ 10.

6 Conclusion

The control of a Magnetic Levitation System via Feedback Error Learning has been presented in this paper. The experimental results have been included to show the excellent performance of the designed control system and it was verified that even in the case of abruptly large changes of the system parameters and disturbances, the control performance were still very satisfactory.

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